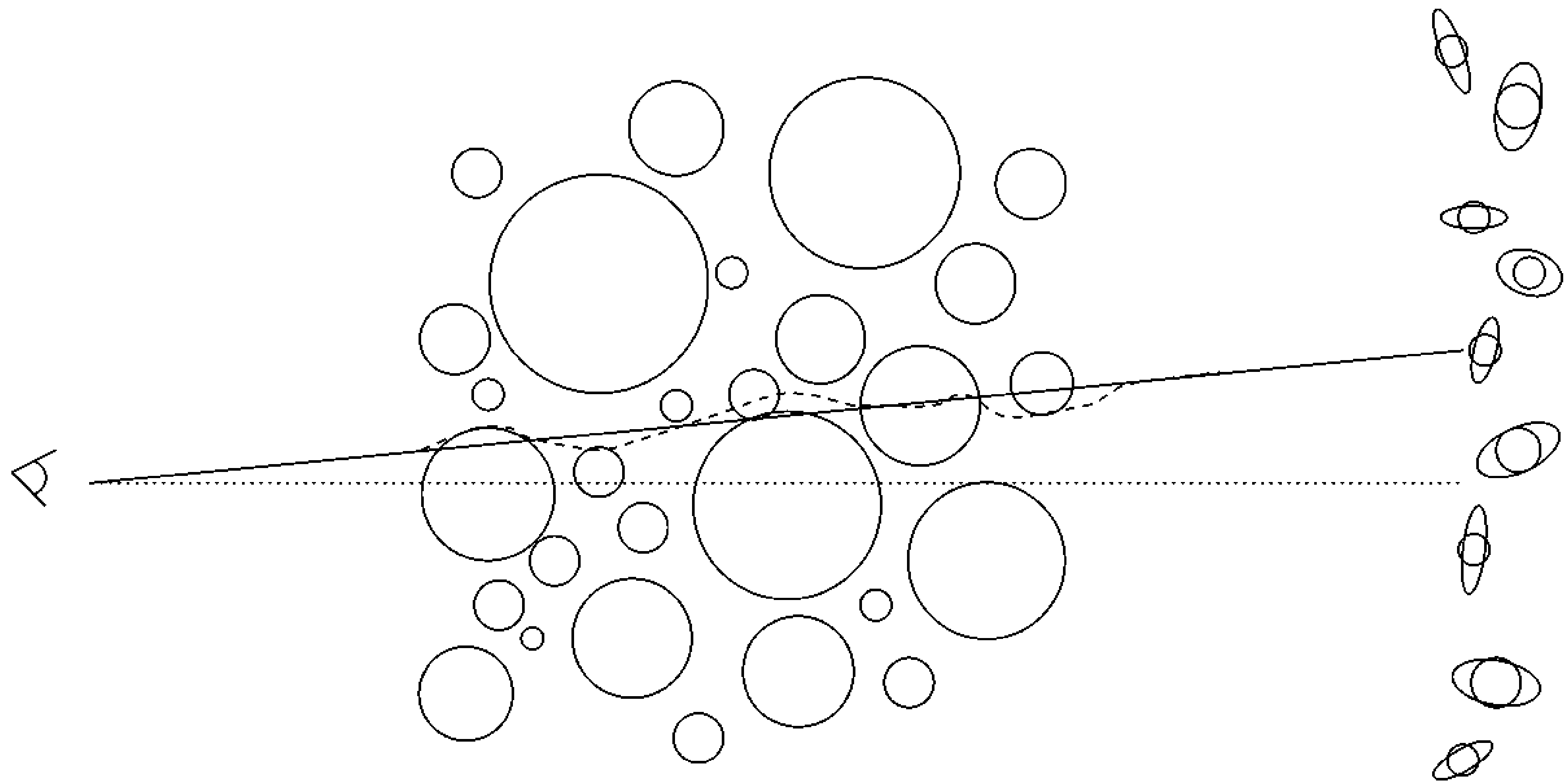
Gravitational lensing - cont'd

Abell 2218 (Hubble Space Telescope image)

Weak lensing



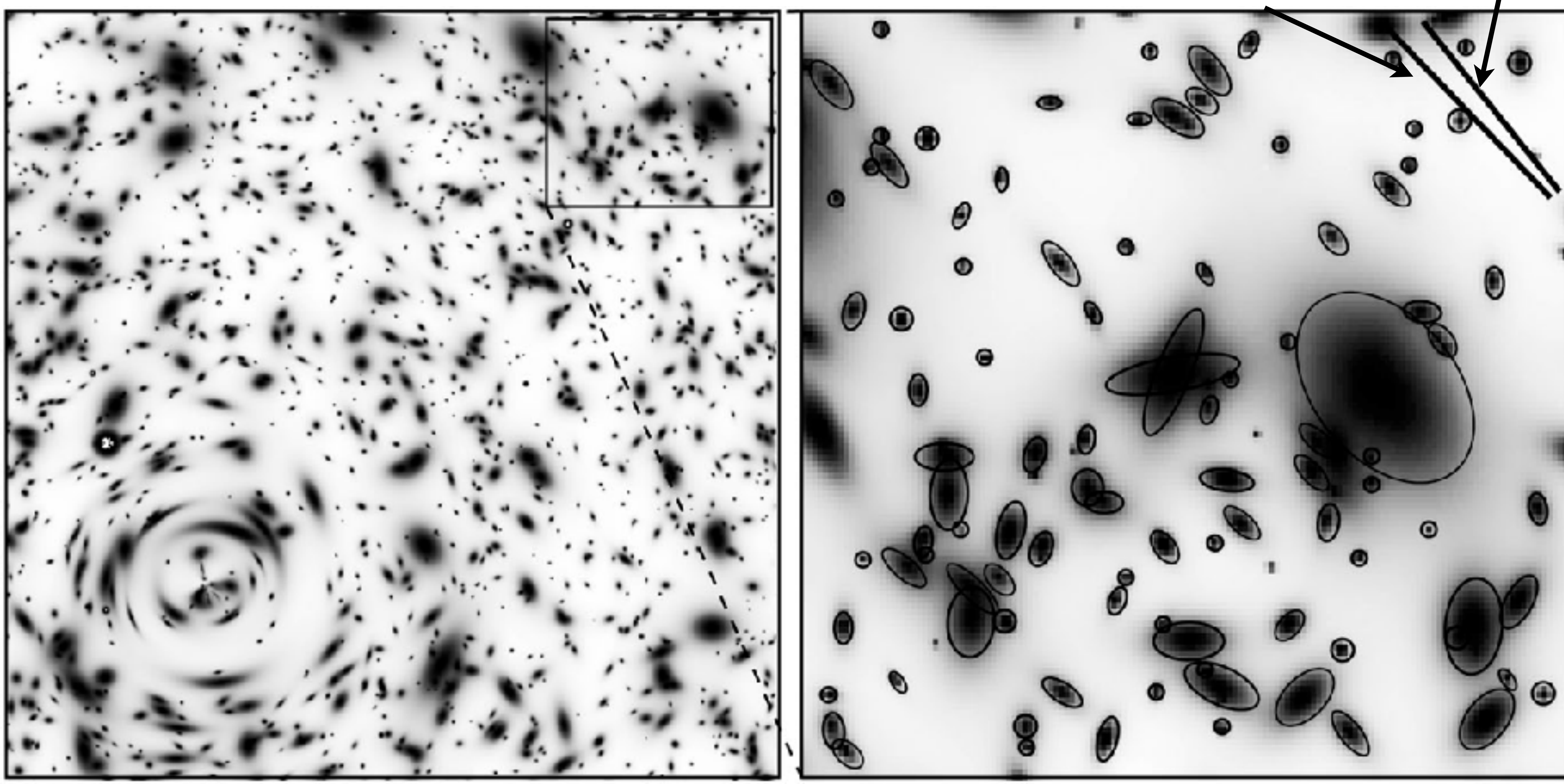
Refregier (2003, ARA&A 41, 645)

