Distance determination

Overview

- How do we determine distances to astronomical objects?
- "Cosmic distance ladder" -

1. Calibrate nearby objects,
2. Identify similar objects associated with more rare objects at greater distances (e.g. in star clusters)
3. Calibrate rare objects
4. Identify rare objects at even greater distances, etc...

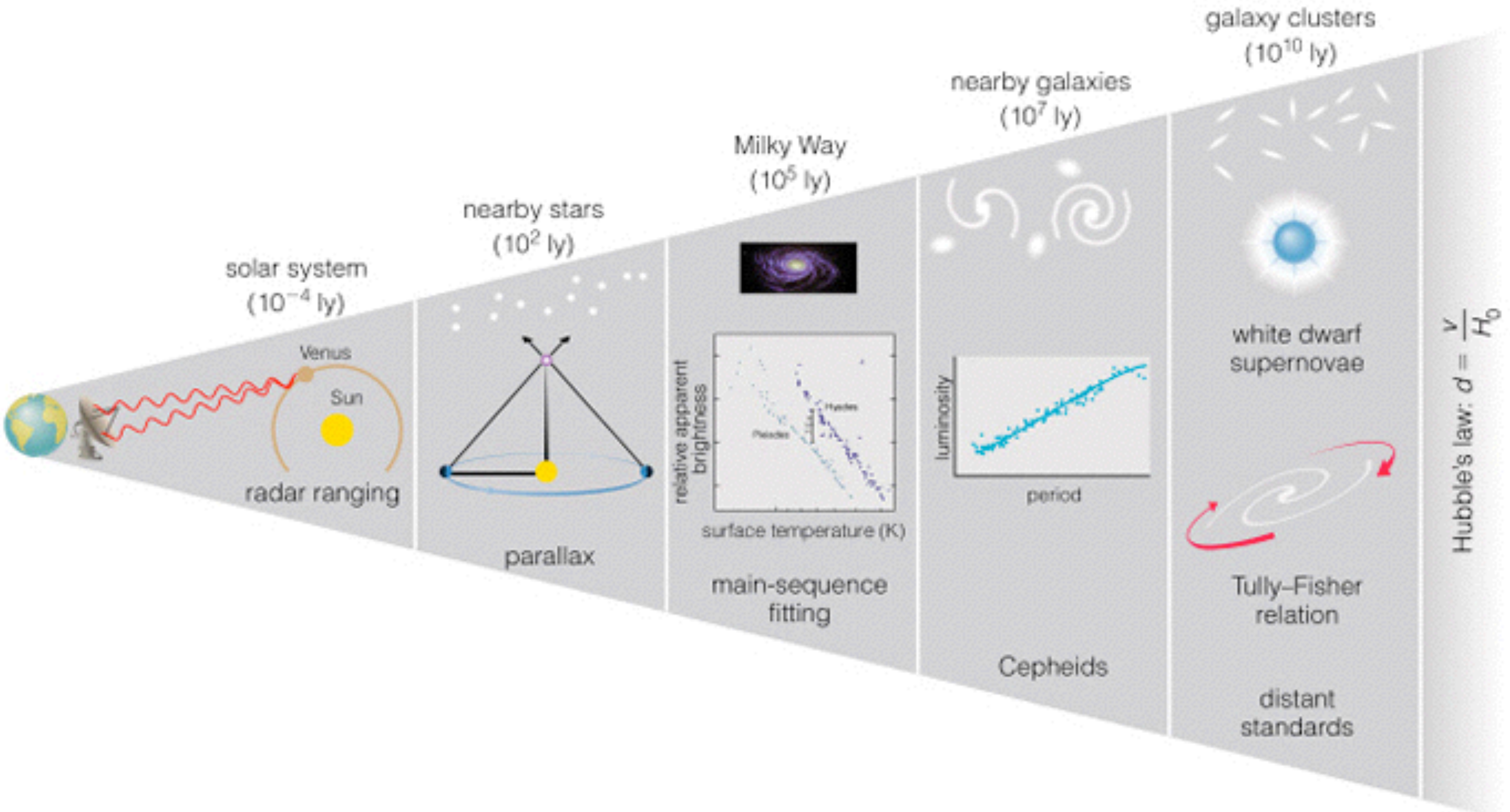


Image from: http://www.daviddarling.info/encyclopedia/C/cosmic_distance_ladder.html

Key Point: methods must overlap in order for more distant ones to be calibrated...

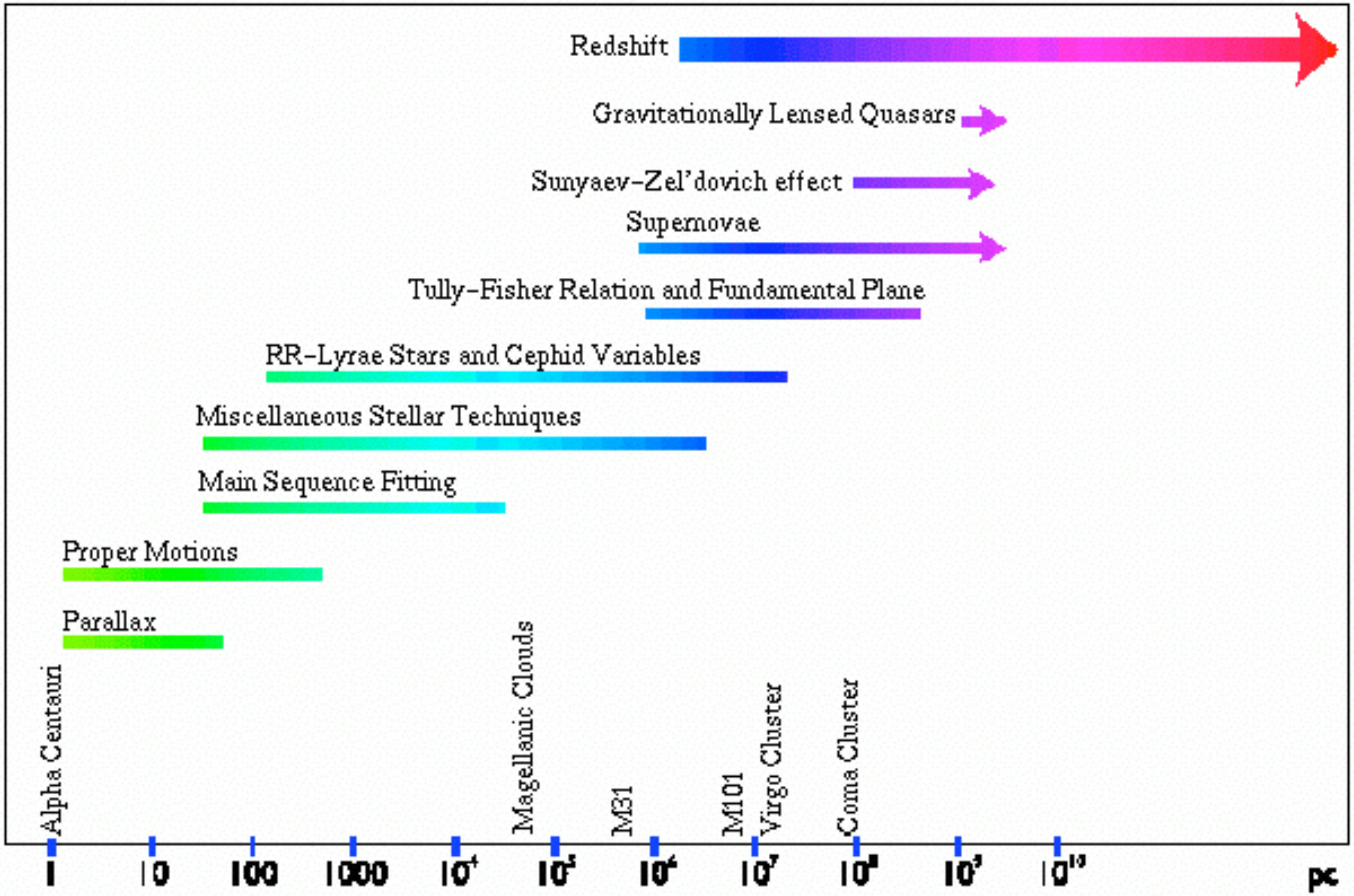


Figure 3.2: *The different distance estimators. This seemingly simple plot shows a grand overview of our efforts to measure distances in the Universe. Adapted from [Rowan-Robinson, 1985] and [Roth and Primack, 1996].*

Image from: http://www.astro.gla.ac.uk/users/kenton/C185/ladder.gif

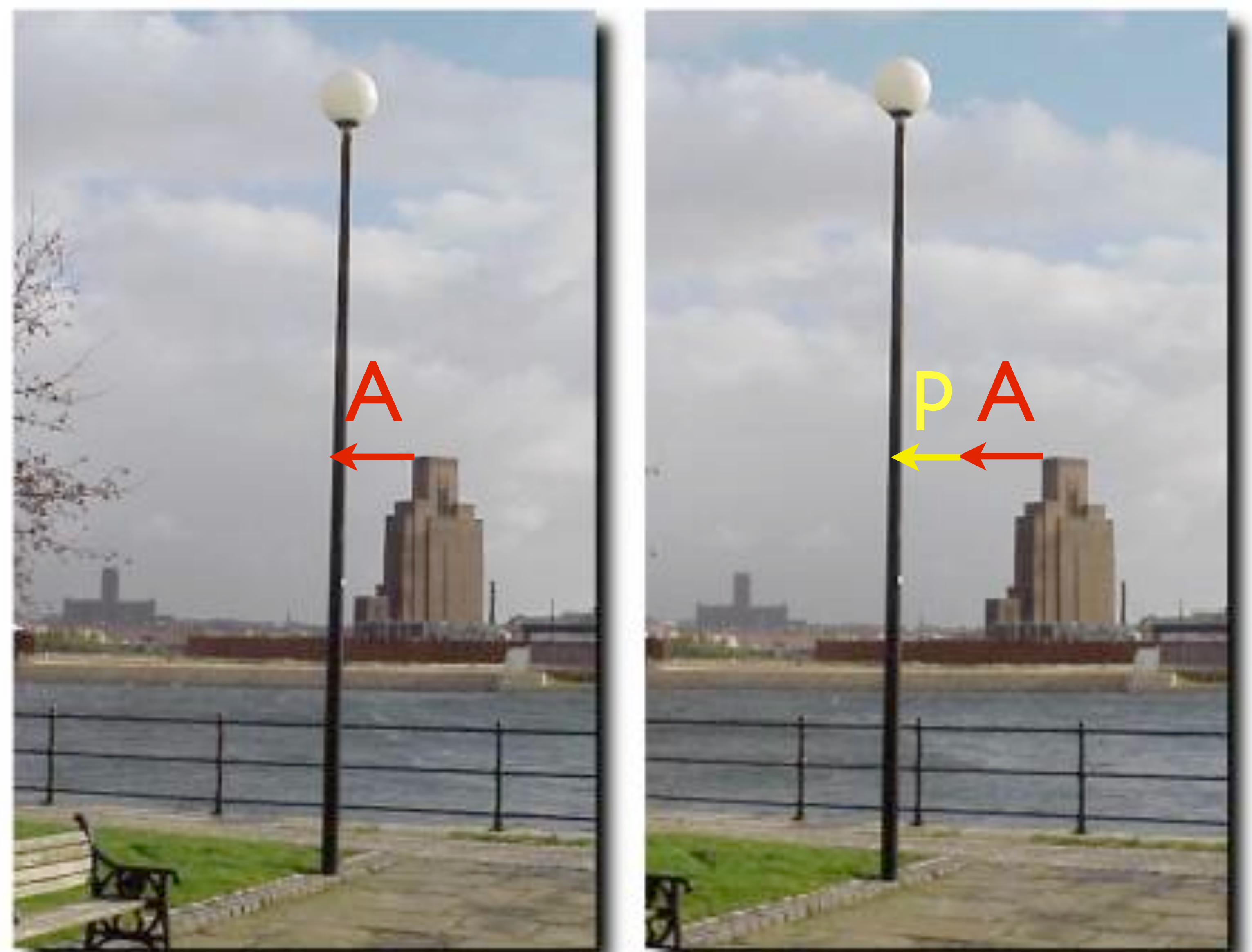
Geometric methods

Trigonometric parallax

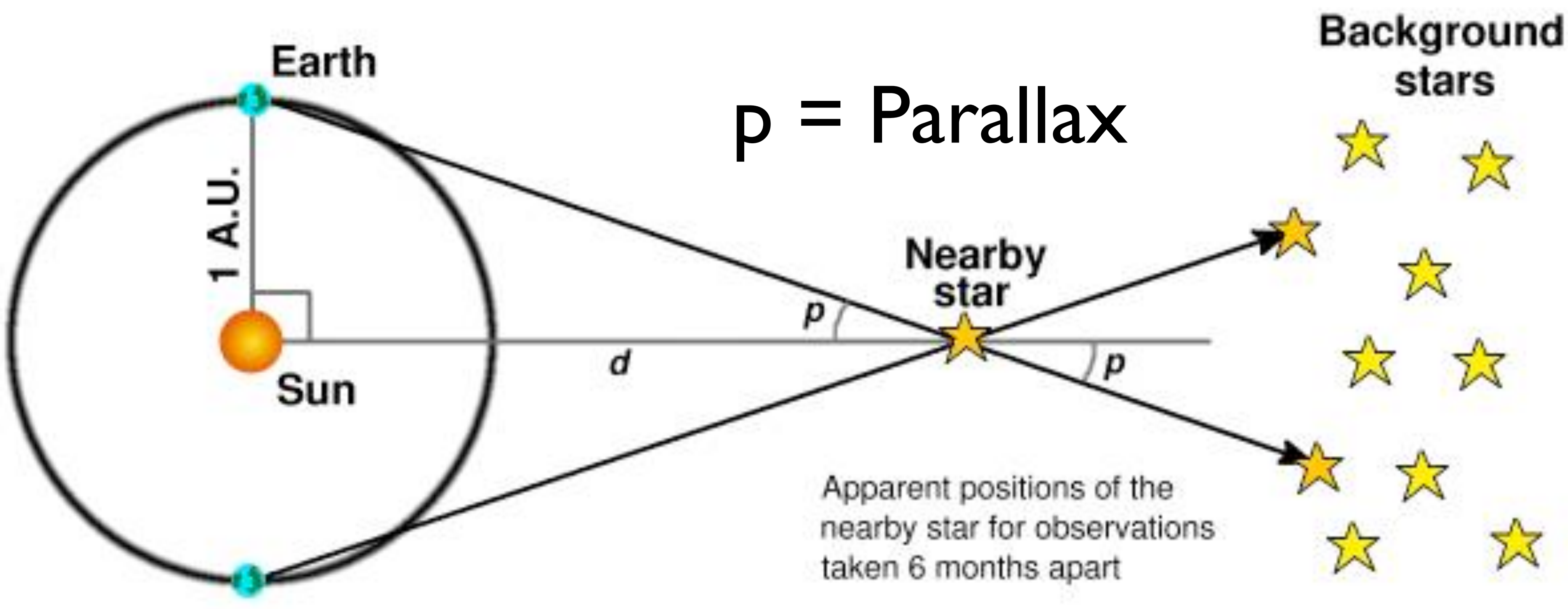
Simple idea:

Nearby objects appear to shift, relative to more distant ones, when the observer moves.

The shift, *p*, depends on the distance



Also works for stars

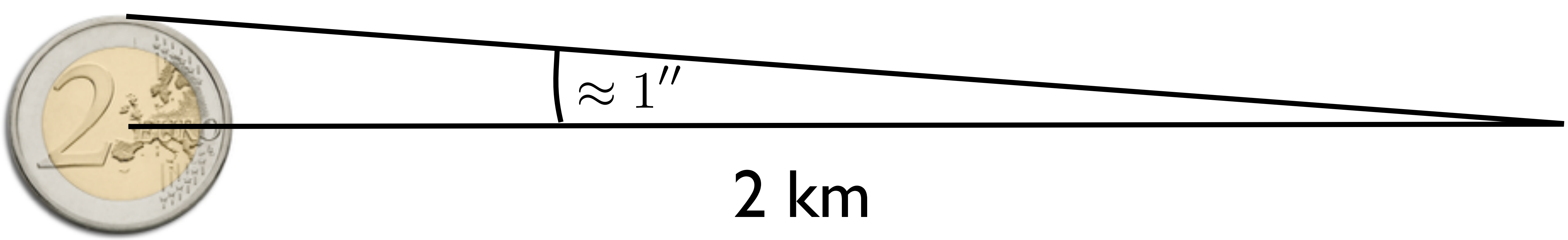


$$1\mathrm{AU} = d \tan p \simeq dp$$

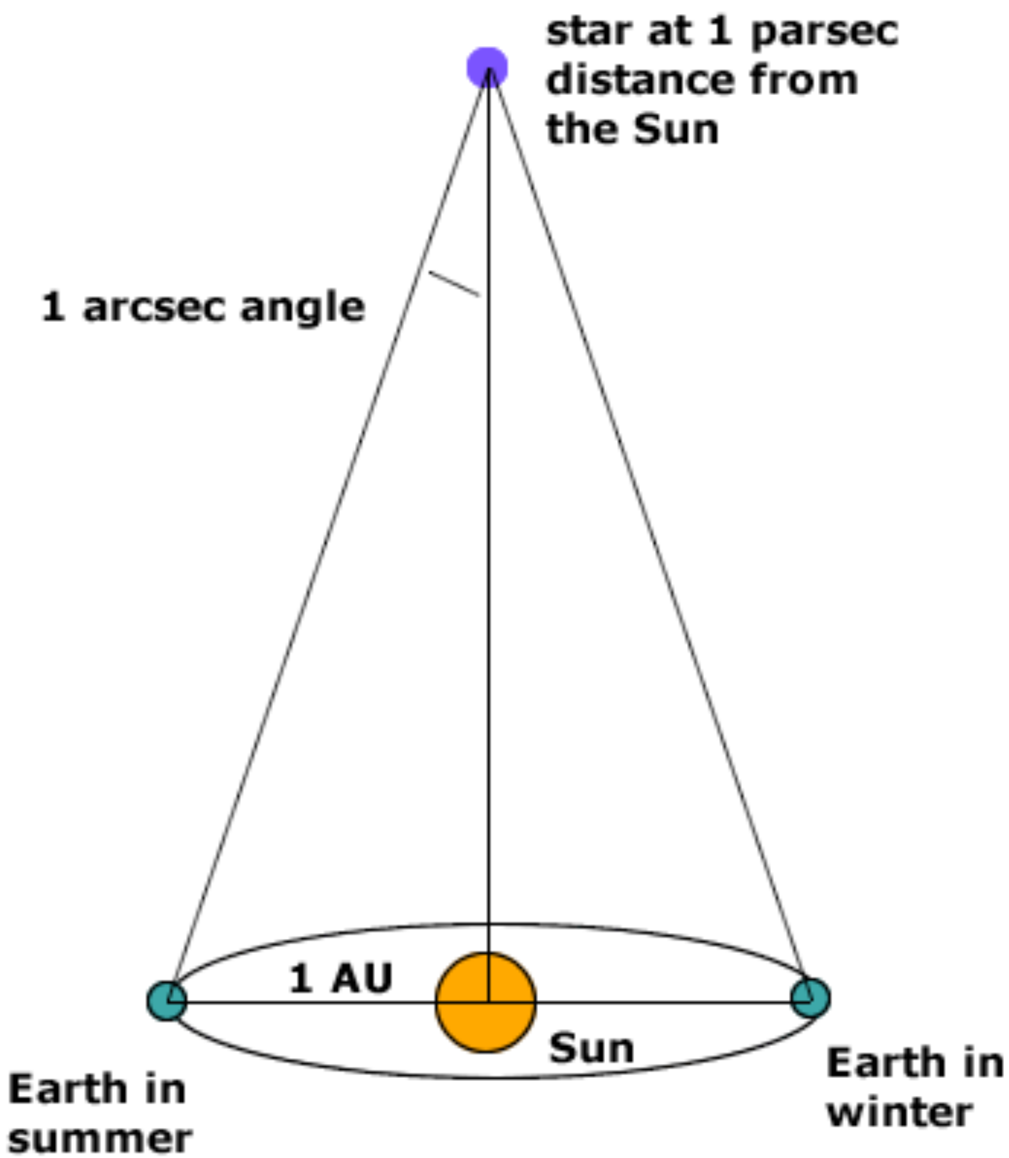
$$d = 1\mathrm{AU}/p$$

Trigonometric parallax

- Principle known to Tycho Brahe (1546-1601) who could not measure it for any star and therefore concluded that the Earth does not move around the Sun
- First measured by F.W. Bessel in 1838 (MNRAS 4, 152) for 61 Cygni (p=0.29")
- Largest parallax is for Alpha Cen (0.742")
- One arcsec (") = 1/3600 deg = 1/206265 radians



