Cosmology

Def.:

Cosmology studies the origin, evolution and eventual fate of the universe.

Old subject:

The questions:

- Why are we here?
- How did we get here?

are as old as mankind.

In the last 50 years cosmology converted these questions in a modern (scientifically more precise) form:

- How did the elements form?
- Why is the universe so smooth?
- How did galaxies form?

Source of incredible excitement:

- physics models addressing these questions
- advance in observational techniques to actually test and falsify models

⇒ era of precision cosmology.

⇒ active research area with bright future!

cosmology : scientific viewpoint :

- Cosmology is a melting pot of physics: It takes input from vastly different research areas
  - field theory and general relativity
  - statistical physics
  - quantum mechanics
  - particle physics and nuclear physics
  - data analysis
  - observational astrophysics

arguably: cosmologists produce the nicest pictures in physics

(megabytes of them !)

Cosmological discoveries (past):

Nobel prizes in physics related to cosmology:

1978: discovery of the cosmic microwave background (CMB)
(Arno Penzias / Robert Wilson)

1983: structure and evolution of stars (S. Chandrasekhar)
formation of chemical elements in the universe
(William Fowler)

1993 discovery of a new type of pulsar and the possibilities of testing the theory of gravitation
(Hulse / Taylor)

2002 detection of cosmic neutrinos (Davis / Koshiba)
discovery of X-ray sources (Giacconi)

2006: black body spectrum and anisotropies in the CMB
(Mather / Smoot)

2011: discovery of the accelerating universe through the observation of supernovae
(Perlmutter / Schmidt / Riess)

cosmological discoveries (future)

- ongoing searches :
  - direct detection of gravitational waves
  - direct dark matter searches

Planed telescopes / satalite missions

- turn cosmology in precise science where quantities can be measured at 1% level

Remark on the future of particle physics :

- Collider physics and cosmology provide two complimentary windows on the standard model of particle physics and physics beyond the standard model. It will be an important scientific challenge to combine the information obtained at the very small (LHC) and very large (telescopes / satelites) into one coherent picture.

1. Introduction

The hot big bang as the cosmological standard model

Essentially, there are 3 pillars supporting the standard model:

- existence of dark matter and dark energy
- evolution of perturbations in an essentially smooth background universe
- inflation as a mechanism for explaining initial conditions

During the course, we will discuss these building blocks in detail. In this introduction, we limit ourselves to a (non-technical) discussion of the main ideas.

We start from the sentence:

" We live in an expanding universe "

Technically, this says that in the past the physical distance between us and a far-away galaxy was smaller than it is today: galaxies move "away from us".

Remark:

- We are talking about average behavior here. The observed velocities are subject to statistical fluctuations. We are focusing on the mean value of samples of galaxies.

This brings us to the important concept of

- distance in cosmology:

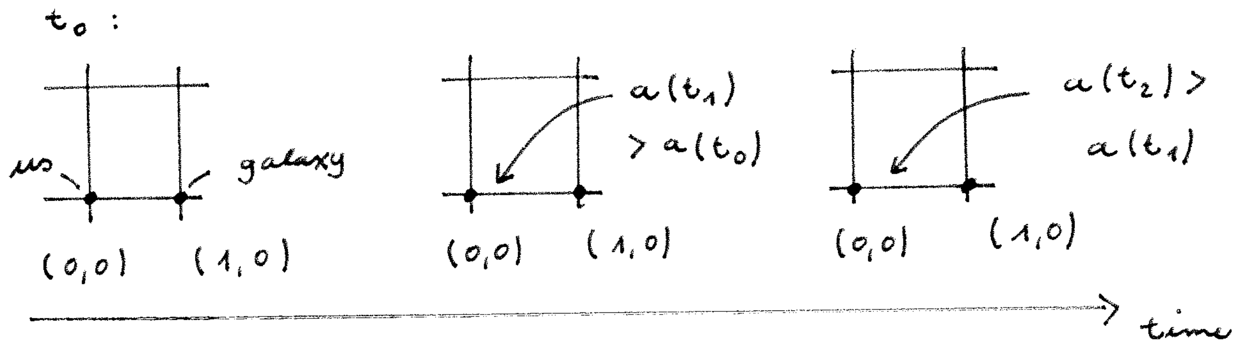
  It is important to distinguish between:

  - coordinate distance / comoving distance
  - physical distance

  to an object.