Weak lensing

- Potential gradients along the line-of-sight modify the brightness, shape and position of distant galaxies
- Sensitive to distribution of *mass* (not just galaxies!)
- Statistical technique - needs large samples of galaxy images.
- Idea dates to lecture by Richard Feynman at Caltech in 1964
- Effect first successfully detected (for galaxy clusters) in 1990; then for "random" field in 2000

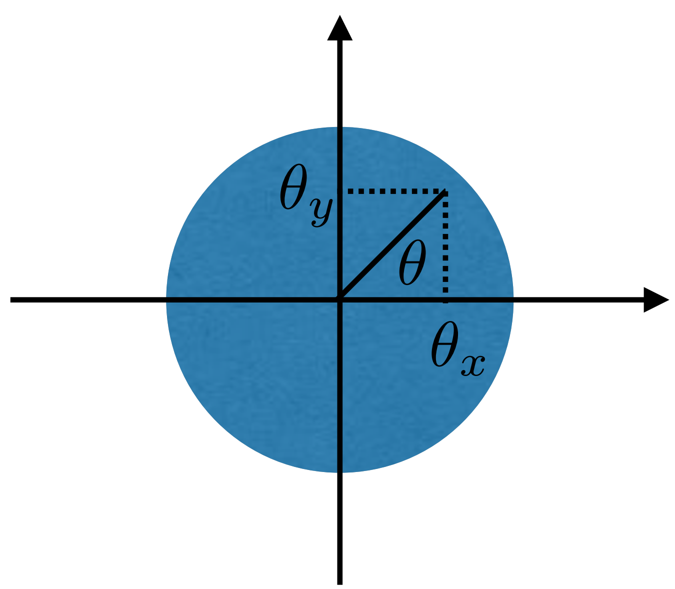
Strong and weak lensing

Mellier 1999, ARA&A 37, 127



Simulation of galaxy cluster at z=0.15 and background galaxies at <z>=1

Quantifying the shear



Define quadrupole moments of light distribution:

$$q_{ij} \equiv \int I_{\rm obs}(\theta)\theta_i\theta_j \mathrm{d}^2\theta$$

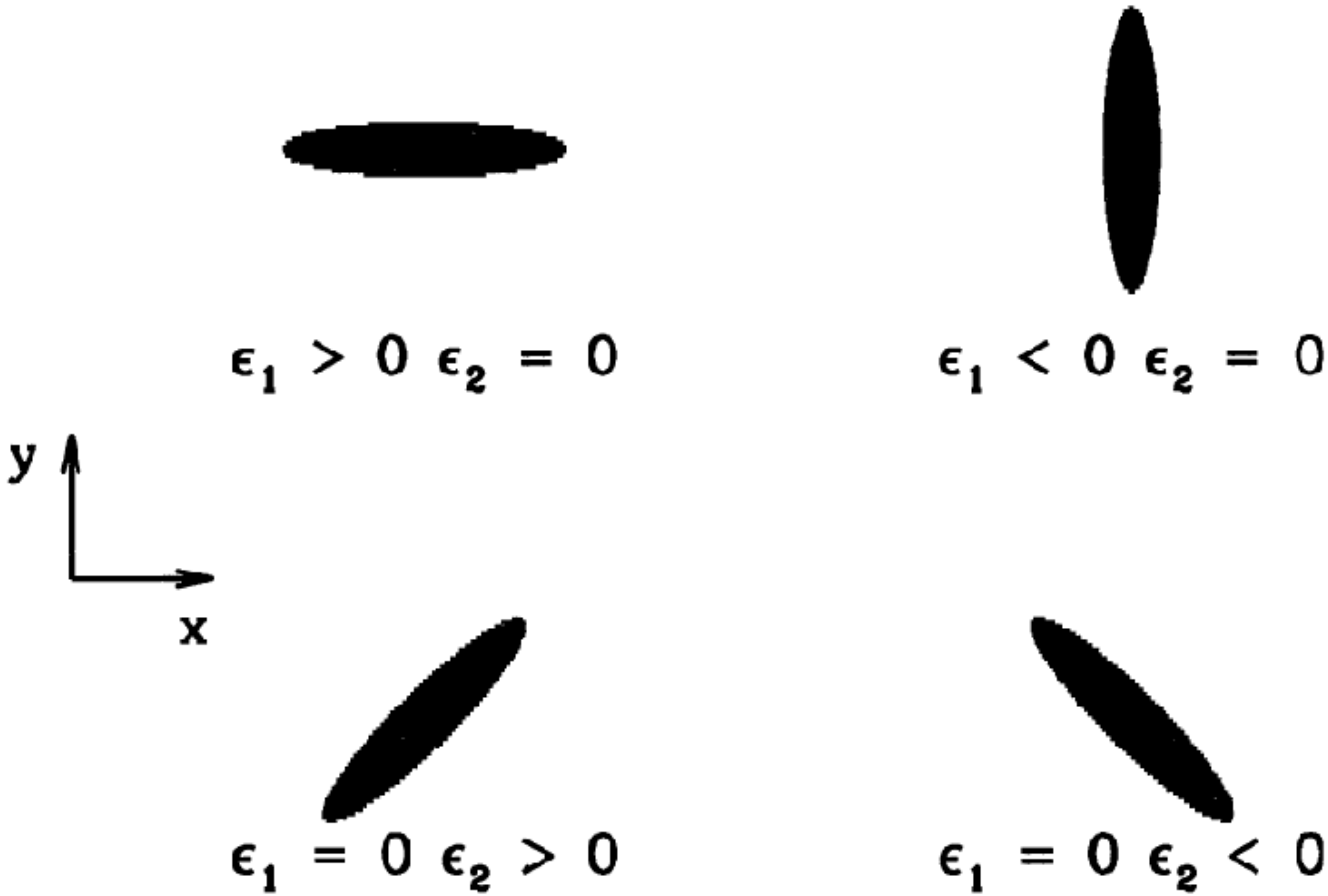
For a circularly symmetric source, we have