Parsec (pc)



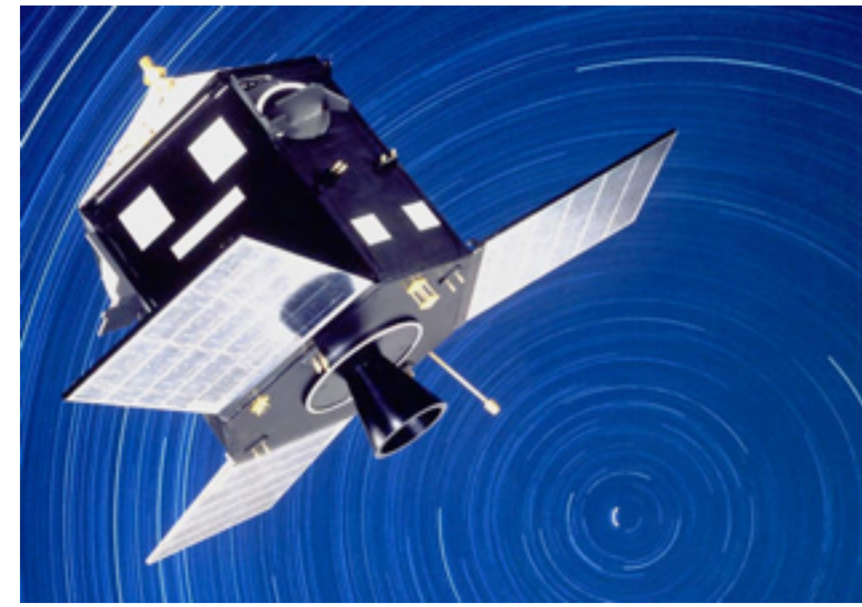
Convenient unit for distance measurements.

1 parsec = 1 “parallax second”

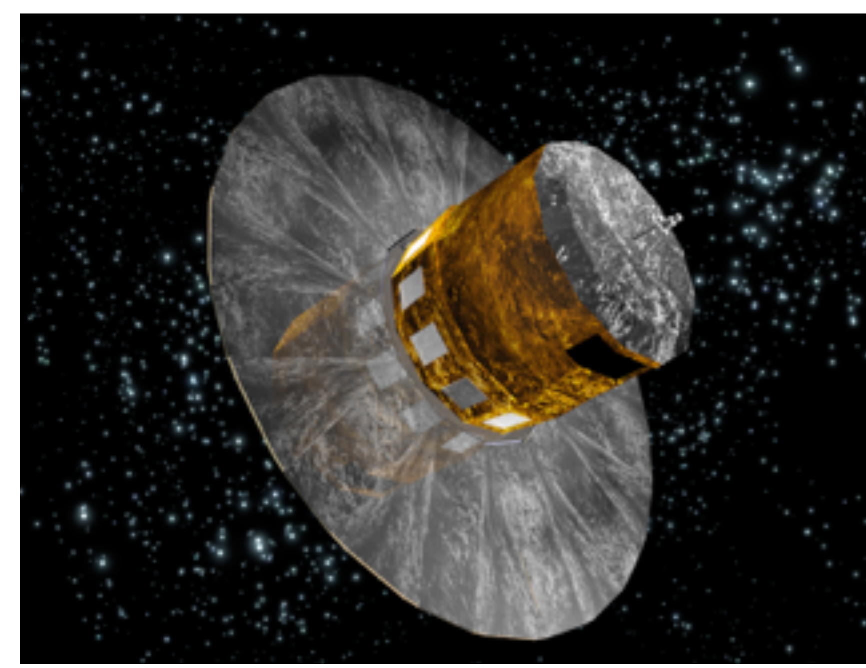
Distance of a star that has a parallax of one arcsecond

1 pc = 206265 AU = 3.09×10^{16} m

Trigonometric parallax



- Best accuracy from the ground : δp ~ 1/50"
  10% accuracy for p = 1/5" or **d = 5 pc**
- *Hipparcos* satellite (1989-1993) measured parallaxes accurate to ~0.001" for 120000 stars.
  10% accuracy for p = 1/100" or **d = 100 pc**
- *Gaia* (launched in October 2013) is measuring parallaxes for about 10^9 stars accurate to 10^{-5} arcsec.
  10% distance accuracy at **d = 10^4 pc** - Galactic centre!



Moving cluster method

- Geometric method, exploiting effect of perspective
- Useful for group of stars with large apparent diameter and coherent space motion
- Best case: Hyades


Penta Hotel

Pleiades

Hyades

APOD 2000,
Sep 29

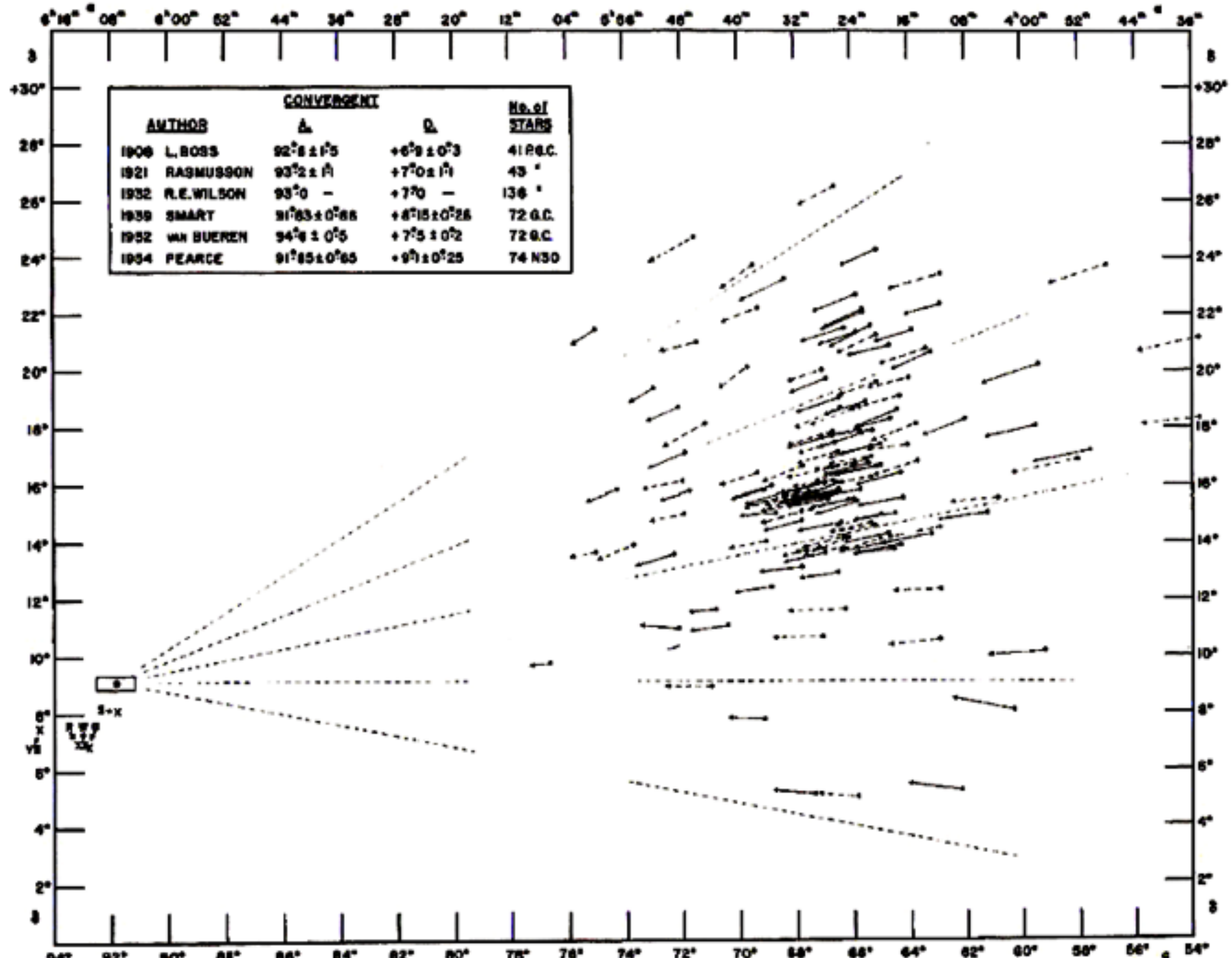
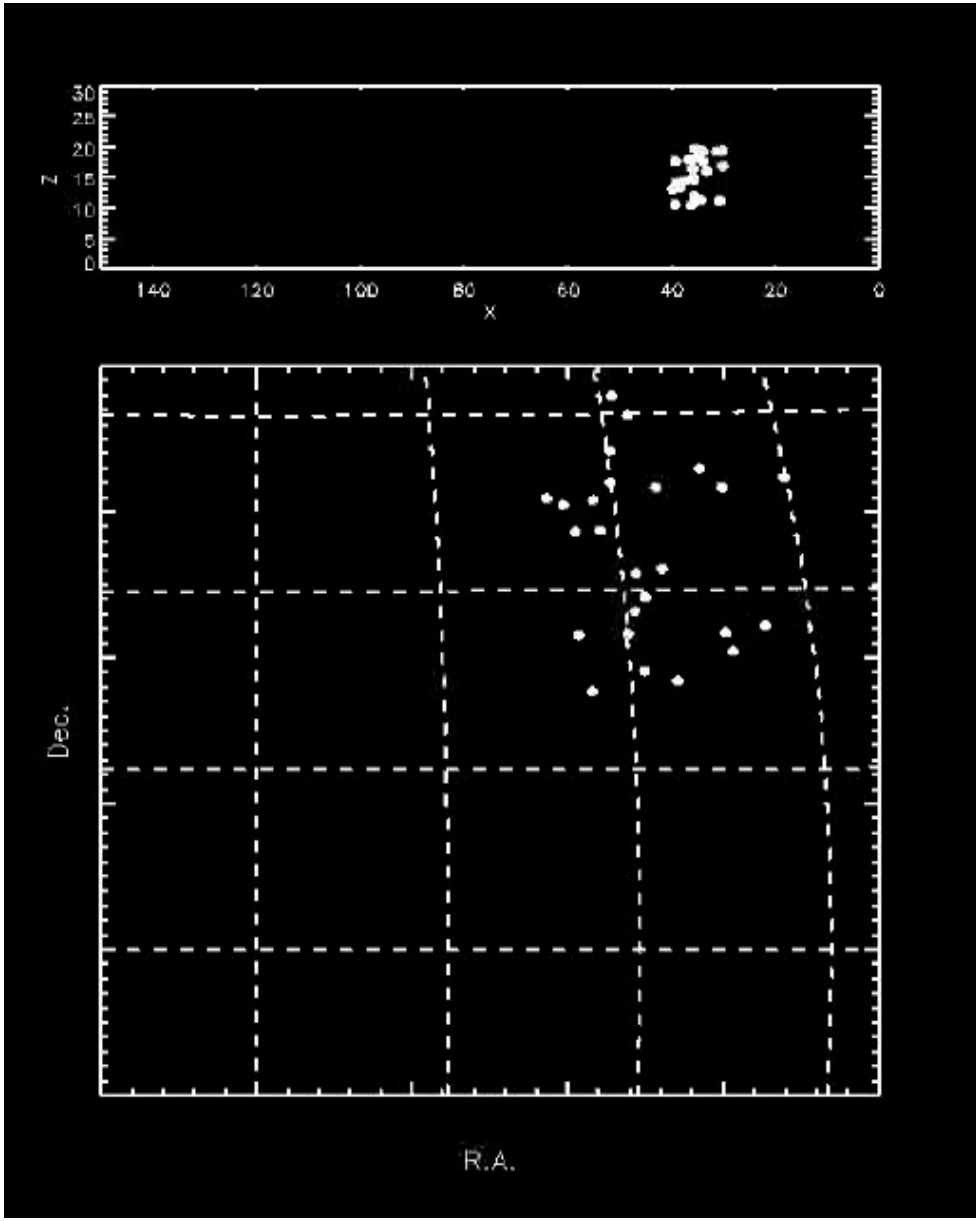


Figure 22.31 The apparent motion of the Hyades across the celestial sphere. (From *Elementary Astronomy* by Otto Struve, Beverly Lynds, and Helen Pillans. Copyright © 1959 by Oxford University Press, Inc. Renewed 1987 by Beverly T. Lynds. Reprinted by permission of the publisher.)



“Moving cluster” demo

Observer at (x,y,z) = (0,0,0)

Top: Side view, projected along y-axis

Bottom: Observer’s view

Distances from moving cluster method

Geometry: $v_t / v_r = \tan(\phi)$

$v_r = \text{radial velocity}$

μ = proper motion: $v_t = \mu D$

$$\frac{\mu D}{v_r} = \tan\phi$$

$$D = \frac{v_r \tan\phi}{\mu}$$

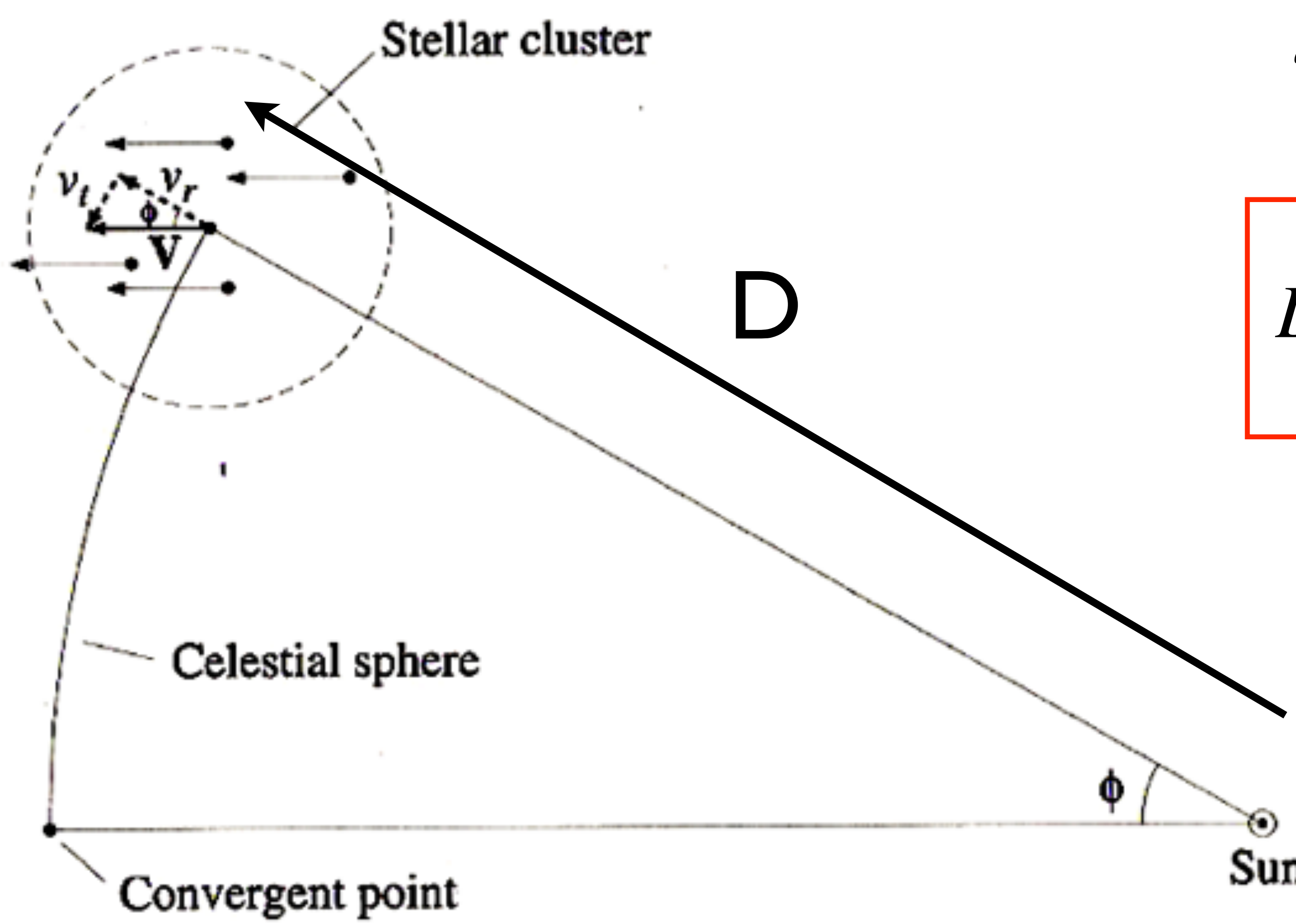
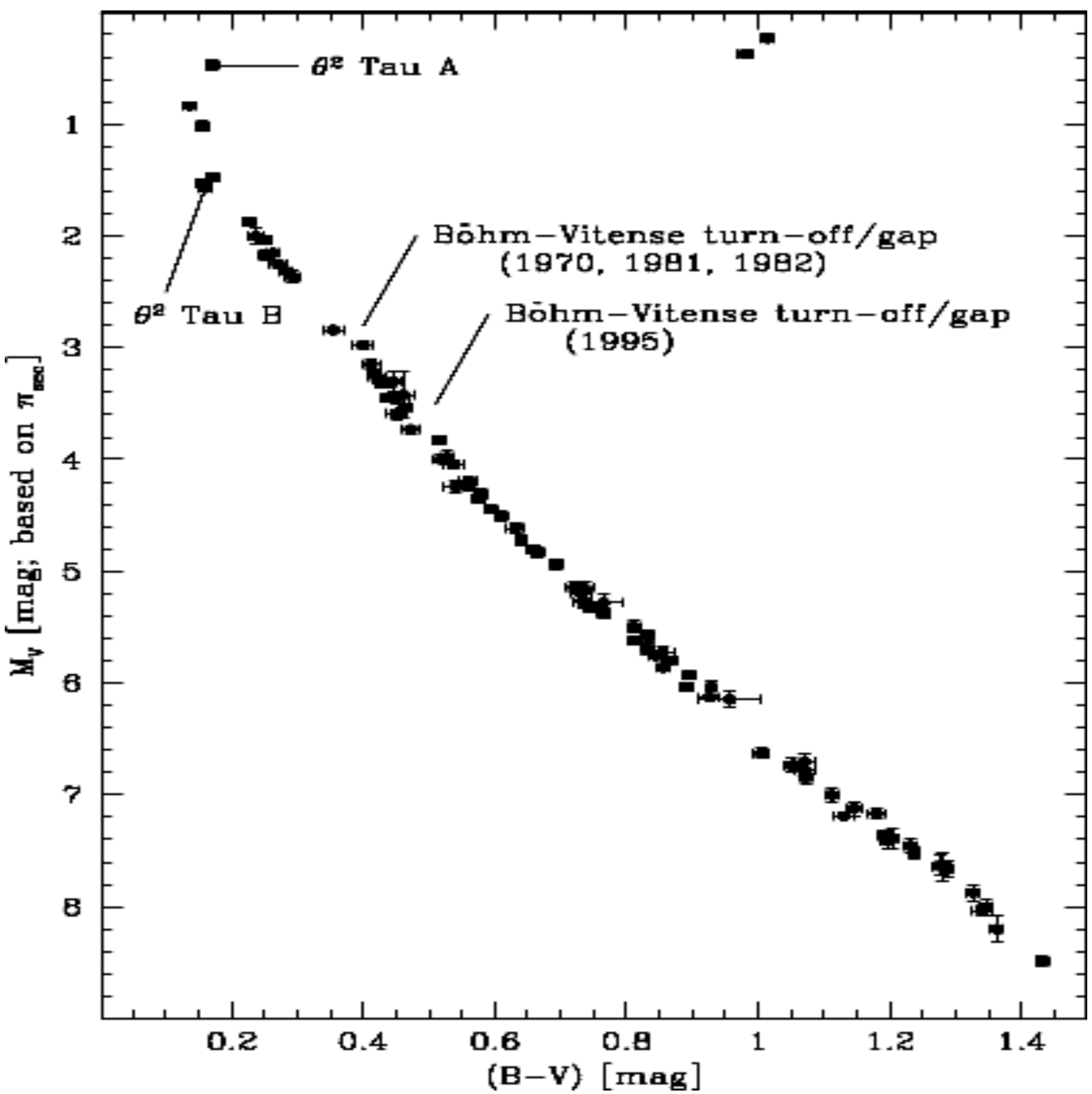


Figure 22.32 The space motion of the cluster is directed toward the convergent point. This velocity vector may be decoupled into its radial and transverse components.