We clarify the difference in the following example:

- cover the two-dimensional plain with a coordinate grid



- we are at origin
- galaxy has fixed coordinates (1, 0) at all times

  (it is comoving with respect to the coordinate system)

$\Rightarrow$ comoving distance is $\Delta x = 1$ for all times.

Introduce a time-dependent scale-factor $a(t)$ describing the physical length of the link

$$d_{Phys} = a(t) \, \Delta x$$

If the scale-factor increases with time, the physical distance to the galaxy increases despite Δx staying the same.

remark :

- The notion of physical distance will be made more precise when we introduce a metric $g_{\mu\nu}(x)$ on the spacetime.

The evolution of a smooth (more precise: homogeneous and isotropic) universe is completely fixed by:

- evolution of scale factor $a(t)$
- the topology of spatial sections of the universe at a fixed instance of time t :

  demanding that the universe is smooth allows only 3 different cases: the spatial slices can be

  - flat ($k = 0$) : particles starting with parallel movement remain to move parallel (plain)
  - closed ($k = 1$) : particles moving parallel initially ~~approach~~ approach each other (sphere)
  - open ($k = -1$) : particles moving parallel initially move away from each other (saddle)

remark :

- again this can be made precise, when we introduce the Friedmann - Robertson - Walker metric underlying the simplest cosmological models.

In the sequel, we use "standard conventions" used in cosmology:

t_0 : cosmological time today

$a_0 \equiv a(t_0)$: today's scale factor

we normalize $a_0 = 1$

For an expanding universe:

$a(t) < 1 \qquad t < t_0$

$a(t) > 1 \qquad t > t_0$

Quantities evaluated at t_0 are distinguished by the subindex "0", e.g. energy density today: ρ_0

The Hubble parameter H

It is convenient to encode the dynamics of the scale-factor in terms of the Hubble parameter

$$H \equiv \frac{1}{a} \frac{da}{dt}$$

remark :

- we call H the Hubble parameter and not Hubble constant, since H typically depends on cosmic time t.

H_0 (Hubble paramater today at time t_0) sets the typical time- and length scales in cosmology (age of the universe, size of observable universe)

$$H_0 = 100 \, h \, km \, sec^{-1} \, MPc^{-1}$$

$$= \frac{h}{9,8 \cdot 10^9 \text{ years}}$$

$$= 2,133 \cdot 10^{-33} \, h \, \frac{eV}{\hbar}$$

The parameter h is determined experimentally

- combine data from CMB, supernovae, baryon acoust. peaks

$$h = 0.6932 \pm 0.0080$$

remark :

- error bar with less than 1% ! Such precise measurements
⇒ of cosmic parameters justify the notion "precision cosmology"

Dynamics of $a(t)$

- determined by Einstein's equations
- depends on the total energy density $\rho(t)$ in the universe

governed by Friedmann - equation:

$$H^2(t) = \frac{8\pi G}{3} \left(\rho(t) + \frac{\rho_{cr} - \rho_0}{a^2} \right) \qquad (*)$$

$G = 6.67 \times 10^{-11} \; m^3 \, kg^{-1} \, s^{-2}$ Newtons constant

$\rho(t)$: total energy density (matter, photons, dark matter, dark energy)

ρ_0 : total energy density today

ρ_{cr} : critical energy density needed for a flat universe

$$\rho_{cr} = \frac{3 H_0^2}{8 \pi G}$$

measurement:

$$\rho_{cr} = h^2 \cdot 1{,}88 \cdot 10^{-29} \; g \, cm^{-3}$$

(comparison: density of water $\rho_{water} = 1 \; g \, cm^{-3}$)

⇒ universe is almost empty.

Observation: $(*)$ is not closed

- we need information how $\rho(t)$ depends on the scale - factor $a(t)$ in order to get a eq. for $a(t)$!

remark:

- here we use some physics arguments to derive this relations. Later on, we will see that Einstein's equations give the same answer!

<u>$\rho(t)$ for massive, non-relativistic matter:</u>

consider n particles of mass m in a box of physical volume V:

$$\rho_m \propto \frac{n \cdot m}{Vol}$$

- number of particles and mass does not change if the universe expands. The physical volume depends on a:

$$Vol \sim a^3$$

Thus

$$\rho_m(t) = \frac{\rho_{m,0}}{a^3}$$

where $\rho_{m,0}$ is the energy density today.