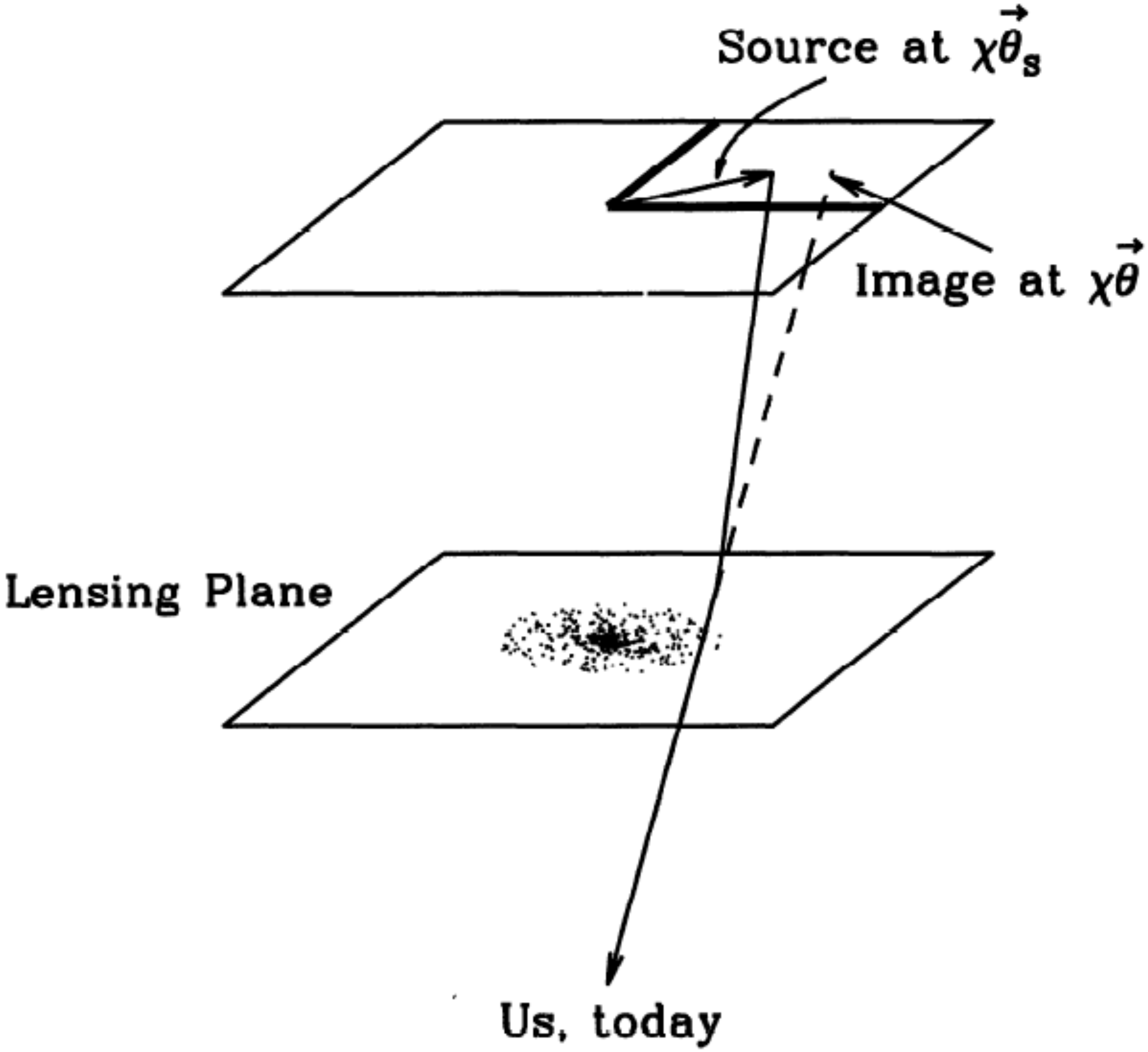
$$q_{xx} = q_{yy} \quad \text{and} \quad q_{xy} = 0$$

Define parameters ε_1 and ε_2:

$$\epsilon_1 \equiv \frac{q_{xx} - q_{yy}}{q_{xx} + q_{yy}} \qquad \epsilon_2 \equiv \frac{2q_{xy}}{q_{xx} + q_{yy}}$$

These will both be zero for symmetric sources


$\epsilon_1 > 0 \ \epsilon_2 = 0$
$\epsilon_1 < 0 \ \epsilon_2 = 0$
y
x
$\epsilon_1 = 0 \ \epsilon_2 > 0$
$\epsilon_1 = 0 \ \epsilon_2 < 0$



Note that the radial coordinate is here called χ (instead of *r*, or really *D*).

Coordinates in plane perpendicular to χ are $x^i = \chi\theta^i$.

Figure credit: Dodelson, *Modern Cosmology*

Geodesics and shear

To study the path of a light ray, we start again from the geodesic eqn:

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}\lambda^2} = -\Gamma^i{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda}$$

Here we are interested in the coordinates perpendicular to the line-of-sight,

$$x^i \equiv \chi\theta^i$$

We have $\frac{\mathrm{d}\chi}{\mathrm{d}\lambda} = \frac{\mathrm{d}\chi}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\lambda}.$ with $\frac{\mathrm{d}\chi}{\mathrm{d}t} = -1/a(t)$

(a light ray travels a co-moving distance (c)dt/a in a small time step dt)

We also have $\frac{\mathrm{d}t}{\mathrm{d}\lambda} = P^0 = p(1-\Psi)$

so $\frac{\mathrm{d}\chi}{\mathrm{d}\lambda} = -\frac{p}{a}(1-\Psi)$

Geodesics and shear

To study the path of a light ray, we start again from the geodesic eqn:

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}\lambda^2} = -\Gamma^i{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda} \qquad x^i \equiv \chi\theta^i$$

Using
$$\frac{\mathrm{d}\chi}{\mathrm{d}\lambda} = -\frac{p}{a}(1-\Psi)$$

we get, for the left-hand side of the geodesic equation:

$$\begin{aligned}\frac{\mathrm{d}^2 x^i}{\mathrm{d}\lambda^2} &= \frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}(\theta^i\chi)\\ &= \frac{p}{a}(1-\Psi)\frac{\mathrm{d}}{\mathrm{d}\chi}\frac{p}{a}(1-\Psi)\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)\end{aligned}$$

and, for small Ψ and θ,
$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}\lambda^2} \simeq \frac{p}{a}\frac{\mathrm{d}}{\mathrm{d}\chi}\frac{p}{a}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)$$

Geodesics and shear

Left-hand side of geodesic equation so far:

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}\lambda^2} \simeq \frac{p}{a}\frac{\mathrm{d}}{\mathrm{d}\chi}\frac{p}{a}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)$$

For photons, we have E=p ~a^{-1}, so *pa* ~ const:

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}\lambda^2} \simeq \frac{p}{a}\frac{\mathrm{d}}{\mathrm{d}\chi}\frac{pa}{a^2}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)$$

$$= p^2\frac{\mathrm{d}}{\mathrm{d}\chi}\left[\frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)\right]$$

Next, the right-hand side of the geodesic equation:

$$p^2\frac{\mathrm{d}}{\mathrm{d}\chi}\left[\frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)\right] = \Gamma^i{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda}$$

Geodesics and shear

Next, the right-hand side of the geodesic equation:

$$p^2 \frac{\mathrm{d}}{\mathrm{d}\chi}\left[\frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)\right] = \Gamma^i{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda}$$

$$= \Gamma^i{}_{\alpha\beta}\left(\frac{\mathrm{d}x^\alpha}{\mathrm{d}\chi}\frac{\mathrm{d}\chi}{\mathrm{d}\lambda}\right)\left(\frac{\mathrm{d}x^\beta}{\mathrm{d}\chi}\frac{\mathrm{d}\chi}{\mathrm{d}\lambda}\right)$$

$$= \left(\frac{p}{a}\right)^2(1-\Psi)^2\Gamma^i{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\chi}\frac{\mathrm{d}x^\beta}{\mathrm{d}\chi}$$