Applications of Moving Cluster method

- Hyades: D ~ 45 pc (Hipparcos: 46 pc)
- Pleiades: D ~ 115 pc
- Ursa Major group: D ~ 24 pc
- Scorpio-Centaurus group: D ~ 170 pc

CMD for Hyades

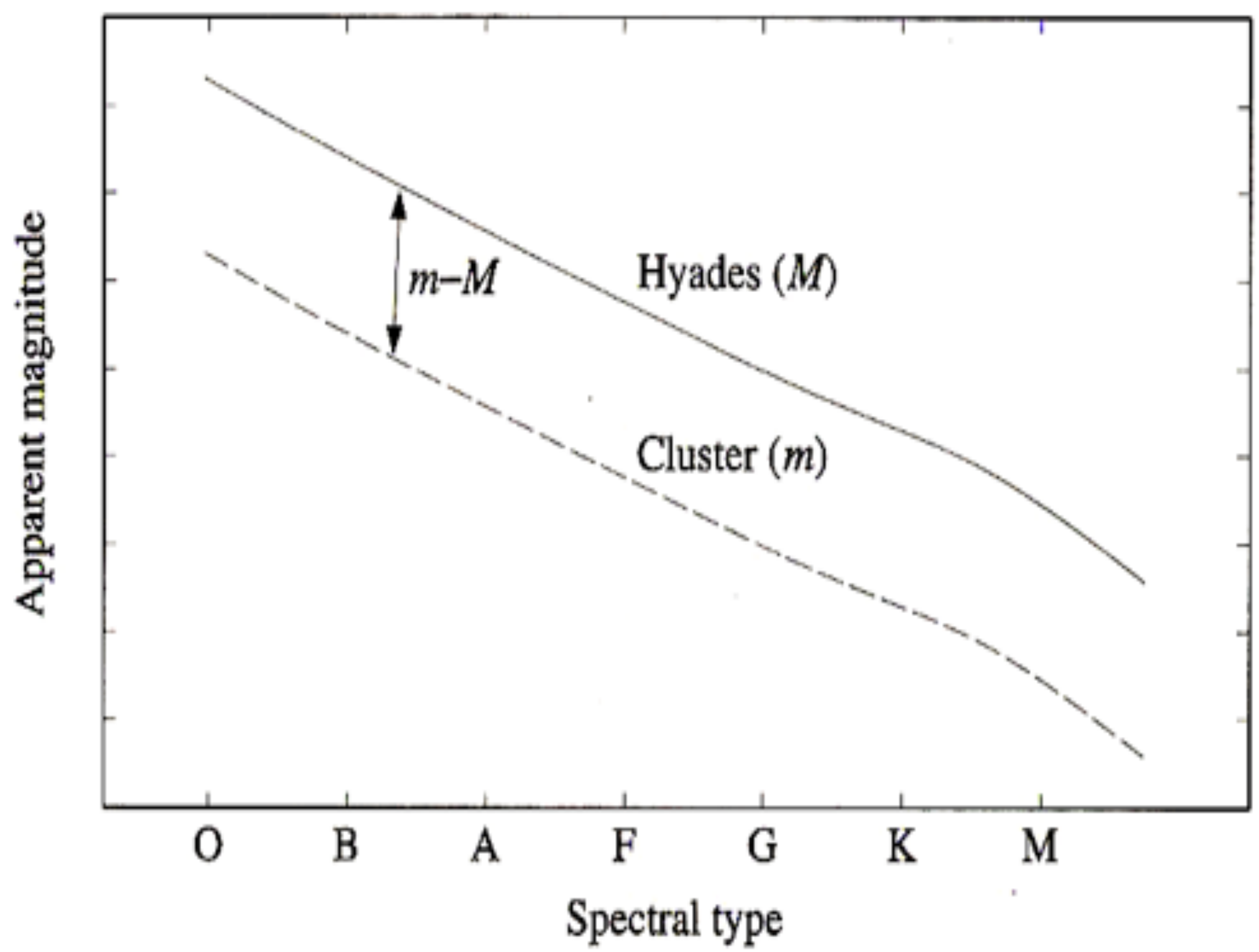


de Bruijne et al. 2001, A&A 367, 111

With distance known, absolute magnitudes can be determined:

$$M = m + 5\log\left(\frac{10\mathrm{pc}}{D}\right)$$

Main Sequence Fitting

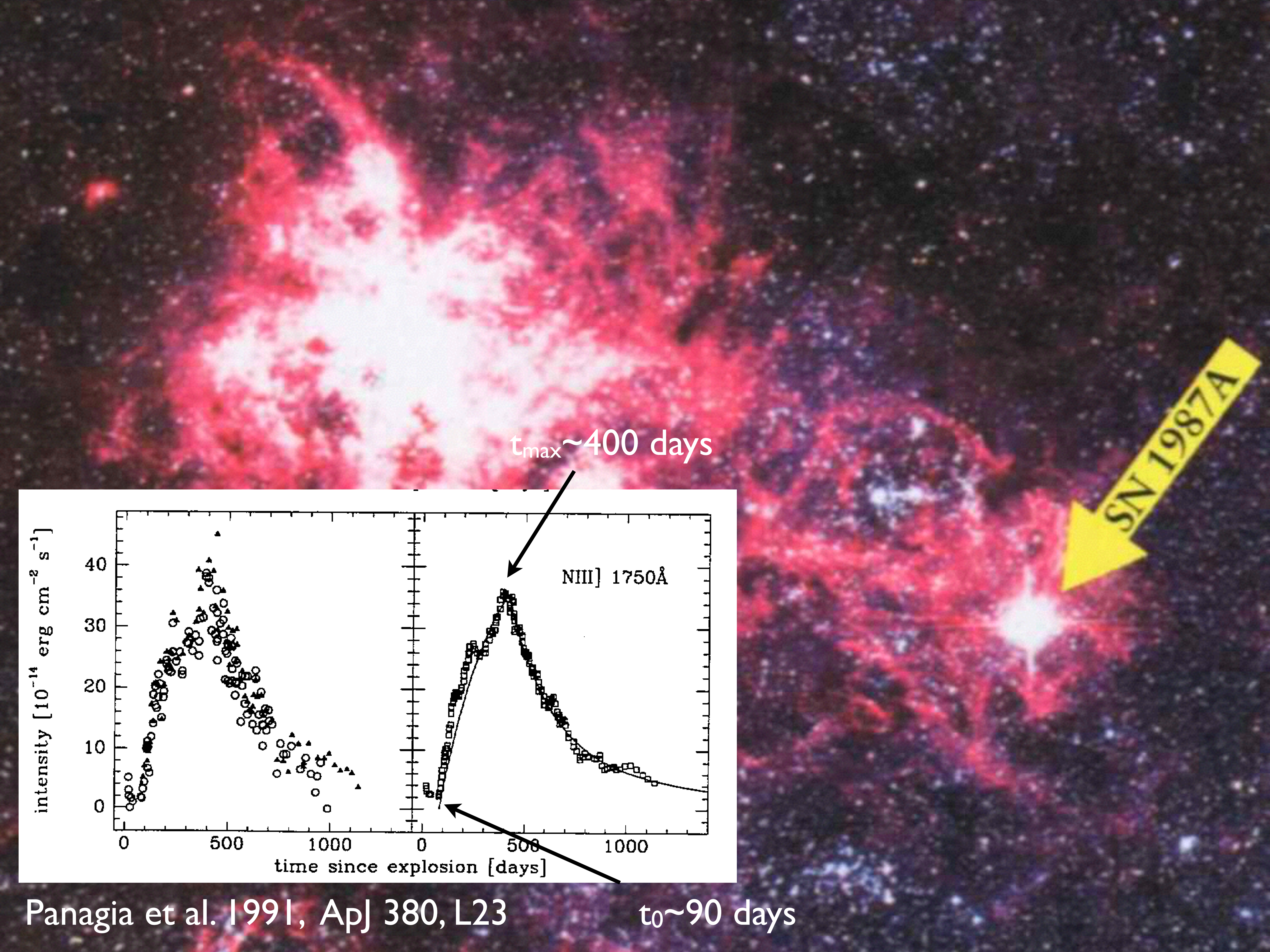


$$D = (10\,\mathrm{pc}) \times 10^{(m-M)/5}$$

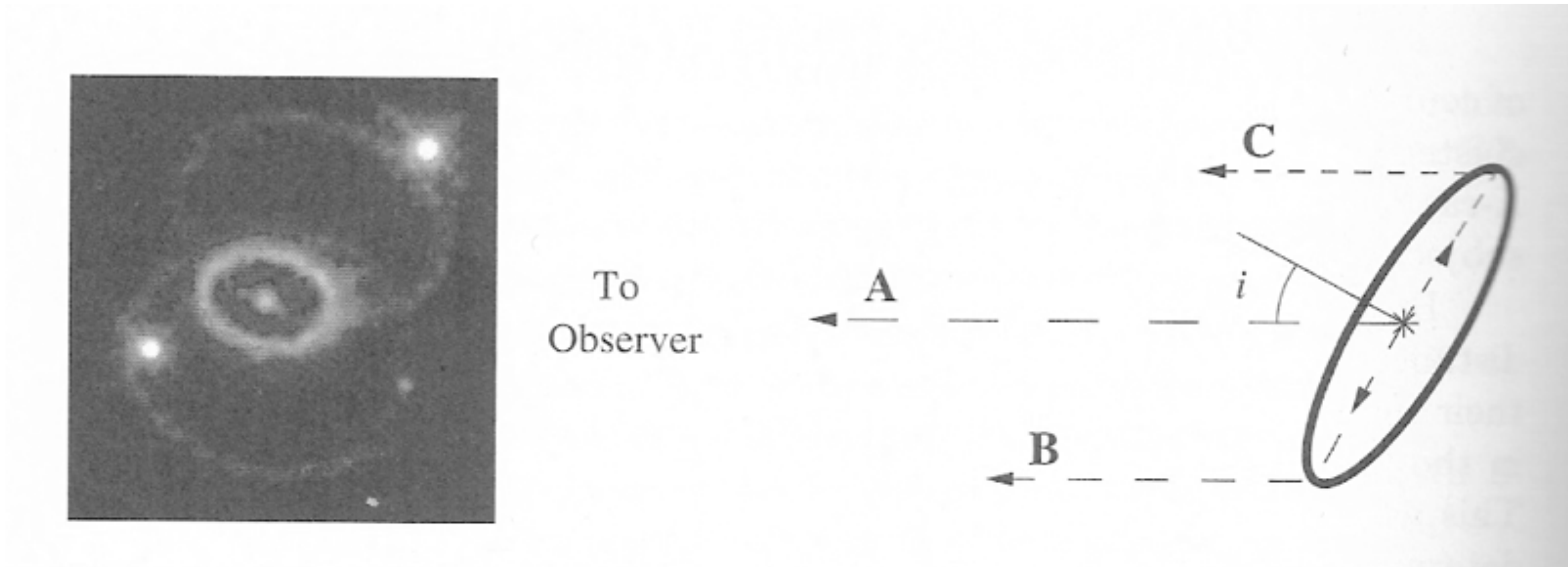
$$= 10^{1+(m-M)/5}$$

Figure 22.33 The distance modulus of a cluster can be determined by shifting the cluster's main sequence vertically in the H–R diagram until it coincides with the known absolute magnitude of the Hyades' main sequence.

Allows calibration of "standard candles" (e.g. Cepheids, RR Lyrae stars) in star clusters


SN 1987A
t_{max}~400 days
intensity [10^{-14} erg cm^{-2} s^{-1}]
0
10
20
30
40
0
500
1000
0
500
1000
N III] 1750Å
time since explosion [days]
Panagia et al. 1991, ApJ 380, L23
t_0~90 days

Distance to the LMC: Light echo of SN1987A



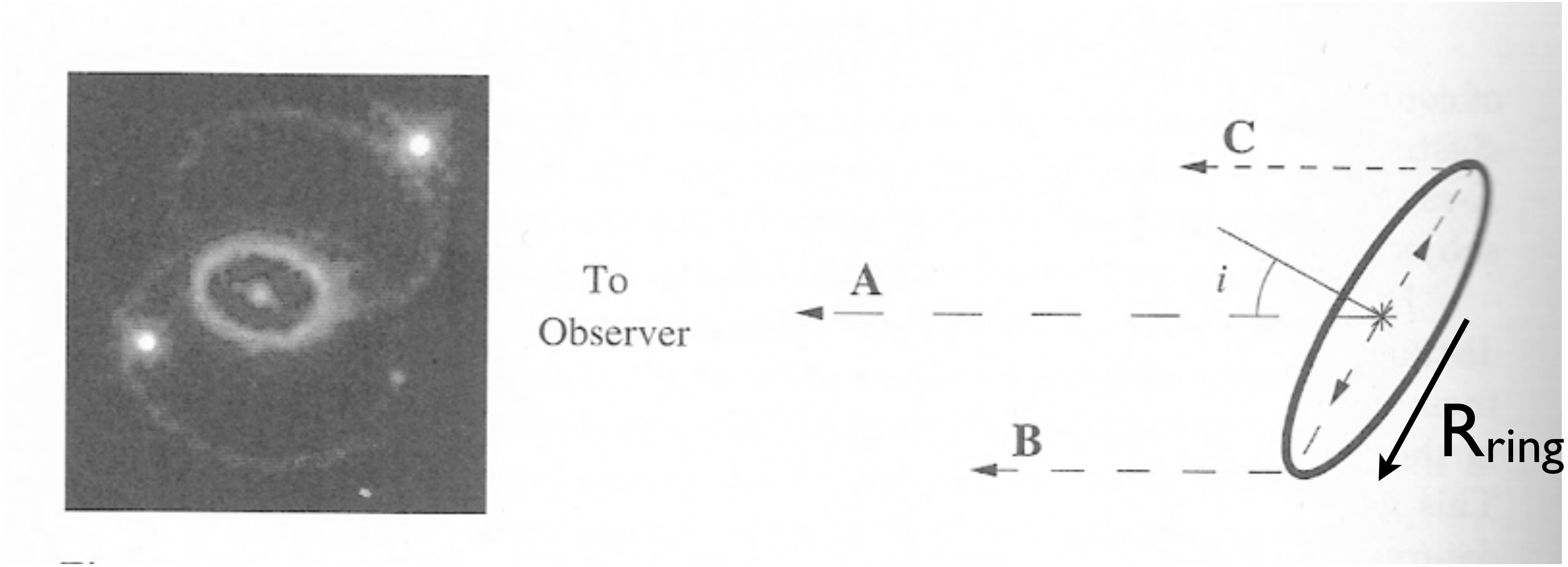
Light from SN travels directly to observer along path A

Light from SN travels to surrounding ring, hits ring and then is re-emitted (in UV) along paths B and C

By comparing arrival times via paths B and C we can determine the size of the ring.

Direct method, does not rely on other calibrators!