<u>$\rho(t)$ for photons (radiation)</u>

- Energy of a photon is determined by its wavelength λ

$$E = \frac{\hbar c}{\lambda}$$

λ is stretched as the universe expands!

energy density of n photons with wavelength λ in a Volume

$$\rho_{rad} = \frac{n\,E}{Vol} = \frac{n\,\hbar\,c}{\lambda\;Vol} = \frac{\rho_{rad,0}}{a^4}$$

Observation :

- in an expanding universe, photons dilute faster than non - relativistic matter !

Based on these results, we can close the Friedmann - eq. and determine evolution of $a(t)$:

- assume $\rho_0 = \rho_{crit}$ (very realistic, supported by obs.)

In a universe populated by non - relativistic matter

$$\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\;\frac{\rho_0}{a^3} = \frac{H_0^2}{a^3}$$

is a separable first order differential equation:

$$\int \sqrt{a}\; da = H_0 \int dt$$

integrate

$$\frac{3}{2}\,a^{3/2} = H_0\,t$$

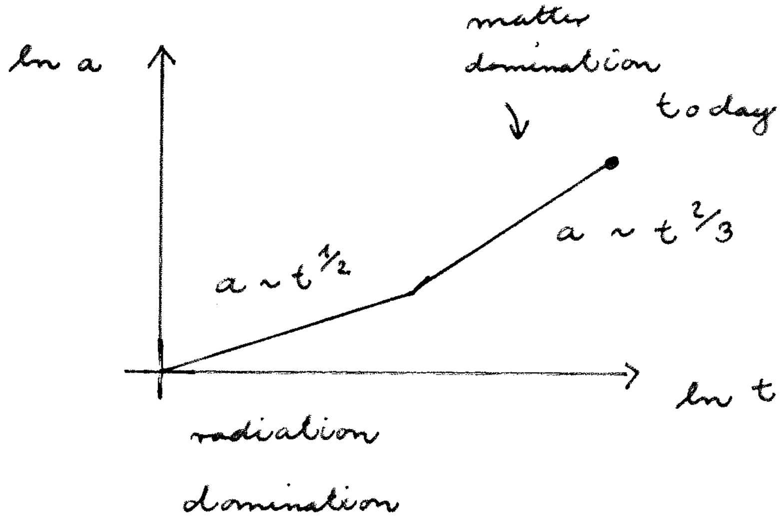
gives

$$a \propto t^{2/3}$$

The analogous computation for photons gives

$$a \propto t^{1/2}$$

Based on these results, we can qualitatively understand the evolution of a universe containing both matter and radiation:



remark:

- often the Friedmann equation is written in terms of relative densities, giving the ratio of a given type of matter (matter, radiation, ...) today and the critical energy density

$$\underbrace{\Omega_m \equiv \frac{\rho_{m,0}}{\rho_{cr}}}_{\text{non-relativistic matter}} , \quad \underbrace{\Omega_r = \frac{\rho_{rad,0}}{\rho_{cr}}}_{\text{radiation}} , \quad \underbrace{\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}}_{\text{cosmological constant.}}$$

Ω_k: contribution due to curvature of spatial slices:

$$\Omega_k = \frac{\rho_{cr} - \sum \rho_i}{\rho_{cr}}$$

Friedmann - eq. in terms of relative densities:

$$\left(\frac{H}{H_0}\right)^2 = \left(\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda + \frac{\Omega_k}{a^2}\right)$$

Evaluating at t_0: $(a_0 = 1)$

$$\sum_i \Omega_i = 1$$

which is always true

(Ω_k encodes the energy contribution from the curvature of the spatial slices, chipping in when the energy budget in the other sectors does not equate to the critical density)

Profound consequences of FRW cosmology (including matter):

1) In an expanding universe photons get redshifted due to the increase of the scale factor:

Definition of redshift z:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{1}{a} \quad , a_0 = 1$$

note: at redshift $z = 1$ the universe had half its present size!

2) There is a time $t = 0$ where the scale-factor vanishes $a = 0$ and $\rho(t = 0)$ diverges

$\Rightarrow$ initial singularity, hot big bang!