We have previously encountered the Christoffel symbols:

$$\Gamma^i{}_{00} = \frac{1}{a^2}\frac{\mathrm{d}\Psi}{\mathrm{d}x^i}$$

$$\Gamma^i{}_{j0} = \delta_{ij}\left[H + \frac{\mathrm{d}\Phi}{\mathrm{d}t}\right]$$

$$\Gamma^i{}_{jk} = \delta_{ij}\frac{\mathrm{d}\Phi}{\mathrm{d}x^k} + \delta_{ik}\frac{\mathrm{d}\Phi}{\mathrm{d}x^j} + \delta_{jk}\frac{\mathrm{d}\Phi}{\mathrm{d}x^i}$$

Geodesics and shear

Right-hand side (χ derivatives):

$$\left(\frac{p}{a}\right)^2 (1-\Psi)^2 \Gamma^i{}_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\chi} \frac{\mathrm{d}x^\beta}{\mathrm{d}\chi}$$

Looking at α=β=0:

using

$$\Gamma^i{}_{00} = \frac{1}{a^2} \frac{\mathrm{d}\Psi}{\mathrm{d}x^i}$$

we get (right hand-side, λ derivatives):

$$\Gamma^i{}_{00} \frac{\mathrm{d}x^0}{\mathrm{d}\lambda} \frac{\mathrm{d}x^0}{\mathrm{d}\lambda} = \frac{1}{a^2} \frac{\mathrm{d}\Psi}{\mathrm{d}x^i} (P^0)^2$$

$$= \frac{p^2}{a^2} \frac{\mathrm{d}\Psi}{\mathrm{d}x^i} (1-\Psi)^2$$

so

$$\left(\frac{p}{a}\right)^2 (1-\Psi)^2 \Gamma^i{}_{00} \frac{\mathrm{d}x^0}{\mathrm{d}\chi} \frac{\mathrm{d}x^0}{\mathrm{d}\chi} = \frac{p^2}{a^2} \frac{\mathrm{d}\Psi}{\mathrm{d}x^i} (1-\Psi)^2$$

or (for Ψ=-ϕ):

$$\Gamma^i{}_{00} \left(\frac{\mathrm{d}t}{\mathrm{d}\chi}\right)^2 = -\frac{\mathrm{d}\Phi}{\mathrm{d}x^i}$$

Geodesics and shear

Right-hand side (χ derivatives): $\left(\frac{p}{a}\right)^2 (1-\Psi)^2 \Gamma^i{}_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\chi} \frac{\mathrm{d}x^\beta}{\mathrm{d}\chi}$

We found
$$\Gamma^i{}_{00} \left(\frac{\mathrm{d}t}{\mathrm{d}\chi}\right)^2 = -\frac{\mathrm{d}\Phi}{\mathrm{d}x^i}$$

The other terms are
$$\Gamma^i{}_{0j} \frac{\mathrm{d}t}{\mathrm{d}\chi} \frac{\mathrm{d}x^j}{\mathrm{d}\chi} = -aH \frac{\mathrm{d}}{\mathrm{d}\chi} \left[\chi\theta^i\right]$$

$$\Gamma^i{}_{jk} \frac{\mathrm{d}x^j}{\mathrm{d}\chi} \frac{\mathrm{d}x^k}{\mathrm{d}\chi} = -\frac{\mathrm{d}\Phi}{\mathrm{d}x^i}$$

Collecting terms, combining with the left-hand side, and leaving out Ψ^2 terms:

$$\frac{\mathrm{d}}{\mathrm{d}\chi}\left[\frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)\right] = \frac{2}{a^2}\left[aH\frac{\mathrm{d}}{\mathrm{d}\chi}\left[\chi\theta^i\right] + \frac{\mathrm{d}\Phi}{\mathrm{d}x^i}\right]$$

Geodesics and shear

The geodesic equation so far:

$$\frac{\mathrm{d}}{\mathrm{d}\chi}\left[\frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)\right] = \frac{2}{a^2}\left[aH\frac{\mathrm{d}}{\mathrm{d}\chi}\left[\chi\theta^i\right] + \frac{\mathrm{d}\Phi}{\mathrm{d}x^i}\right]$$

The left-hand side expands to

$$\frac{\mathrm{d}}{\mathrm{d}\chi}\left[\frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi)\right] = (2H/a)\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi) + \frac{1}{a^2}\frac{\mathrm{d}^2}{\mathrm{d}\chi^2}(\theta^i\chi)$$

so the first terms on each side cancel, and we get

$$\boxed{\frac{\mathrm{d}^2}{\mathrm{d}\chi^2}(\theta^i\chi) = 2\frac{\mathrm{d}\Phi}{\mathrm{d}x^i}}$$

Note: if there is no potential gradient, then $\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi) = \frac{\mathrm{d}x^i}{\mathrm{d}\chi} = \mathrm{const}$

The light ray travels along a straight line, as it should.

Geodesics and shear

$$\boxed{\frac{\mathrm{d}^2}{\mathrm{d}\chi^2}(\theta^i\chi) = 2\frac{\mathrm{d}\Phi}{\mathrm{d}x^i}}$$

Now integrate (twice) over χ to find angle at source (θ_S):

$$\frac{\mathrm{d}}{\mathrm{d}\chi}(\theta^i\chi) = \frac{\mathrm{d}x^i}{\mathrm{d}\chi} = 2\int_0^{\chi}\frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i}\mathrm{d}\chi' + \mathrm{const}$$

$$\theta^i\chi = 2\int_0^{\chi}\mathrm{d}\chi''\int_0^{\chi''}\frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i}\mathrm{d}\chi' + \mathrm{const}$$

Outer integral: over all χ'' from observer (χ''=0) to source (χ''=χ).

Inner integral: over all χ' from observer (χ'=0) up to χ'=χ''.

Divide by χ:

$$\theta_S^i = \theta^i + \frac{2}{\chi}\int_0^{\chi}\mathrm{d}\chi''\int_0^{\chi''}\mathrm{d}\chi'\frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i}$$