Light echo of SN1987A



IUE observed (UV) line emission at t_0=90 days after SN explosion, increasing in strength until t_{max}=400 days

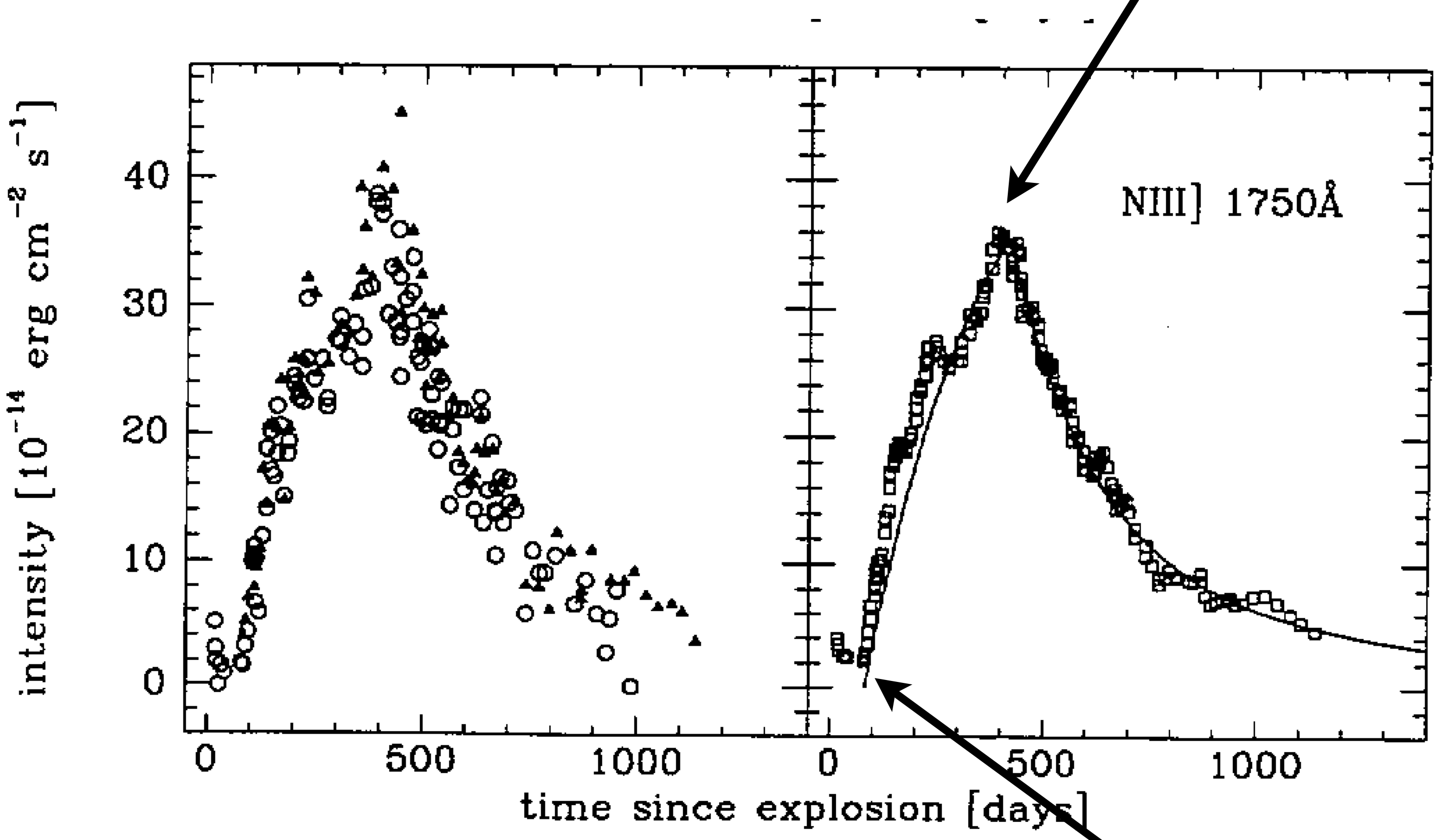
Path B: $$t_0 = t_B - t_A \quad = (D_B + R_{\rm ring} - D_A)/c$$

$$= (-R_{\rm ring} \sin i + R_{\rm ring})/c = (1 - \sin i) R_{\rm ring}/c$$

Path C: $$t_{\rm max} = t_C - t_A = (1 + \sin i) R_{\rm ring}/c$$

Panagia et al. 1991, ApJ 380, L23

t_{max}~400 days



t_0~90 days

$$t_0 = (1 - \sin i) R_{\rm ring}/c$$

$$t_{\rm max} = (1 + \sin i) R_{\rm ring}/c$$

$$t_{\rm max} + t_0 = 2 R_{\rm ring}/c \qquad R_{\rm ring} = 0.21\,{\rm pc}$$

$$t_{\rm max} - t_0 = 2 \sin i R_{\rm ring}/c$$

$$\sin i = (t_{\rm max} - t_0)/(t_{\rm max} + t_0) \qquad i \approx 40^\circ$$

Measured on HST images: radius = r_{ring} = 0.83” = 4.0×10^{-6} rad

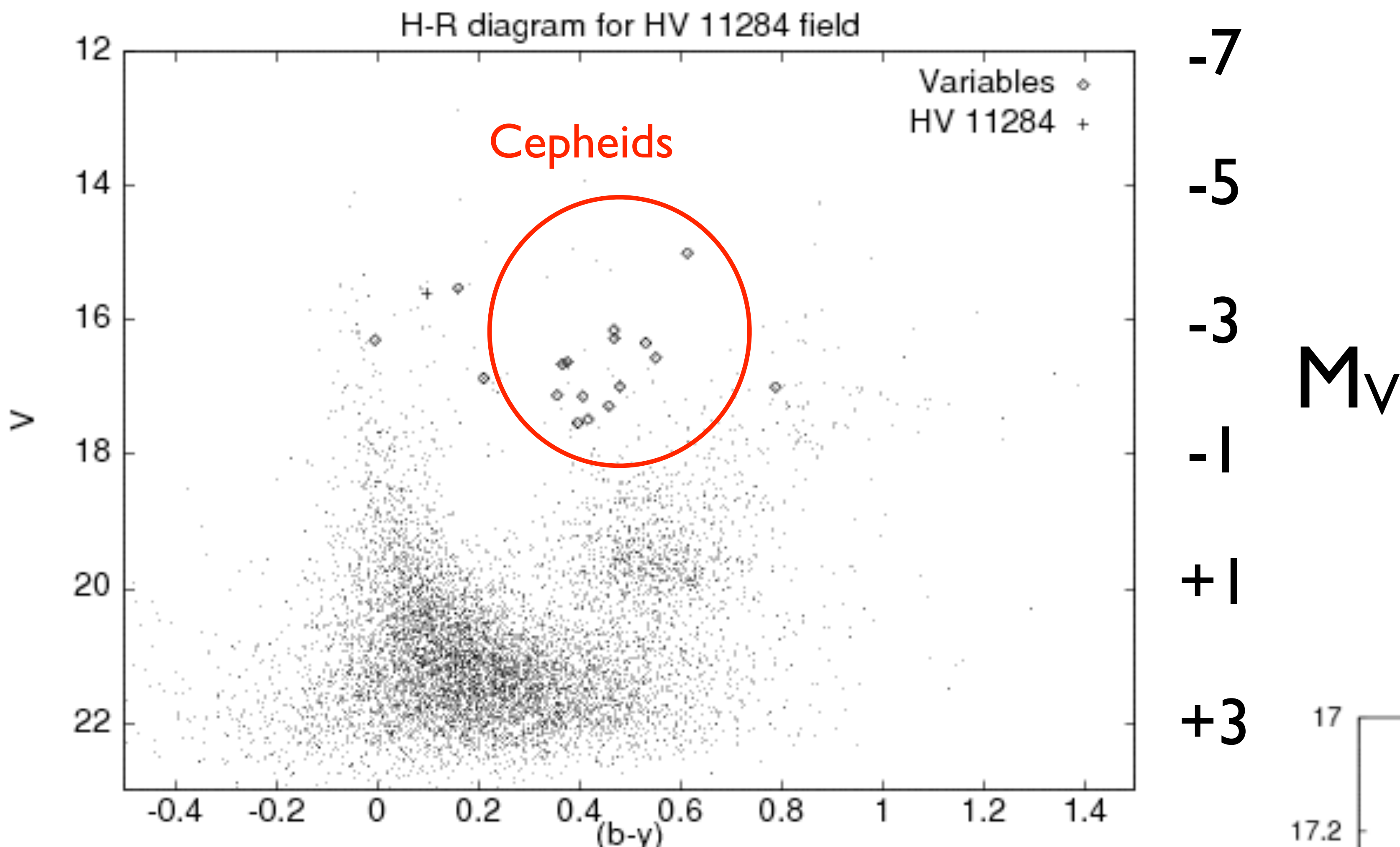
Distance = R_{ring}/r_{ring} = 52 kpc

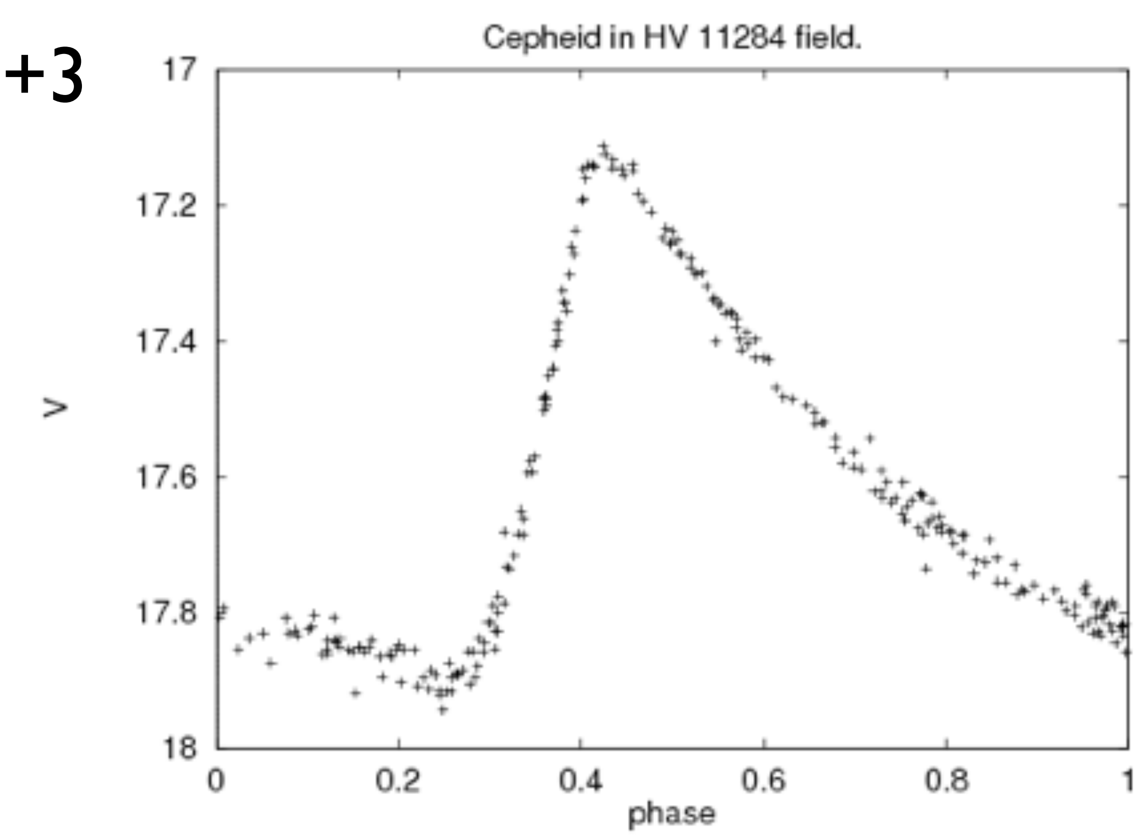
Table 7.2 LMC distance estimates

Method	Distance/kpc
Main Sequence Fitting	50 ± 5
Cepheids	50 ± 2
RR Lyrae	44 ± 2
SN1987a time delay	52 ± 3
SN1987a Baade–Wesselink method	55 ± 5

Binney & Merrifield, "Galactic Astronomy"

Cepheids


H-R diagram for HV 11284 field
Variables
HV 11284
Cepheids
V
(b-y)
12
14
16
18
20
22
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
-7
-5
-3
-1
+1
+3
M_V


Cepheid in HV 11284 field.
V
phase
17
17.2
17.4
17.6
17.8
18
0
0.2
0.4
0.6
0.8
1

P-L relations for LMC Cepheids

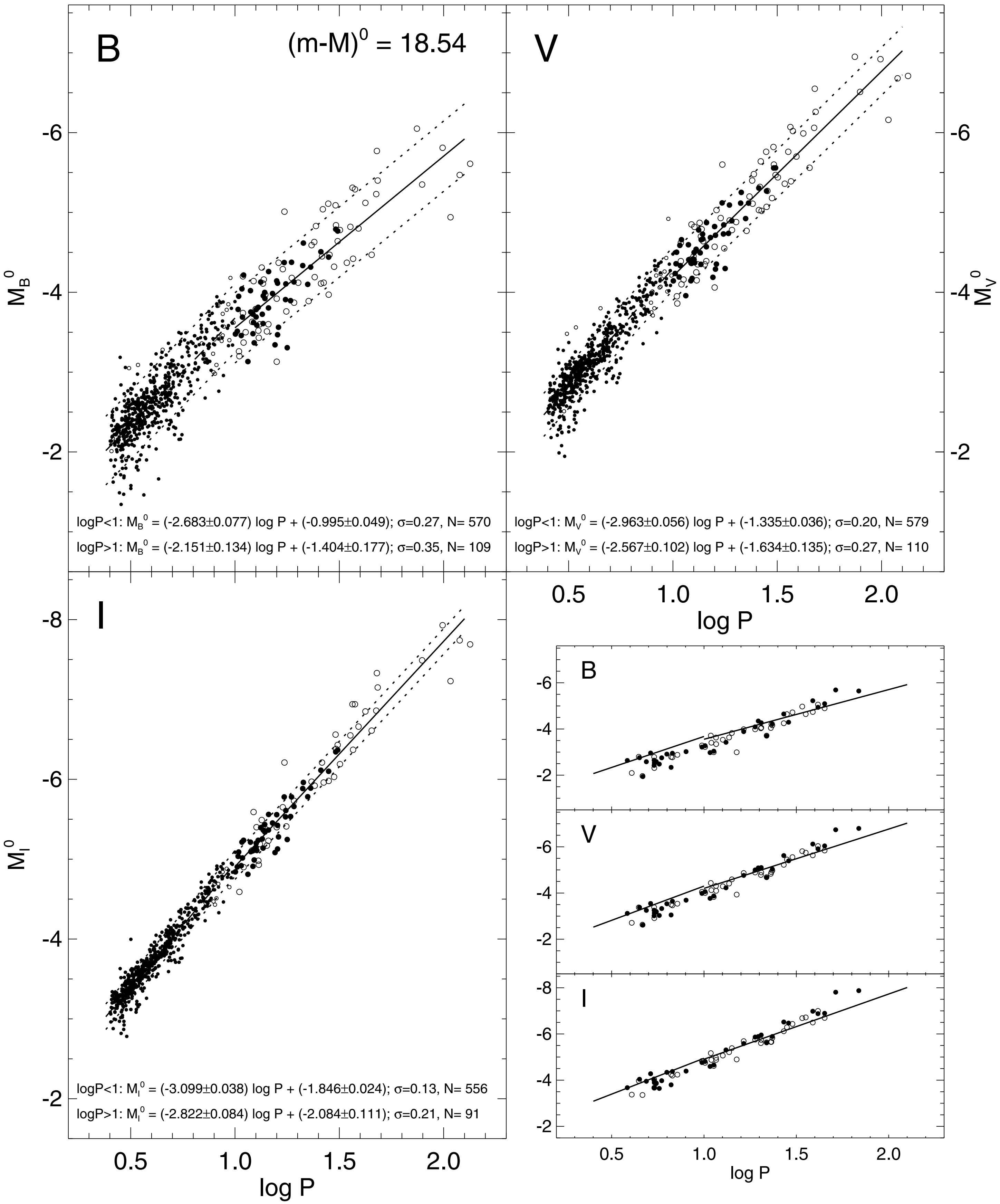


Fig. 4. The P-L relations in *B*, *V*, and *I* of LMC Cepheids. The data in each color are fitted with two linear regressions breaking at log $P = 1.0$. Symbols as in Fig. 1a. For the dashed intrinsic boundaries see text (6.2). Comparison with the revised Galactic calibration in Sect. 4.2.1 (Eqs. (16)−(18)) are in the lower right panel. The individual Galactic Cepheids with known absolute magnitudes (cf. Sect. 4.2.1) are the data points. The LMC mean relations are the solid lines.

Sandage et al. (2004)

Cepheids

Pulsating variables, driven by "Eddington valve"

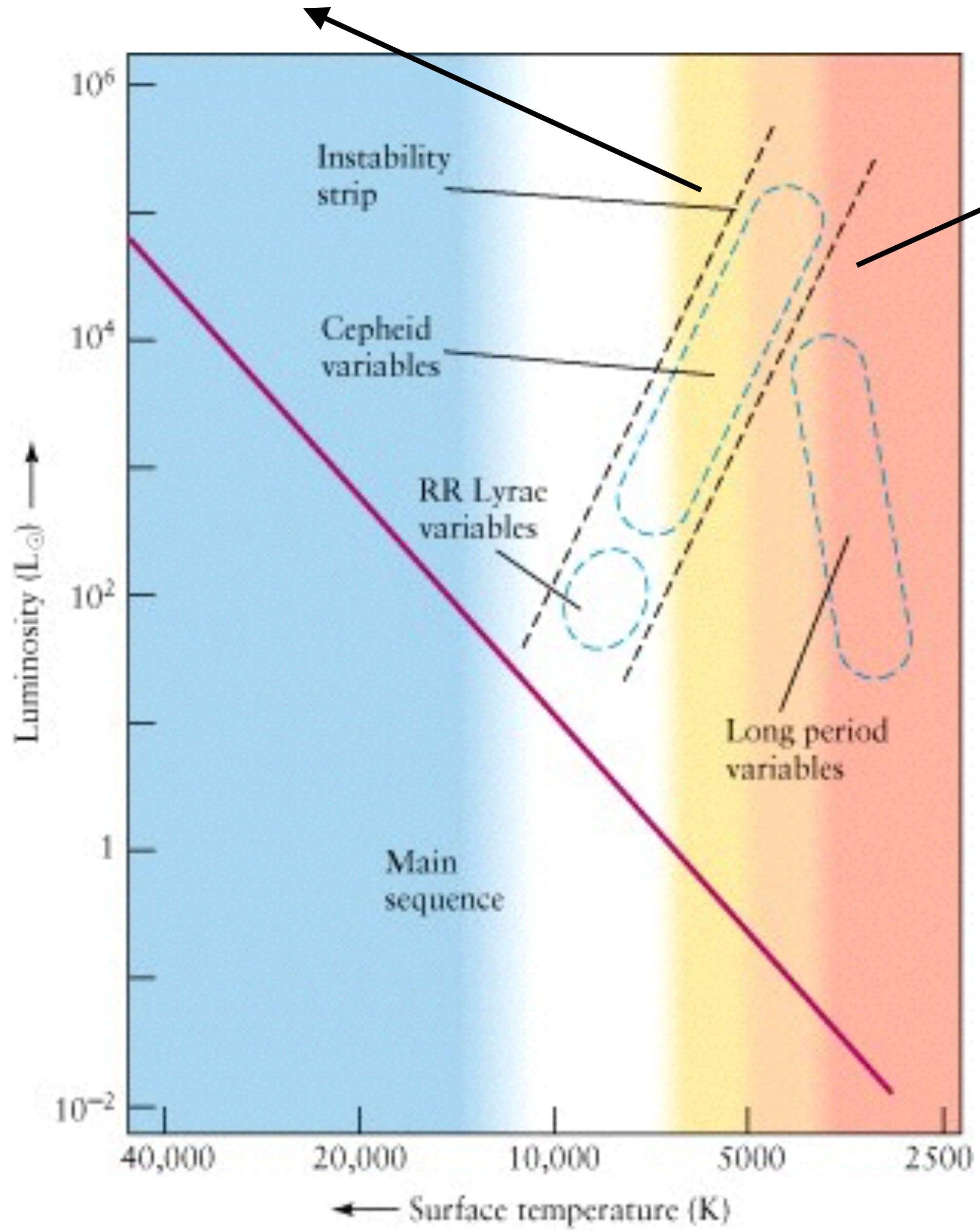
Generally: Stars are in hydrostatic equilibrium (pressure/gravity balance).
If "compressed", ρ, T, and P will increase, and the star will "bounce back" toward equilibrium.