Observational evidence for the big bang model:

How do we know the universe is expanding?

1. The Hubble diagram (1929!)

Physical distance to a galaxy at coordinate distance x:

$$d = a x$$

assume that the coordinate position of the galaxy remains fixed

(This can be proven from the geodesic equation for the FRW universe)

The relative velocity of the galaxy with respect to us is

$$v \equiv \dot{d} = \dot{a} x = H d$$

$v \equiv$ measured by Doppler shift of spectral lines

$d \equiv$ distance to object (determined via the distance ladder)

use "standard candles" Type I a supernovae

(possible redshifts $z \simeq 1.93$)

$\Rightarrow$ determine H:

- H is non-zero and positive (the universe expands)
- H is not constant (hence Hubble parameter)

the expansion accelerates!

2. Big bang nucleosynthesis

- Evolution of the energy density of a FRW universe containing non-relativistic matter only:

$$\rho_m(t) = \frac{\rho_{m,0}}{a^3} = \frac{\rho_{m,0}}{t^2}$$

⇒ in the past the universe was denser and hotter!

⇒ universe cools down as it expands.

When the temperature is of the order of the binding energy of nuclei (~ 1 MeV) protons and neutrons can bind to light nuclei

$$p + n \rightarrow D \quad \text{(deuterium)}$$

- we will compute the amount of deuterium created in this process

- This amount can be measured very accurately (redshifts $z = 3-4$ where stellar evolution has not yet altered the primordial deuterium density)

⇒ Theory and observations agree to very good accuracy

The contribution of baryons to the total energy budget of the universe can be determined very accurately:

(5% of the critical energy density)

3. Formation of the cosmic microwave background

- At redshift $z \simeq 1100$ photons and electrons where in thermal equilibrium
- as temperature drops photons decouple and stream freely through the universe

⇒ process predicts a background of cosmic photons following a black body spectrum with intensity:

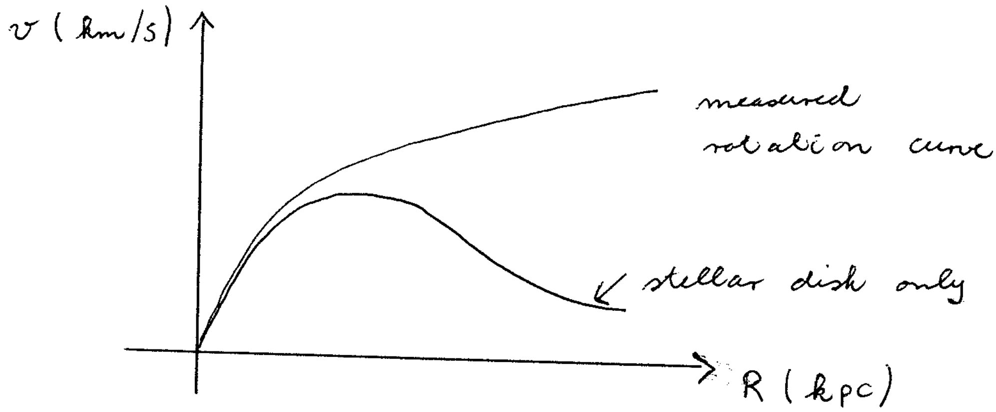
$$I_\nu = \frac{4\pi\hbar\,\nu^3/c^2}{\exp\left(2\pi\hbar\nu/k_B T\right) - 1}$$

Observed by satelite experiments (COBE, WMAP)

minimal fluctuations $\sim 10^{-5}$

4. Galaxy rotation curves:

- relation between gravitating matter in a galaxy and the orbital velocity of a star at distance r from the center (see exercise)
- if the grav. field of the galaxy was created by baryonic matter only one would expect a fall-off of the orbital velocity at large distances



=> non-baryonic matter must contribute to gravitational binding in a galaxy

=> indirect evidence for dark matter

Summary:

As a result of

- precise cosmological observations
- theoretical model building

we now have a good idea what populates the universe:

- ordinary standard model matter : 4,9%
- dark matter : 26.8%
- dark energy : 68.3%

Energy density matches the critical density to more than 1% accuracy (=> flat universe)

Goal of lecture series:

- teach the theoretical and observational foundations which lead to these fantastic insights.