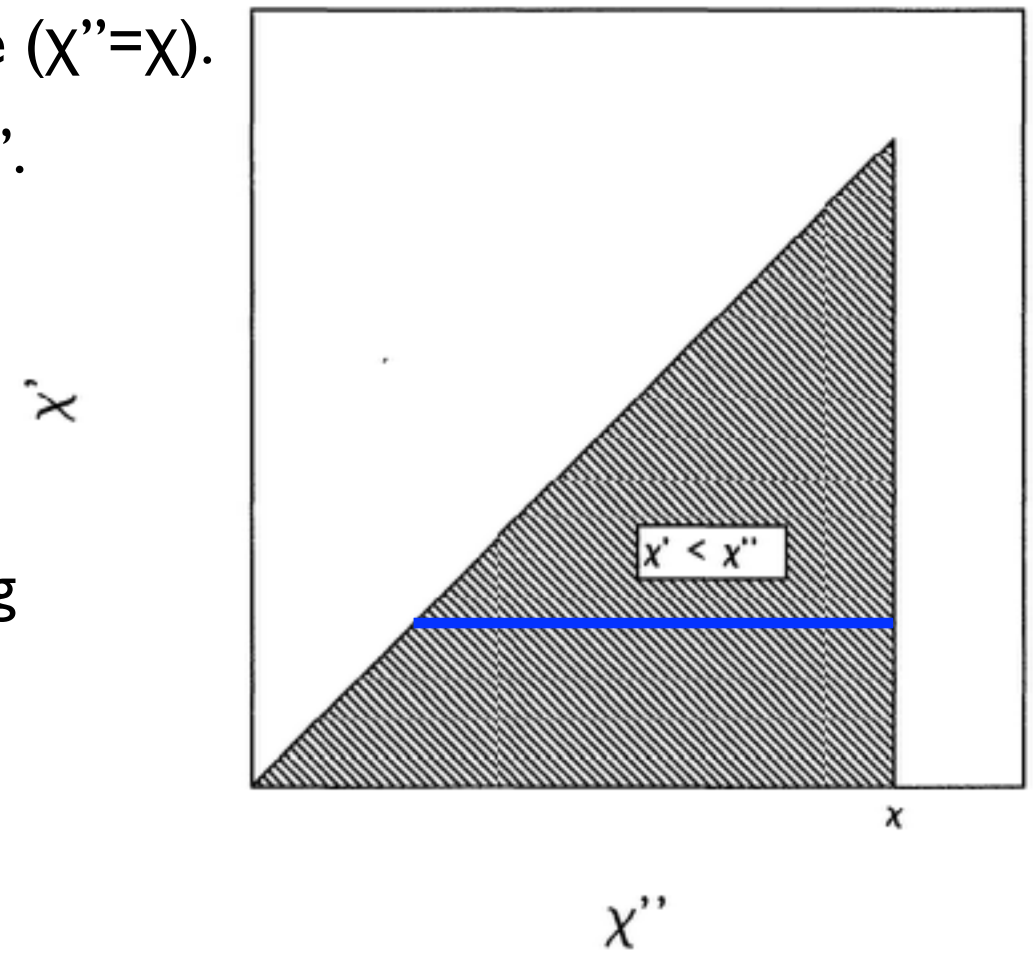
We see that the constant must be θ^i, since $\theta_S^i=\theta^i$ if there is no potential (gradient)

Geodesics and shear

Outer integral: over all χ'' from observer (χ''=0) to source (χ''=χ).

Inner integral: over all χ' from observer (χ'=0) up to χ'=χ''.

$$\theta_S^i = \theta^i + \frac{2}{\chi}\int_0^{\chi} \mathrm{d}\chi'' \int_0^{\chi''} \mathrm{d}\chi' \frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i}$$



We can change the order of integration, with χ'' then going from χ' to χ:

$$\begin{aligned}\theta_S^i &= \theta^i + \frac{2}{\chi}\int_0^{\chi} \mathrm{d}\chi' \int_{\chi'}^{\chi} \mathrm{d}\chi'' \frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i} \\ &= \theta^i + \frac{2}{\chi}\int_0^{\chi} \mathrm{d}\chi' \frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i}(\chi - \chi')\end{aligned}$$

or

$$\boxed{\theta_S^i = \theta^i + 2\int_0^{\chi} \mathrm{d}\chi' \frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i}\left(1 - \frac{\chi'}{\chi}\right)}$$

The transformation matrix

We now define the transformation matrix A_{ij}:

$$A_{ij} \equiv \frac{\partial \theta_S^i}{\partial \theta^j}$$

A maps angles at the *source* to those seen by the *observer*.

Usually written as

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

with *convergence* κ (describing the magnification) and *shear* (γ_1 ,γ_2), describing the distortion.

The shear parameters are thus

$$\gamma_1 = -\frac{A_{11}-A_{22}}{2} = -\frac{1-\kappa-\gamma_1-(1-\kappa+\gamma_1)}{2}$$

$$\gamma_2 = -A_{12}$$

The transformation matrix

Shear parameters:

$$\gamma_1 = -\frac{A_{11} - A_{22}}{2}$$

$$\gamma_2 = -A_{12}$$

To determine A_{ij} we need the derivatives $\frac{\mathrm{d}\theta_S^i}{\mathrm{d}\theta^j}$

Since $\theta_S^i = \theta^i + 2\int_0^\chi \mathrm{d}\chi' \frac{\mathrm{d}\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i}\left(1 - \frac{\chi'}{\chi}\right)$

these depend on the second derivatives of the potential,

$$\frac{\mathrm{d}}{\mathrm{d}\theta^j}\frac{\mathrm{d}\Phi(\vec{x})}{\mathrm{d}x^i} = \frac{\mathrm{d}x^j}{\mathrm{d}\theta^j}\frac{\mathrm{d}}{\mathrm{d}x^j}\frac{\mathrm{d}\Phi(\vec{x})}{\mathrm{d}x^i} = \frac{\mathrm{d}^2\Phi(\vec{x})}{\mathrm{d}x^i\mathrm{d}x^j}\chi$$

so

$$A_{ij} = \frac{\partial\theta_S^i}{\partial\theta^j} = \delta_{ij} + 2\int_0^\chi \mathrm{d}\chi' \frac{\mathrm{d}^2\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i\mathrm{d}x^j}\chi'\left(1 - \frac{\chi'}{\chi}\right)$$

The transformation matrix

Shear parameters: $\gamma_1 = -\frac{A_{11} - A_{22}}{2}$ $\gamma_2 = -A_{12}$

with

$$A_{ij} = \frac{\partial \theta_S^i}{\partial \theta^j} = \delta_{ij} + 2\int_0^\chi \mathrm{d}\chi' \frac{\mathrm{d}^2\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i \mathrm{d}x^j} \chi' \left(1 - \frac{\chi'}{\chi}\right)$$

Can also be written as

ψ = distortion tensor

$$\begin{aligned} A_{ij} - \delta_{ij} &\equiv \psi \\ &= \begin{pmatrix} -\kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa + \gamma_1 \end{pmatrix} \end{aligned}$$

$$\boxed{\psi = 2\int_0^\chi \mathrm{d}\chi' \frac{\mathrm{d}^2\Phi(\vec{x}(\chi'))}{\mathrm{d}x^i \mathrm{d}x^j} \chi' \left(1 - \frac{\chi'}{\chi}\right)}$$

We can now calculate the shear parameters, given the potential (variations) along the line-of-sight.

Ellipticity as an estimator of shear

Still to do: connect the theoretical shear parameters (γ_1 ,γ_2) to the observed ellipticities (ε_1 and ε_2).

We start from the definitions:

$$\epsilon_1 = \frac{q_{xx} - q_{yy}}{q_{xx} - q_{yy}}$$

Filling in the definitions of the *q*'s, we get

$$\epsilon_1 = \frac{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\theta_x\theta_x - \int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\theta_y\theta_y}{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S\theta_x\theta_x + \int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\theta_y\theta_y}$$

$$= \frac{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\left[\theta_x\theta_x - \theta_y\theta_y\right]}{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S\left[\theta_x\theta_x + \theta_y\theta_y\right]}$$

- but mix of angles in the source and observer planes

Ellipticity as an estimator of shear

$$\epsilon_1 = \frac{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\,[\theta_x\theta_x - \theta_y\theta_y]}{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\,[\theta_x\theta_x + \theta_y\theta_y]}$$

For small angles, we can write

$$\theta_S^x = \frac{\partial\theta_S^x}{\partial\theta^x}\theta^x + \frac{\partial\theta_S^x}{\partial\theta^y}\theta^y$$

$$\theta_S^y = \frac{\partial\theta_S^y}{\partial\theta^x}\theta^x + \frac{\partial\theta_S^y}{\partial\theta^y}\theta^y$$