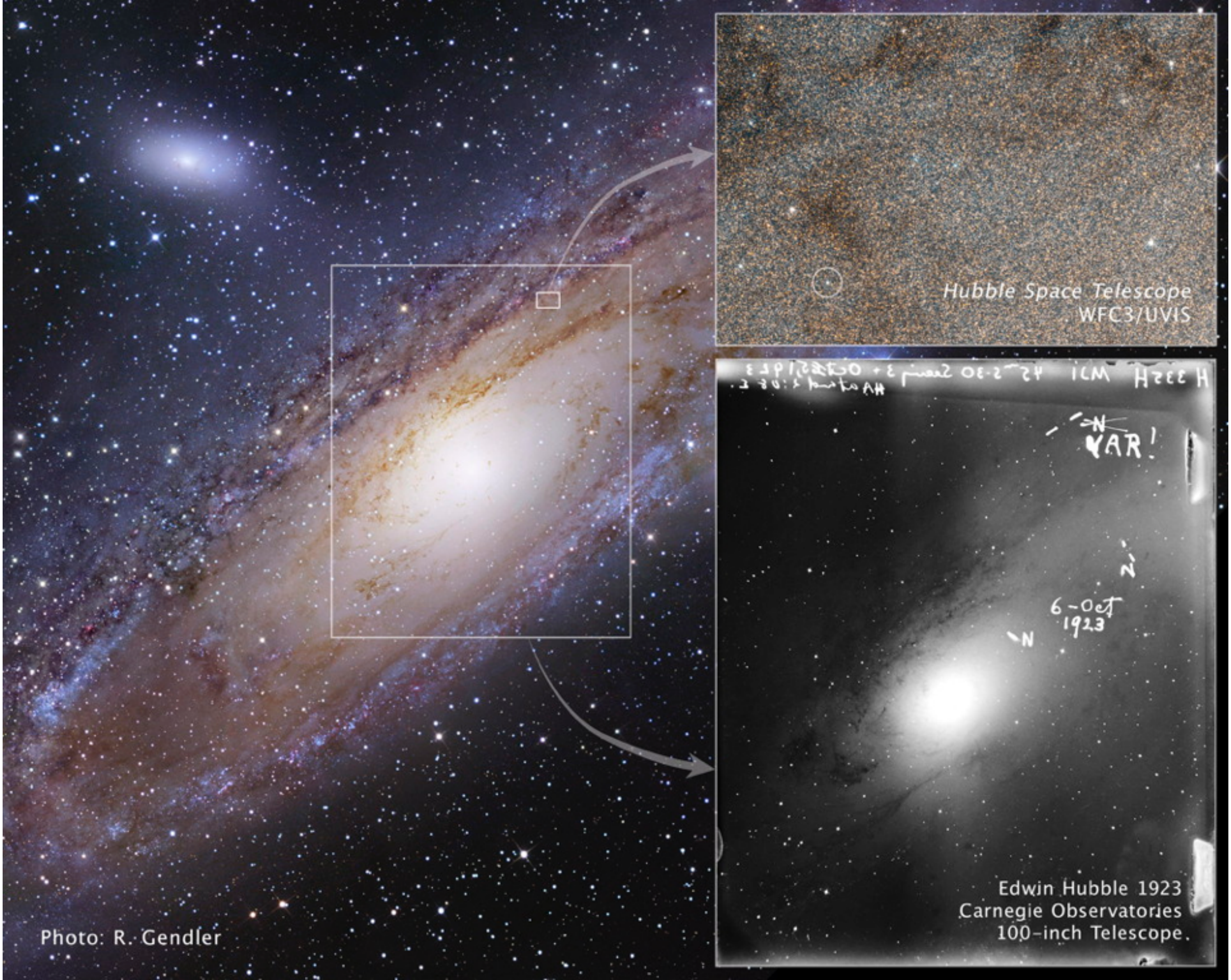
Cepheids: "*partial He II ionization zone*" drives oscillations:
Compression → increasing ρ, T → sudden ionization of He → increase in opacity → radiation is "trapped" and pushes outer layers back.

The instability strip

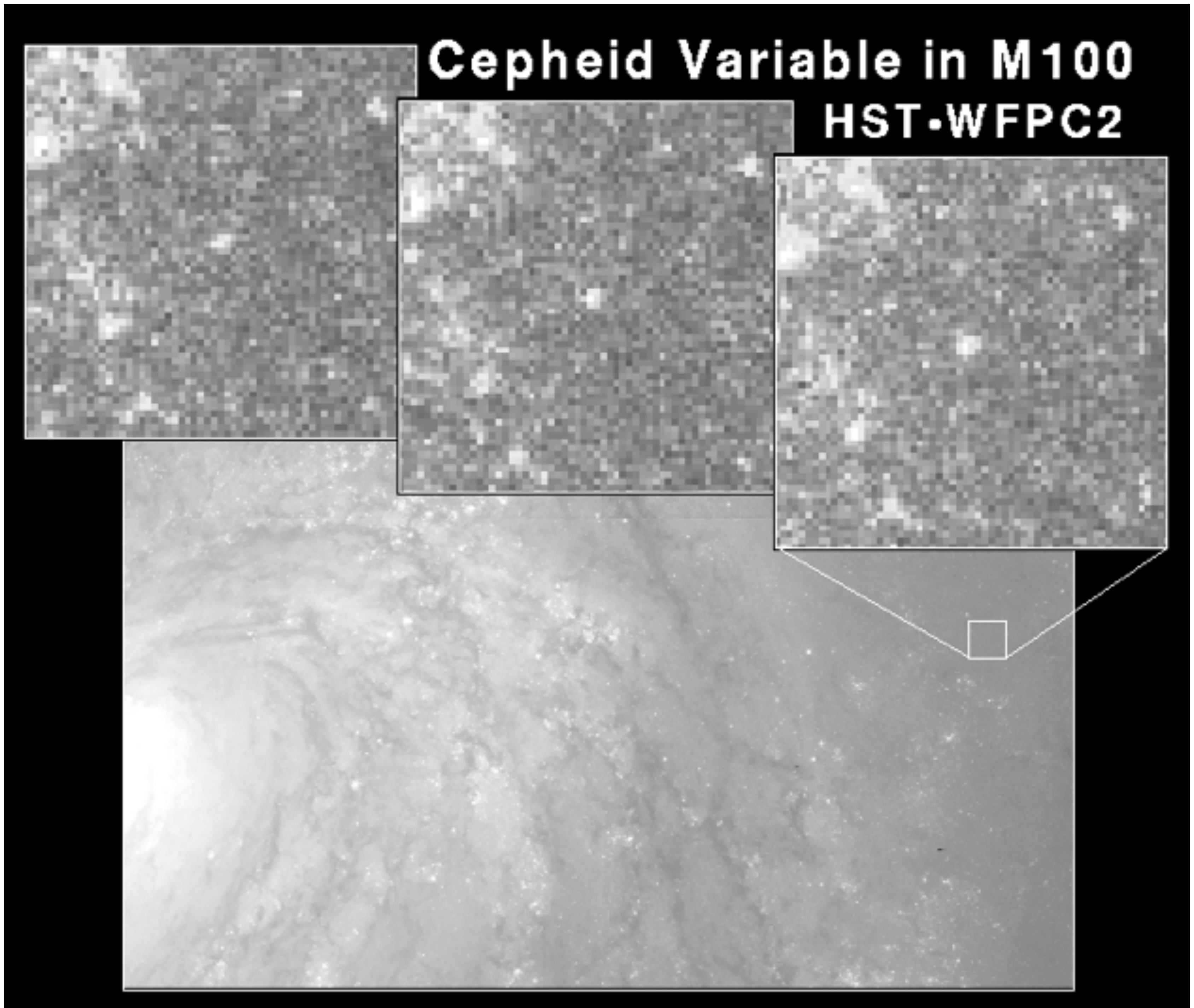
Hot stars: P.I.Z. too close to surface;
Eddington valve not effective

Cool stars: atmospheres become convective;
Eddington valve not effective




Hubble Space Telescope
WFC3/UVIS
VAR!
N
N
6-Oct
1923
N
Edwin Hubble 1923
Carnegie Observatories
100-inch Telescope
Photo: R. Gendler

Cepheid in M100 (in Virgo cluster)



APOD 1996, Jan 10

Cepheids in M100

Ferrarese, Freedman et al. 1996, ApJ 464, 568

52 Cepheids in M100

m_V=25-26.5

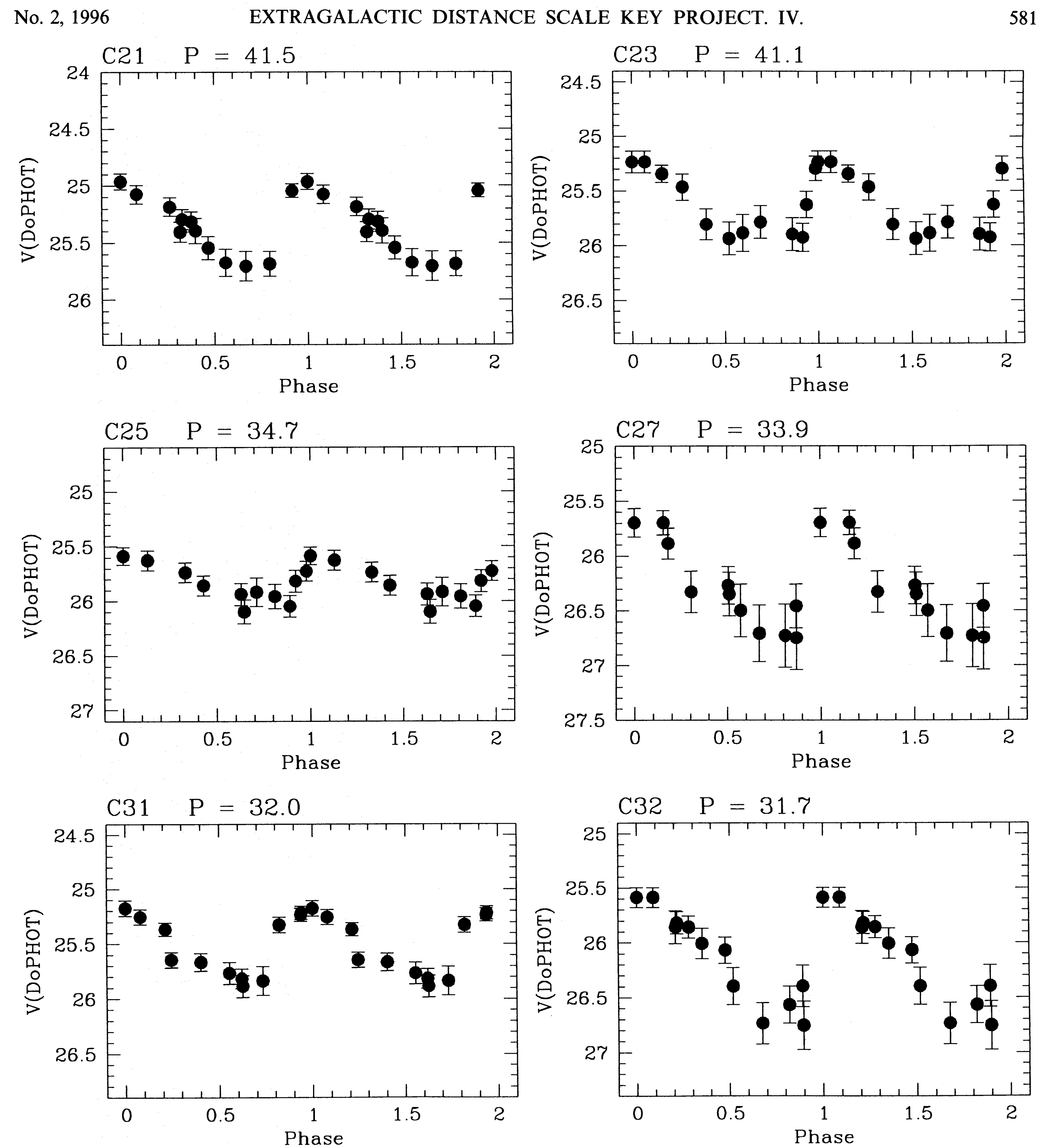


Fig. 5d

Cepheids in M100

Distance = 16.1 +/- 1.3 Mpc
(Ferrarese et al. 1996)

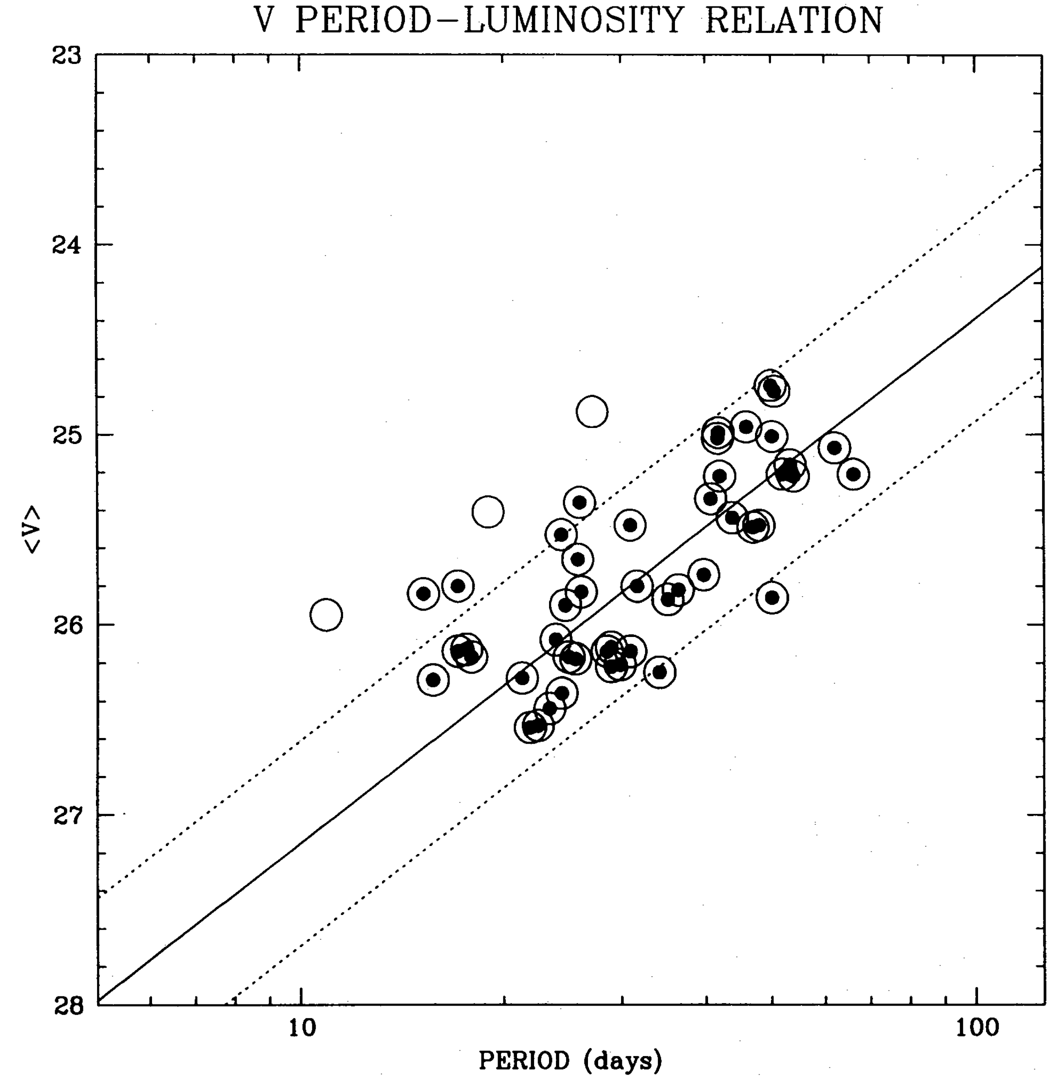


FIG. 7.—*V* PL relation for the sample of Cepheids listed in Table 5. For reasons discussed in the text, only the Cepheids with periods between 8 and 70 days are plotted. The solid line represents the best unweighted fit to the Cepheids with periods between 20 and 70 days, using phase weighted mean magnitudes, and corresponds to an apparent distance modulus of 31.31 ± 0.06 mag. The dashed lines, drawn at ± 0.54 mag, reflect the finite width of the Cepheids instability strip, and thus the expected intrinsic 2 σ scatter around the best-fitting PL relation. The three points plotted as open circles mark outliers falling more than 4 σ away from the mean, in either the *V* or the *I* PL plots.

Baade-Wesselink

- Uses pulsating stars to get size (and therefore distance) information
- Combines information about *linear* changes in sizes of variable stars (from radial velocities) with *relative* changes (from light curves) to get distances
- Applied to RR Lyrae stars (GCs, LMC, SMC) Cepheids, Miras (long period variables), and (in modified form) SNe

Compares *difference* of radii with *ratio* of radii at two different points in the pulsation cycle

--> two equations with two unknowns (voila -- easily solved)

It's fairly straightforward to find the *difference* in radii:

Absolute size change: $$\Delta R = \int_{t_0}^{t_0+\Delta t} \frac{dR}{dt} dt = -p \int_{t_0}^{t_0+\Delta t} v_{los} \, dt \qquad p \approx 1.5$$

Finding the *ratio* is a little more complicated (but not too much)

we can use:

Stephan-Boltzmann law: $L = 4\pi R^2 \sigma T_{eff}^4$

Magnitudes:
$$M_{\rm bol} = -2.5 \log L + C' = -5 \log R - 10 \log T_{\rm eff} + C$$

At two epochs with same $T_{\rm eff}$: difference in L yields *relative* size change

$$\Delta m_V = \Delta m_{bol} = \Delta M_{bol} = -5\left[\log(R_0 + \Delta R) - \log R_0\right]$$

Ratio!

$$\longrightarrow \quad \Delta m_{bol}/5 = -\log\frac{R_0 + \Delta R}{R_0} \longrightarrow 10^{\Delta m_{bol}/5} = H = \frac{R_0}{R_0 + \Delta R}$$

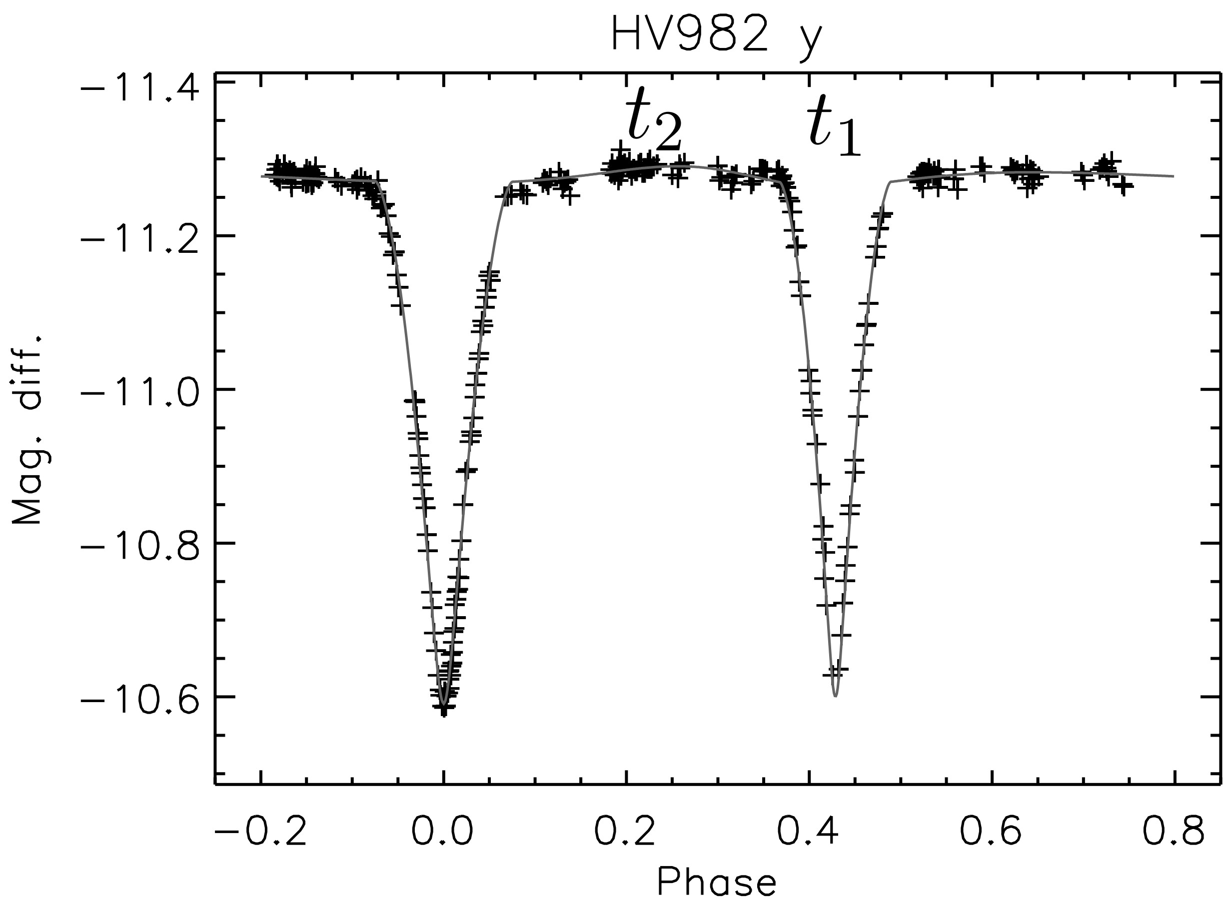
$$1/H = 1 + \Delta R/R_0 \rightarrow \boxed{R_0 = \frac{\Delta R}{1/H - 1} = \frac{H \Delta R}{1 - H}}$$

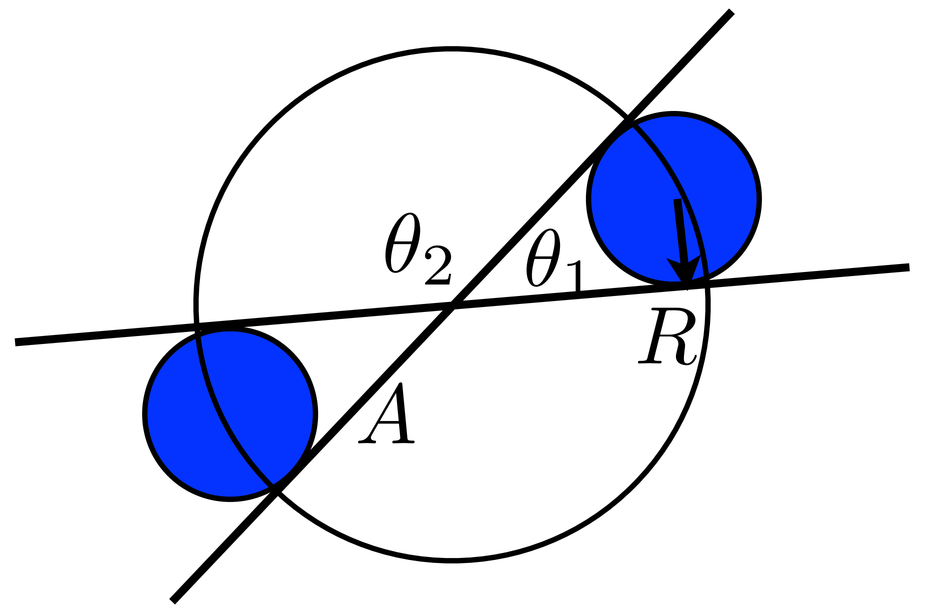
R_0 now known -> L follows from S-B law, and distance can be obtained from *apparent* magnitude.

Caveats:

- Region forming absorption lines may not exactly trace surface seen in continuum - factor p between v_{los} and dR/dt uncertain
- Pulsations may be non-radial
- Not trivial to identify points of constant T_{eff} on light curve

Eclipsing Binaries - I





R = stellar radius
A = radius of orbit

For two identical stars, circular orbit: $R/A = \sin(\theta_1/2)$

$$\theta_1/\theta_2 = t_1/t_2 \qquad \theta_1 + \theta_2 = \pi \;\Rightarrow\; \theta_2 = \pi - \theta_1$$

$$(\pi - \theta_1)/\theta_1 = t_2/t_1 \quad \Rightarrow\; \pi/\theta_1 = t_2/t_1 + 1$$

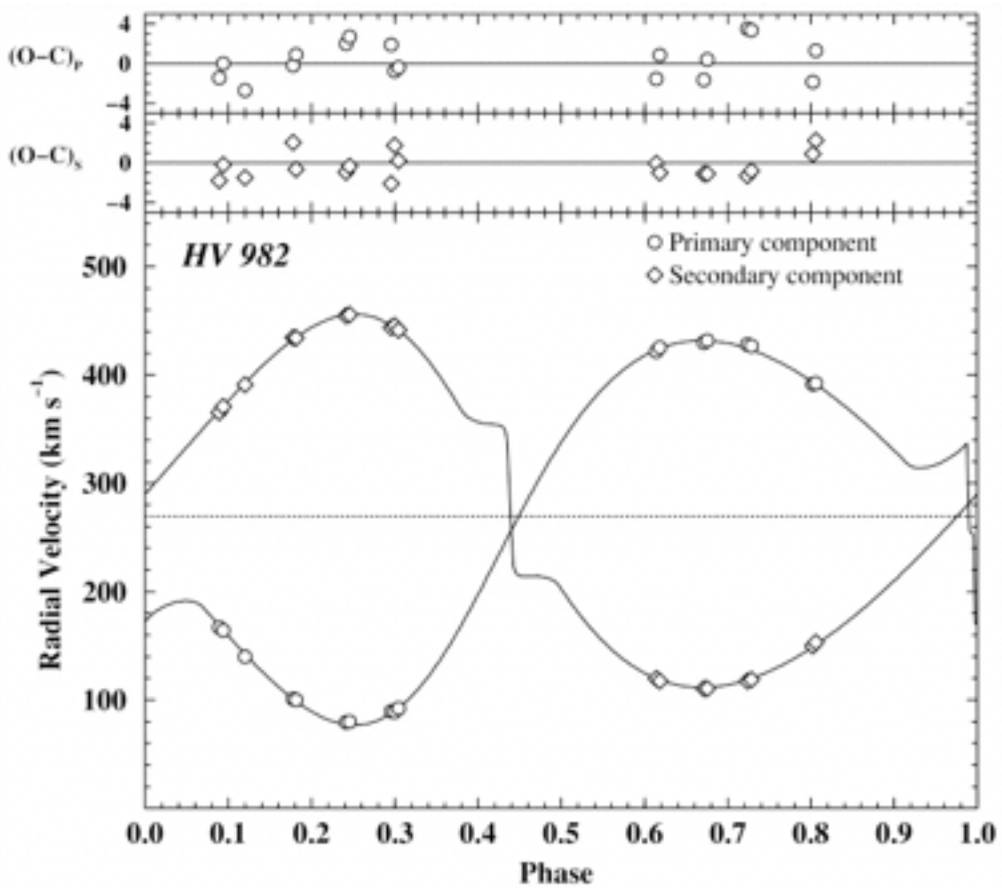
Relative dimensions

$$\Rightarrow\; \theta_1 = \frac{\pi}{t_2/t_1 + 1}$$

so

$$R/A = \sin\left(\frac{1}{2}\frac{\pi}{t_2/t_1 + 1}\right)$$

Eclipsing Binaries - II



Absolute dimensions from radial velocities:

Circumference of orbit

$$2\pi A = v_{\rm orb} P$$

P = period

Combine with

$$R/A = \sin\left(\frac{1}{2}\frac{\pi}{t_2/t_1+1}\right)$$

Yields stellar radius R

Finally, L follows from

$$L = 4\pi R^2 \sigma T_{eff}^4$$

$$M_V = -5\log R/R_\odot - 10\log T_{eff}/T_{eff\odot} + M_{bol\odot} - BC$$

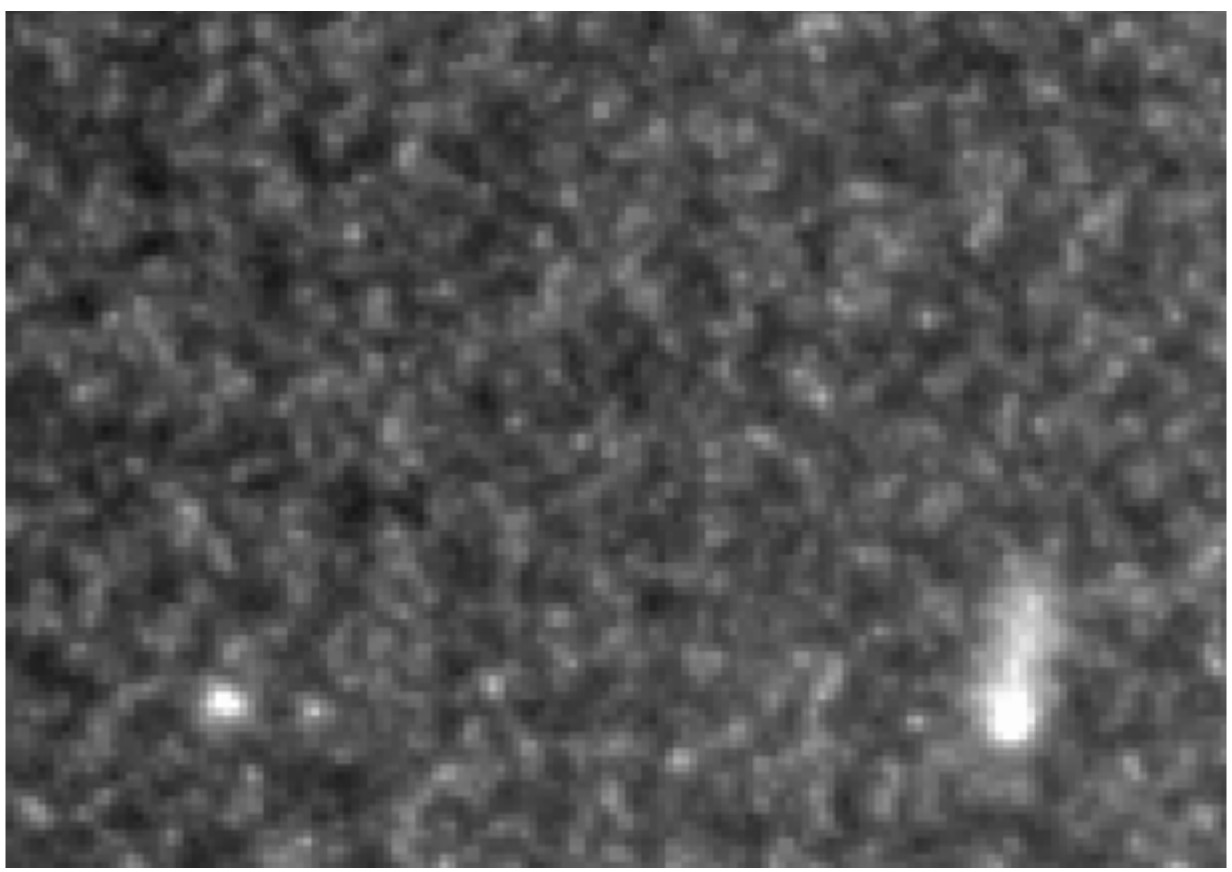
$$V_{obs} = M_V + A_V + 5\log\left(\frac{D}{10pc}\right)$$

Eclipsing binaries - caveats:

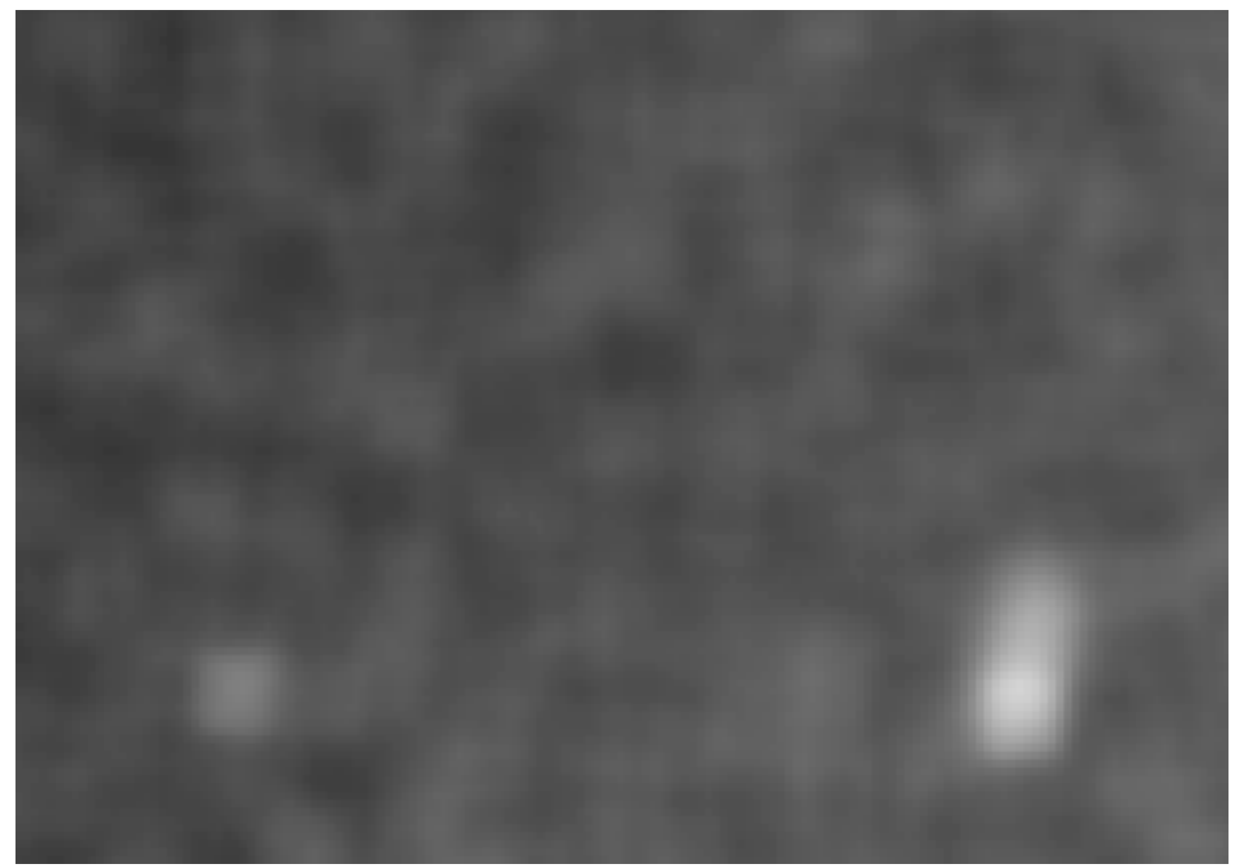
- Stars are generally not identical (although often quite similar)
- Orbits may not be circular
- Stars may not be spherical
- Line-of-sight may not lie in orbital plane
- Reflection effects, star spots, etc.

Surface brightness fluctuations

Finite number of stars per "resolution element" leads to "granular" appearance of distant galaxies



HST image of nearby galaxy: NGC 3384

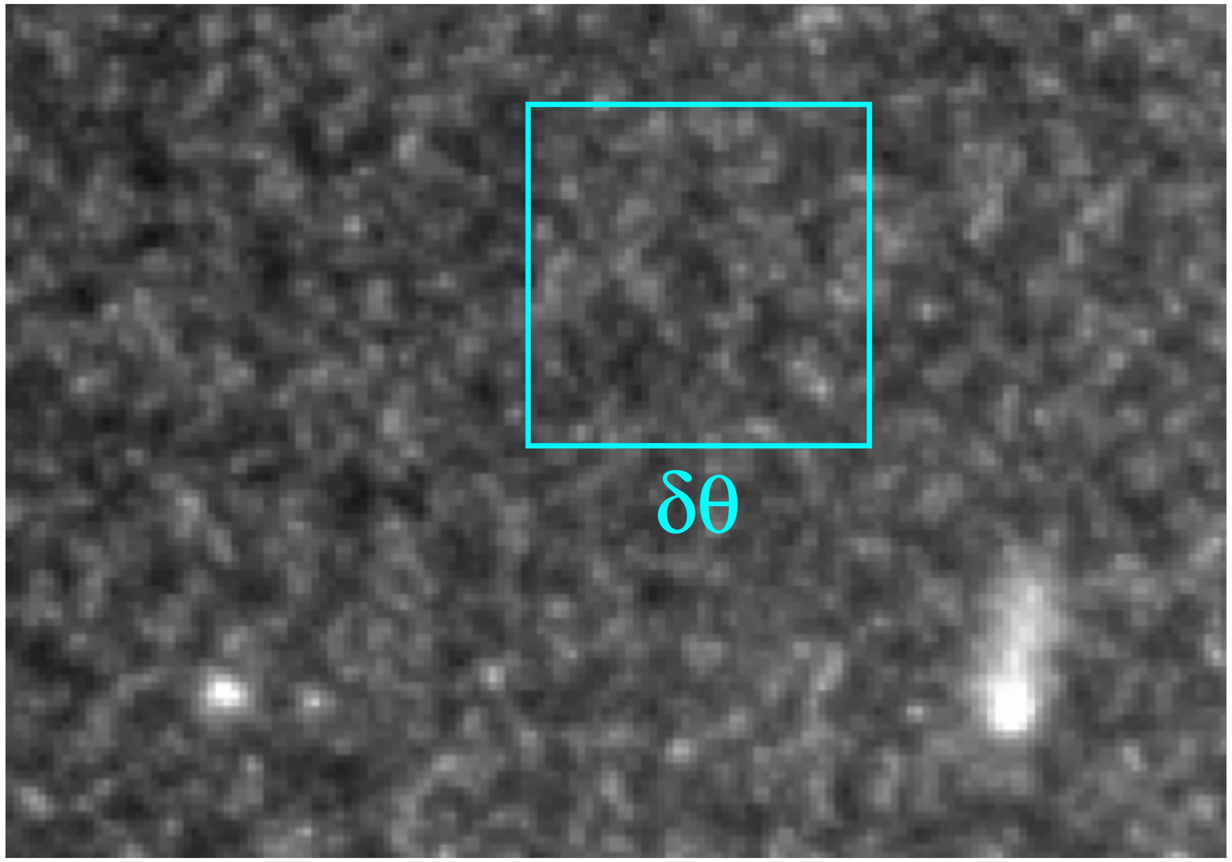


Smoothed to simulate appearance of more distant galaxy

Small resolution element: $(\delta\theta)^2$

Mean surface density of stars: n pc^{-2}

Each star has luminosity L



Number of stars per resolution element: $N = n(D\delta\theta)^2$

Flux per resolution element= $F = NL/(4\pi D^2) = nL(\delta\theta)^2/4\pi$

Independent of D

Number fluctuations $\sigma N = \sqrt{N} = \sqrt{n(D\delta\theta)^2} = D\delta\theta\sqrt{n}$