In matrix form,

$$\vec{\theta}_S = \mathbf{A}\vec{\theta}$$
$$\vec{\theta} = \mathbf{A}^{-1}\vec{\theta}_S$$

so

$$\theta_x = (A^{-1})_{xx}\theta_x^S + (A^{-1})_{yx}\theta_y^S$$

$$\theta_y = (A^{-1})_{xy}\theta_x^S + (A^{-1})_{yy}\theta_y^S$$

Ellipticity as an estimator of shear

$$\epsilon_1 = \frac{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\, [\theta_x\theta_x - \theta_y\theta_y]}{\int \mathrm{d}^2\vec{\theta}_{\mathrm{obs}}\, I_{\mathrm{true}}(\vec{\theta}_S)\, [\theta_x\theta_x + \theta_y\theta_y]}$$

Filling in

$$\theta_x = (A^{-1})_{xx}\theta_x^S + (A^{-1})_{yx}\theta_y^S$$

$$\theta_y = (A^{-1})_{xy}\theta_x^S + (A^{-1})_{yy}\theta_y^S$$

one gets

$$\epsilon_1 = \frac{\sum_{ij} \left[(A^{-1})_{xi}(A^{-1})_{xj} - (A^{-1})_{yi}(A^{-1})_{yj}\right] \int \mathrm{d}^2\vec{\theta}_{\mathrm{S}}\, I_{\mathrm{true}}(\vec{\theta}_S)\theta_S^i\theta_S^j}{\sum_{ij} \left[(A^{-1})_{xi}(A^{-1})_{xj} + (A^{-1})_{yi}(A^{-1})_{yj}\right] \int \mathrm{d}^2\vec{\theta}_{\mathrm{S}}\, I_{\mathrm{true}}(\vec{\theta}_S)\theta_S^i\theta_S^j}$$

For a circular source, the integrals are non-zero only for *i*=*j* (and cancel out) so this reduces to

$$\epsilon_1 = \frac{(A^{-1})^2_{xx} - (A^{-1})^2_{yy}}{(A^{-1})^2_{xx} + 2(A^{-1})^2_{xy} + (A^{-1})^2_{yy}}$$

We already see that this depends only on A.

Ellipticity as an estimator of shear

$$\epsilon_1 = \frac{(A^{-1})^2_{xx} - (A^{-1})^2_{yy}}{(A^{-1})^2_{xx} + 2(A^{-1})^2_{xy} + (A^{-1})^2_{yy}}$$

In general, the inverse of a 2x2 matrix *A* is

$$A^{-1} = \begin{pmatrix} A_{xx} & A_{yx} \\ A_{xy} & A_{yy} \end{pmatrix}^{-1} = \frac{1}{A_{xx}A_{yy} - A_{xy}A_{yx}} \begin{pmatrix} A_{yy} & -A_{yx} \\ -A_{xy} & A_{xx} \end{pmatrix}$$

The factor in front cancels out, so we only need

$$A^{-1} \propto \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix}$$

Filling this in above gives

$$\epsilon_1 = \frac{2\gamma_1(1-\kappa)}{(1-\kappa)^2 + \gamma_1^2 + \gamma_2^2}$$

and, for small distortions,

$$\boxed{\epsilon_1 \simeq 2\gamma_1}$$

(a similar relation exists for ϵ_2 and γ_2)

Weak lensing - summary

- Measurements of galaxy shapes (corrected for instrumental effects) yield the *ellipticity* parameters ε_1 and ε_2
- These are (very!) closely related (on average) to the *shear* parameters, γ_1 and γ_2
- The shear parameters depend on variations in the potential along the line-of-sight (really along the slightly curved light ray)

Weak lensing - applications

- Mapping of mass in clusters (e.g. bullet cluster) - independent of assumptions about virial equilibrium, x-ray properties, etc.
- Statistical studies of large-scale structure - still in its infancy, but several upcoming surveys (e.g. *Euclid*, to be launched by ESA in 2019)
- Weak lensing measurements are sensitive to intrinsic *galaxy alignment* - a whole research discipline by itself.

Weak lensing and large scale structure

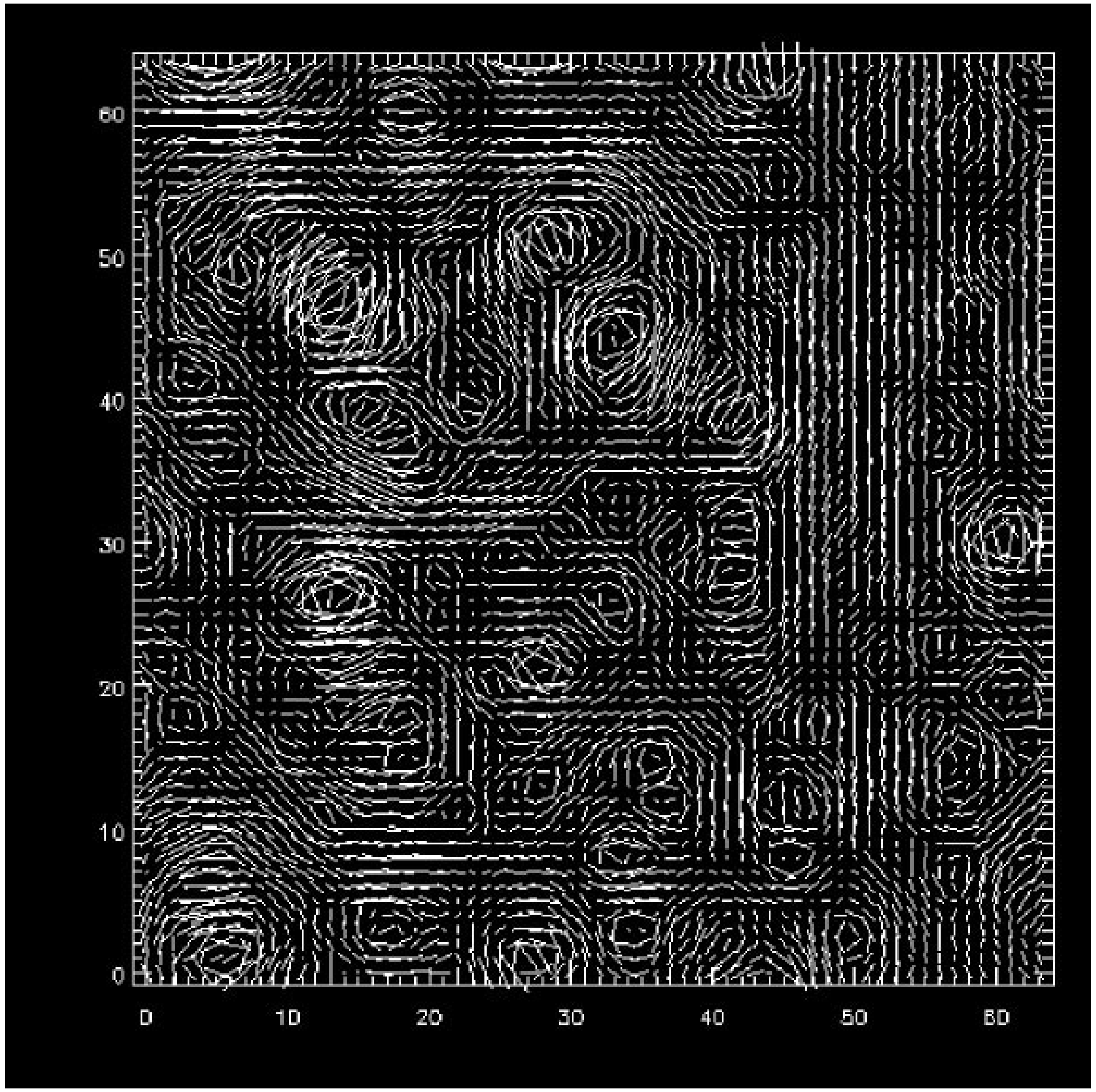


Figure 3 Shear map derived by ray-tracing simulations by Jain, Seljak & White (2000). The size and direction of each line gives the amplitude and position angle of the shear at this location on the sky. The displayed region is $1^\circ \times 1^\circ$ for an SCDM (Einstein-De Sitter) model. Tangential patterns about the overdensities corresponding to clusters and groups of galaxies are apparent. A more complex network of patterns is also visible outside of these structures. The root-mean-square shear is approximately 2% in this map (from Jain et al. 2000).