Flux fluctuations $\sigma F = L\sigma N/(4\pi D^2) = L\delta\theta\sqrt{n}/(4\pi D)$

Relative fluctuations $$\frac{(\sigma F)^2}{F} = \frac{L^2(\delta\theta)^2 n}{(4\pi D)^2}\frac{4\pi}{nL(\delta\theta)^2} = \boxed{\frac{L}{4\pi D^2}}$$

SBF technique

- Introduced by Tonry & Schneider (1988, AJ 96, 807)
- Requires accurate (to ~1%) surface brightness measurements - CCD photometry
- 3-D structure of Virgo cluster, distance to Coma cluster (~100 Mpc)
- Useful for dwarf galaxies: few alternatives

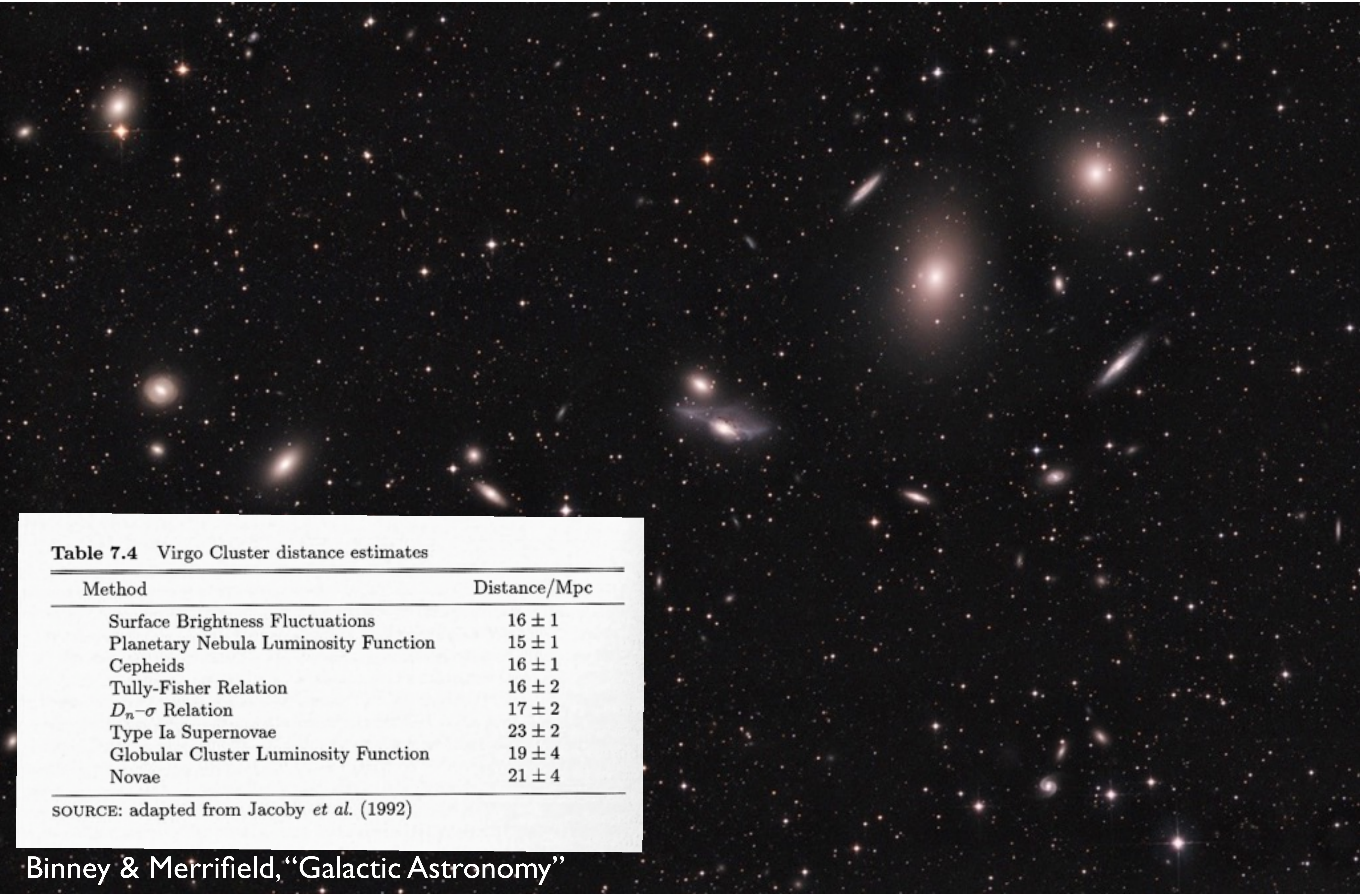


Table 7.4 Virgo Cluster distance estimates

Method	Distance/Mpc
Surface Brightness Fluctuations	16 ± 1
Planetary Nebula Luminosity Function	15 ± 1
Cepheids	16 ± 1
Tully-Fisher Relation	16 ± 2
D_n–σ Relation	17 ± 2
Type Ia Supernovae	23 ± 2
Globular Cluster Luminosity Function	19 ± 4
Novae	21 ± 4

SOURCE: adapted from Jacoby *et al.* (1992)

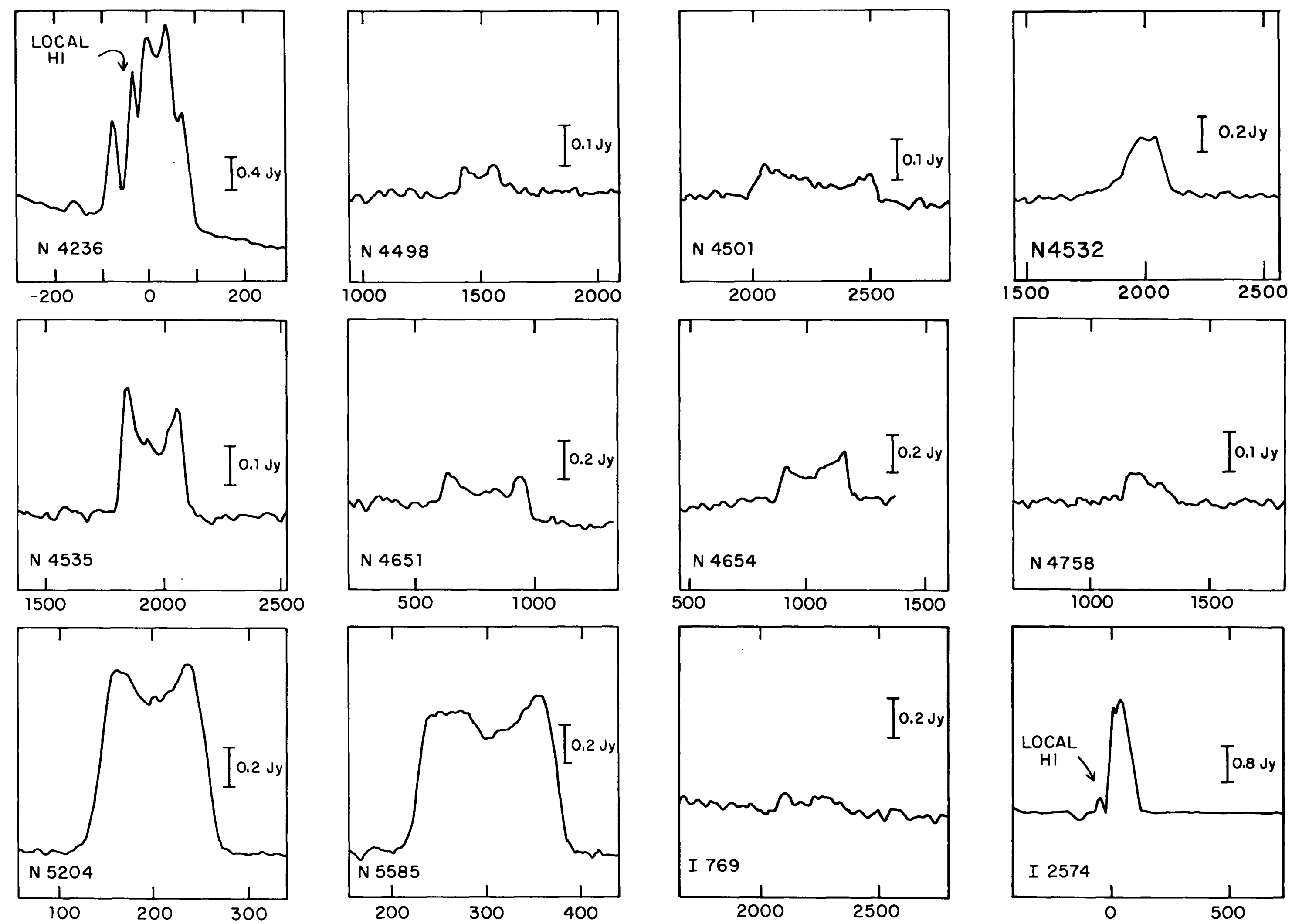
Binney & Merrifield, "Galactic Astronomy"

Tully-Fisher Relation

Relation between *HI 21 cm line width* and galaxy luminosity

Tully & Fisher 1977, A&A 54, 661

Applies to galaxies with rotating gas disks (i.e. *spirals*).



21 cm profiles (Tully & Fisher 1977)

Tully-Fisher Relation

Notes:
ΔV must be corrected for inclination (sin i) and random gas motions

Magnitude must be corrected for internal absorption - better to use magnitudes that are less affected by extinction (e.g. near-IR)

$$L \propto \Delta V(o)^{2.5 \pm 0.3}$$

R. B. Tully and J. R. Fisher: Distances to Galaxies

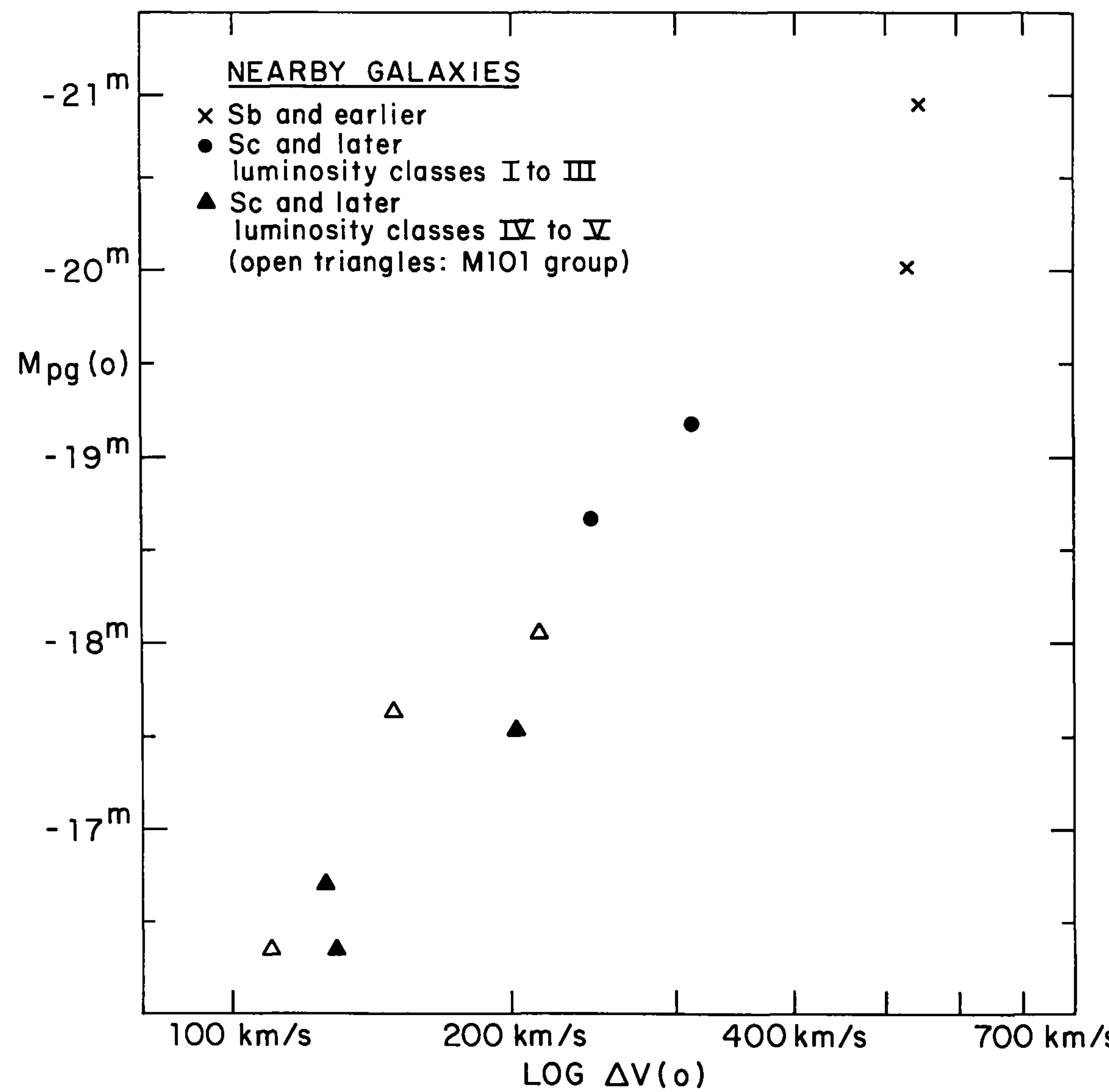
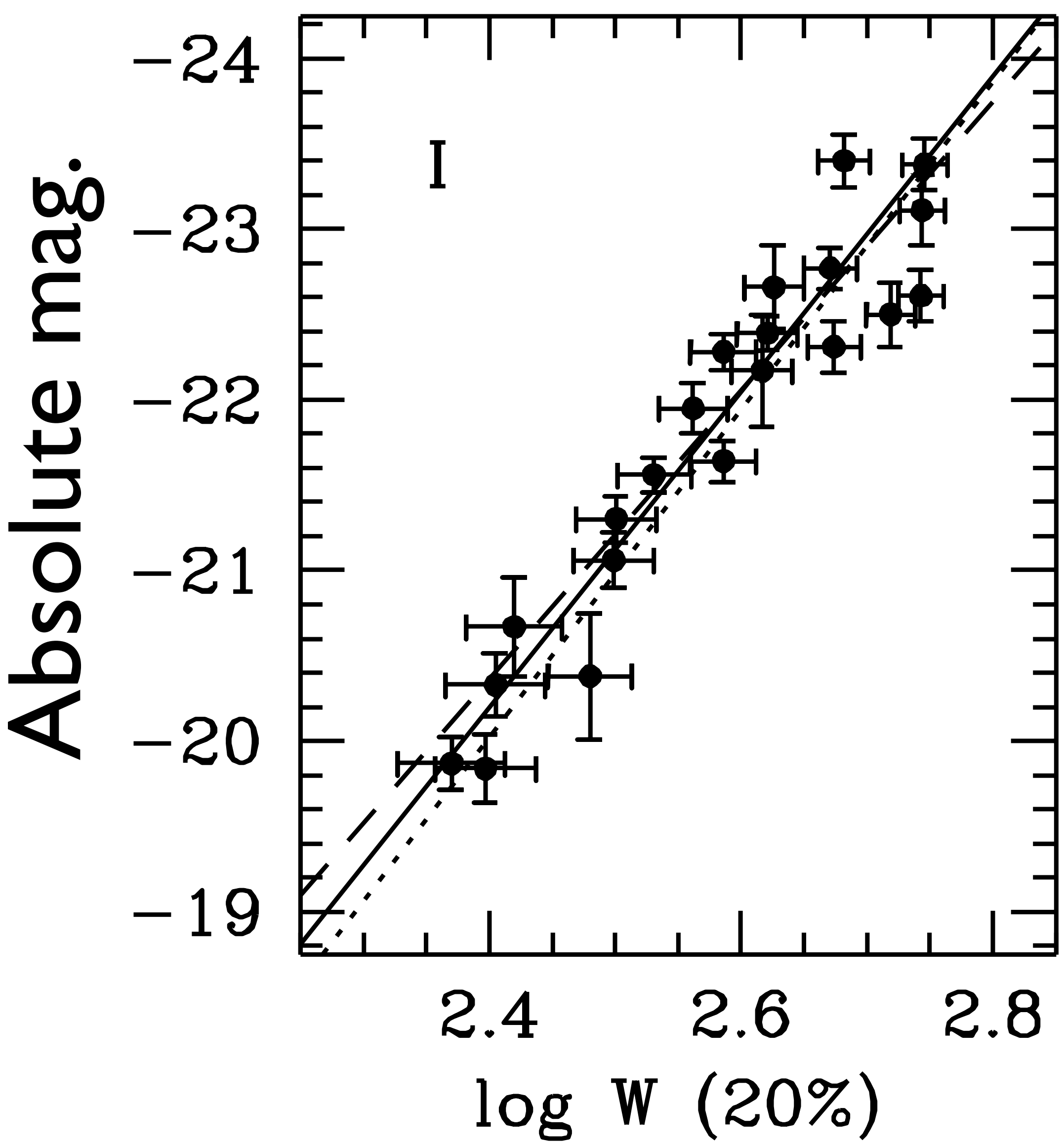
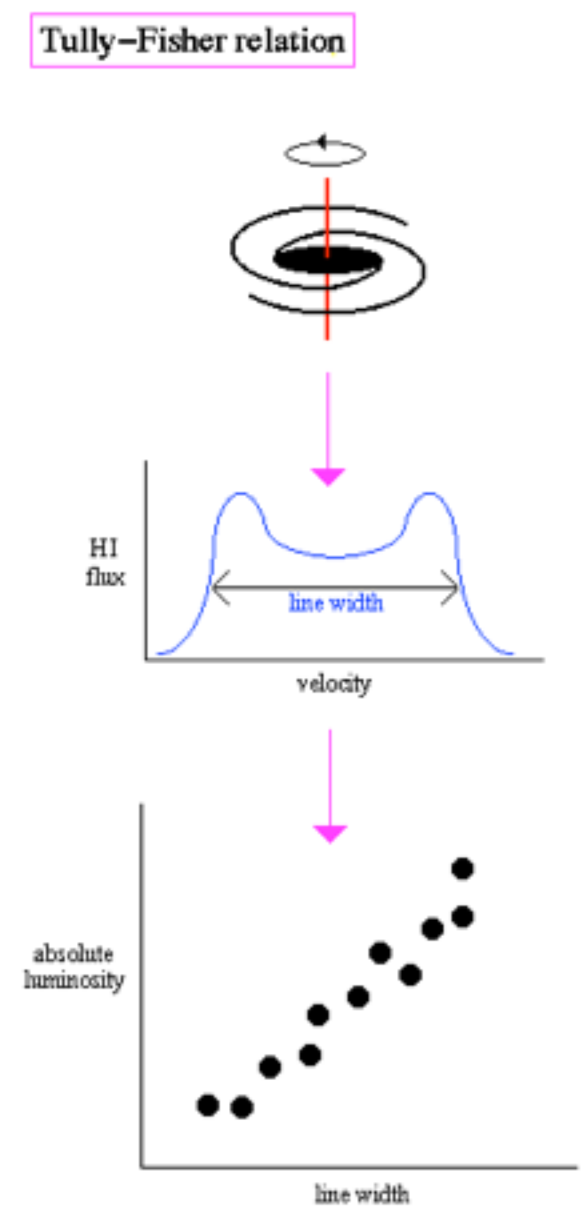


Fig. 1. Absolute magnitude – global profile width relation for nearby galaxies with previously well-determined distances. Crosses are M31 and M81, dots are M33 and NGC 2403, filled triangles are smaller systems in the M81 group and open triangles are smaller systems in the M101 group

Tully-Fisher relation



Sakai et al. (2000)



Spiral galaxies rotate, doppler shift varies across disc.

Width *W* of emission line profile (e.g. from HI) can be measured, independently of distance.

a plot of line width versus absolute luminosity of a galaxy is called the Tully−Fisher relation. When calibrated using galaxies with Cepheid distances, the TF relation is used to determine Hubble's constant.

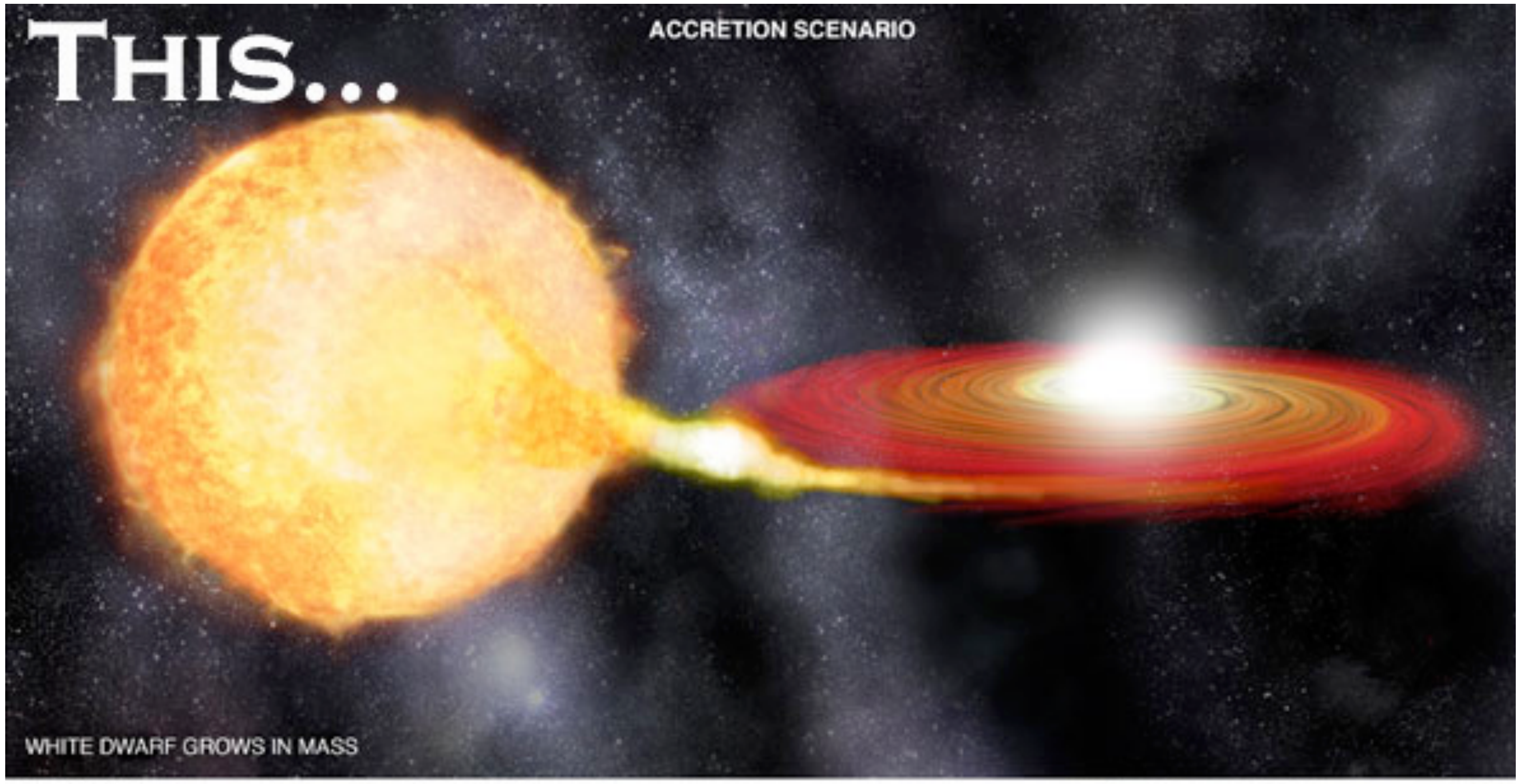
Adapted from J. Schombert, Univ. of Oregon

http://abyss.uoregon.edu/~js/ast123/lectures/lec13.html

Supernovae

- Supernovae of Type 1a (thermonuclear explosions of white dwarfs) are believed to be good *standard candles*.
- Absolute magnitude at maximum is M_B=-19.3 - comparable to a whole galaxy!
- SNe can be observed at cosmological distances

Supernova in NGC 4526

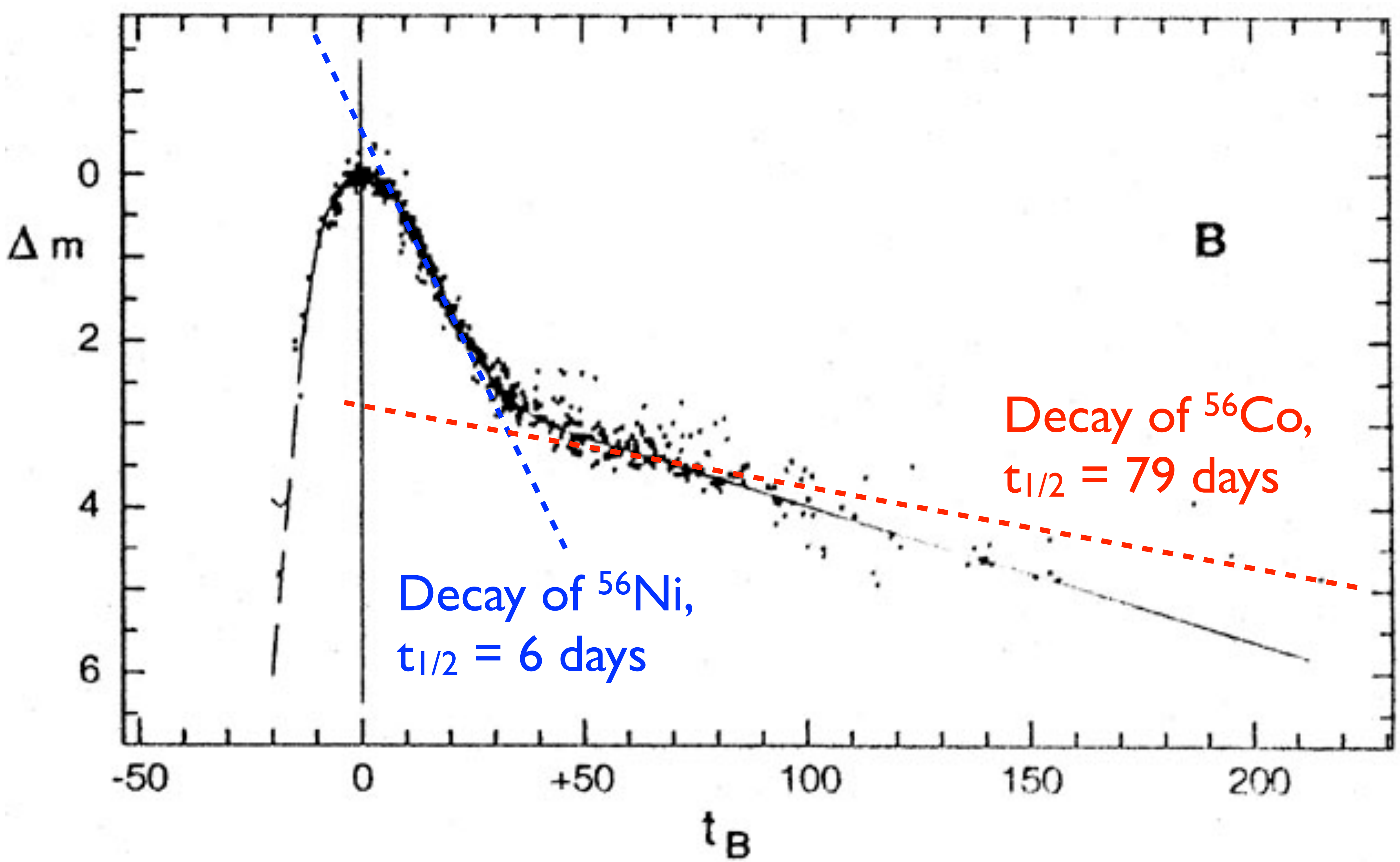


Type Ia SNe are believed to occur when a white dwarf becomes more massive than the Chandrasekhar limit (1.4 $M_\odot$).

The WD then becomes unstable and explodes.

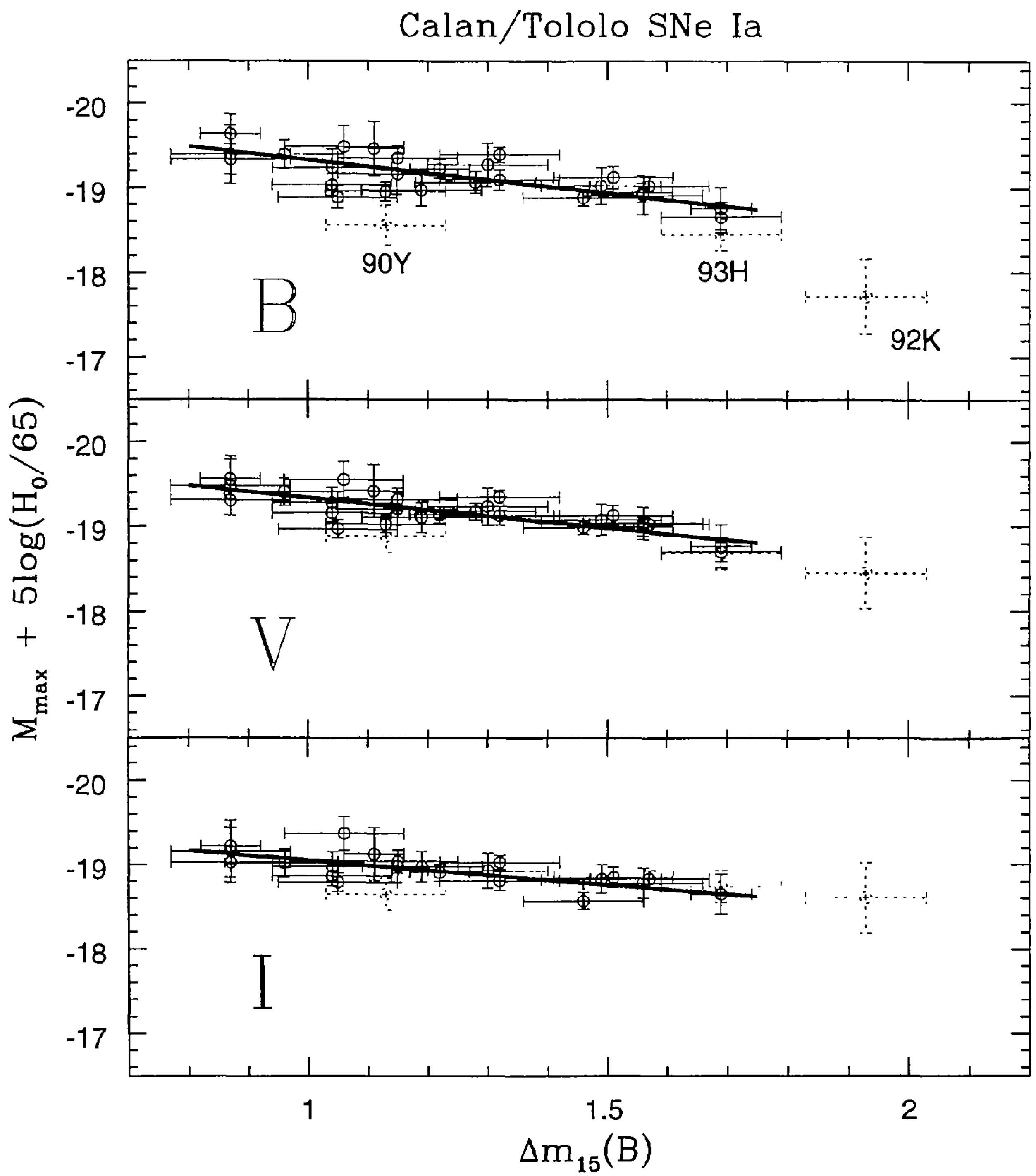
Exact physics of the accretion and explosion itself uncertain. However, light curves are empirically shown to be well behaved.

SN Ia light curve



Branch & Tammann 1992

Refining SN Ia as standard candles



Δm_{15} = fading after 15 days.

Strong correlation between absolute magnitude at maximum and Δm_{15}

When this is taken into account, the scatter is only ~0.15 mag.

Sufficient to constrain cosmological models (when many data points are used)

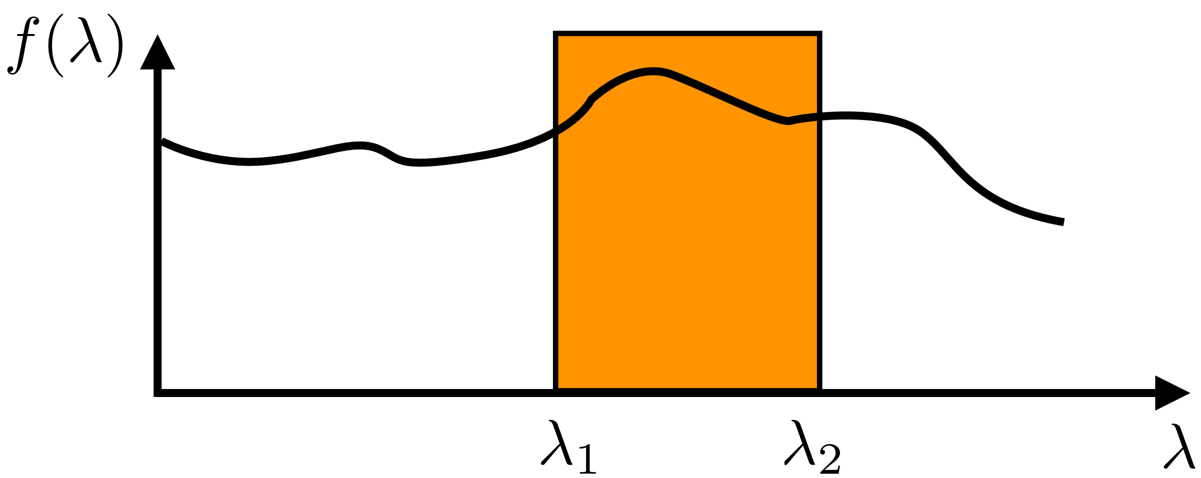
Hamuy et al. (1996)

Complication: K-corrections

- Fixed photometric filters used for observations (e.g. B, V, R) correspond to different *rest-frame* wavelengths for SNe at different redshifts.
- Two consequences:
  - A different (bluer) part of the spectrum is seen at non-zero redshift
  - Filter width $\Delta\lambda$ mapped to a smaller rest-frame wavelength range, $\Delta\lambda/(1+z)$
- Corrections based on the spectral energy distributions of SNe

K-correction

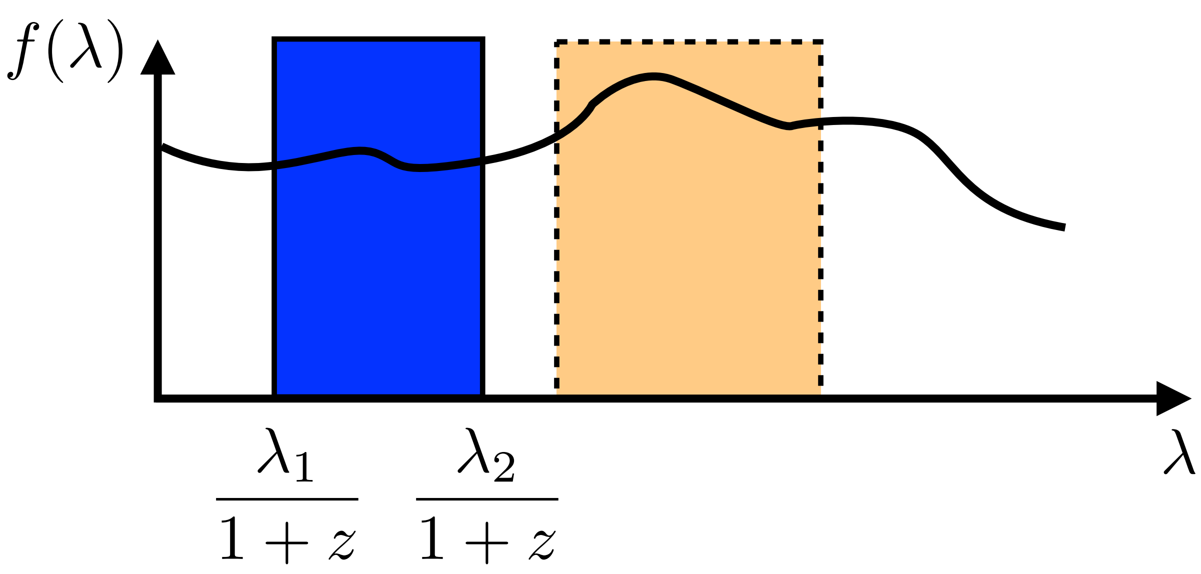
Redshift z=0



Flux density of rest-frame spectrum over filter bandpass *S*:

$$f_0 = \frac{\int f(\lambda)\, S(\lambda)\, \mathrm{d}\lambda}{\int S(\lambda)\, \mathrm{d}\lambda}$$

Redshift z:



Flux density of red-shifted spectrum:

$$f_z = \frac{\int f[\lambda/(1+z)]\, S(\lambda)\, \mathrm{d}\lambda}{\int S(\lambda)\, \mathrm{d}\lambda}$$

In magnitudes:

$$m_0 = m_z - 2.5\log_{10}\left(\frac{\int f(\lambda)S(\lambda)\mathrm{d}\lambda}{\int f[\lambda/(1+z)]S(\lambda)\mathrm{d}\lambda}\right) - 2.5\log_{10}(1+z)$$

where the 'K'-correction is

$$K(z) = 2.5\log_{10}\left(\frac{\int f(\lambda)S(\lambda)\mathrm{d}\lambda}{\int f[\lambda/(1+z)]S(\lambda)\mathrm{d}\lambda}\right) + 2.5\log_{10}(1+z)$$

Shift of bandpass

correction for (1+z) "compression" of wavelength scale

Important: depends on the spectrum of the source, *f(λ)*!

(Oke & Sandage 1968)

The Hubble constant

- Accurate calibration of distance scale and determination of H_0 one of the main goals of the *Hubble Space Telescope*
- Outcome of HST Key Project:

$$H_0 = (72 \pm 8)\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

(Freedman et al. 2001)