The only meaningful model-data comparison is a *statistical* one.

Various shear statistics are being used.

For example, power-spectrum C_l:

$$\sum_{i=1}^{2} \langle \tilde{\gamma}_i(\vec{l}) \tilde{\gamma}_i(\vec{l'}) \rangle = (2\pi)^2 \delta(\vec{l} - \vec{l'}) C_l$$

- here, the Fourier components of the shear are being used

Refregier (2003)

Weak lensing and large scale structure

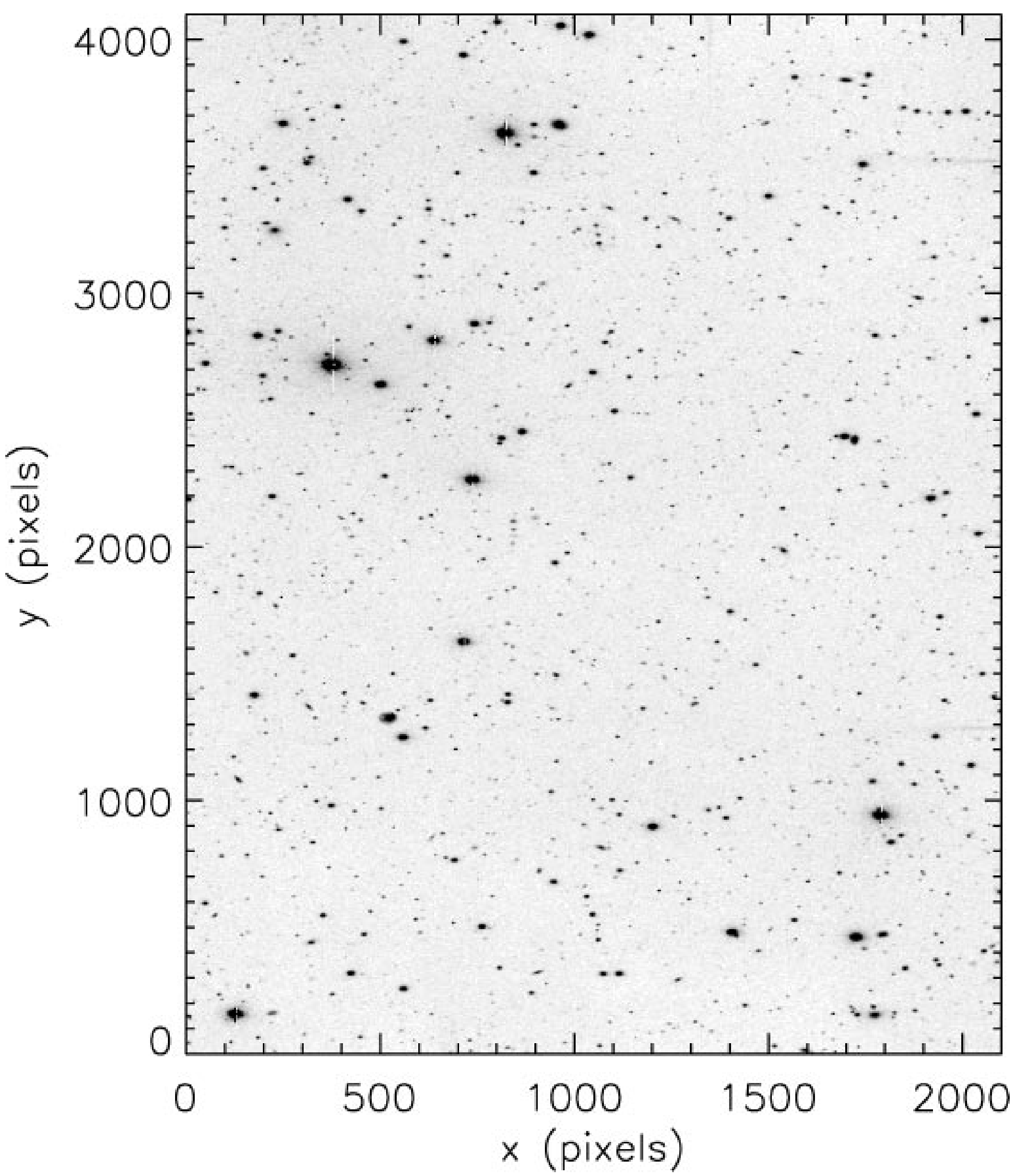


Figure 5 Example of an deep image in the cosmic-shear survey by Bacon, Refregier & Ellis (2000). This corresponds to a 1 h exposure with the EEV camera on the William Herschel telescope (WHT). The field of view is $8' \times 16'$ and achieves a magnitude depth of $R \simeq 26$ (5σ detection). The bright objects are saturated stars. The faint objects comprise approximately 200 stars and approximately 2000 galaxies that are usable for the weak-lensing analysis (from Bacon, Refregier & Ellis 2000).

Refregier (2003)

Weak lensing and large scale structure

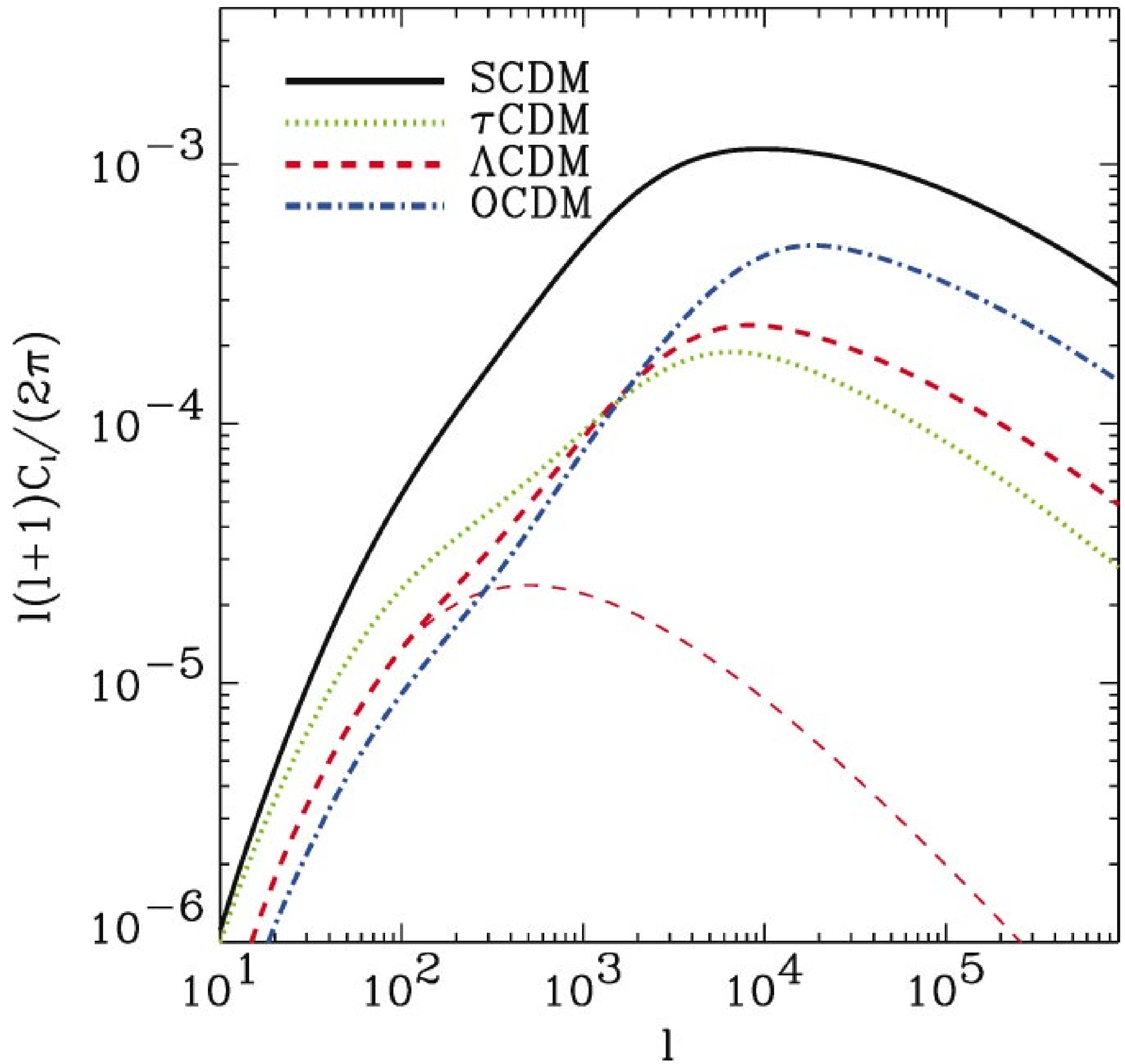


Figure 4 Shear power spectrum for different cosmological models and for source galaxies at $z_s = 1$. The SCDM model is COBE normalized and thus has a higher amplitude than the three cluster-normalized models ΛCDM, OCDM, and τCDM. The thin dashed line shows the ΛCDM spectrum for linear evolution of structures. Notice that for $l > 1000$ (corresponding approximately to angular scales $\theta < 10'$) the lensing power spectrum is dominated by nonlinear structures.

Different cosmological models predict different power spectra.

$$\sum_{i=1}^{2} \langle \tilde{\gamma}_i(\vec{l}) \tilde{\gamma}_i(\vec{l'}) \rangle = (2\pi)^2 \delta(\vec{l} - \vec{l'}) C_l$$

Refregier (2003)

Results - 2PCF

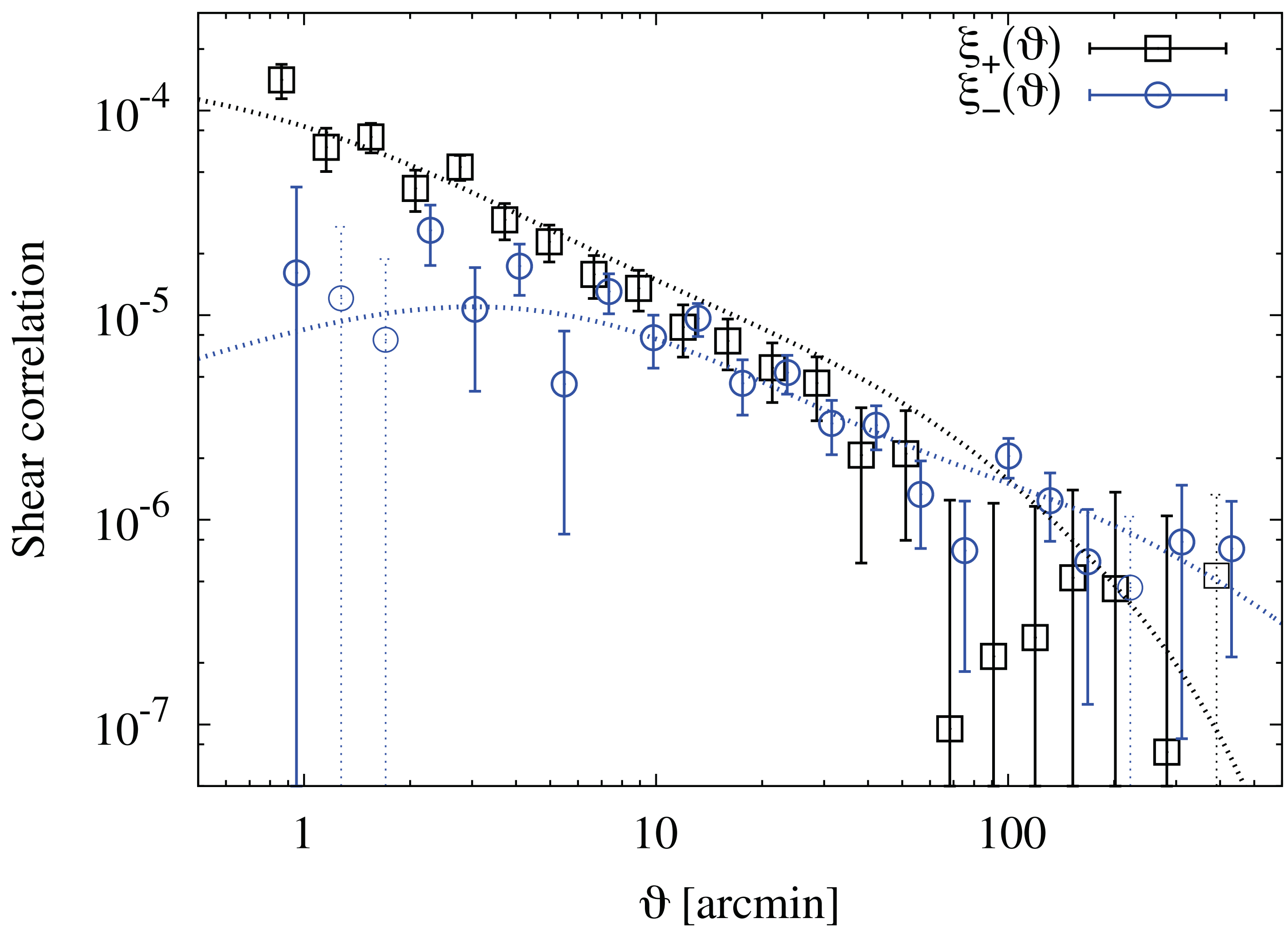


Figure 4. 2PCF components ξ_+ and ξ_- (32) measured in CFHTLenS. The dotted lines show the WMAP7 model prediction (Komatsu *et al* 2011). From Kilbinger *et al* (2013). © 2013 Oxford Universiy Press.

Complex shear:

$$\gamma \equiv \gamma_1 + i\gamma_2$$

2-point correlation functions:

$$\xi_+(\theta) = \langle \gamma\gamma^* \rangle(\theta)$$

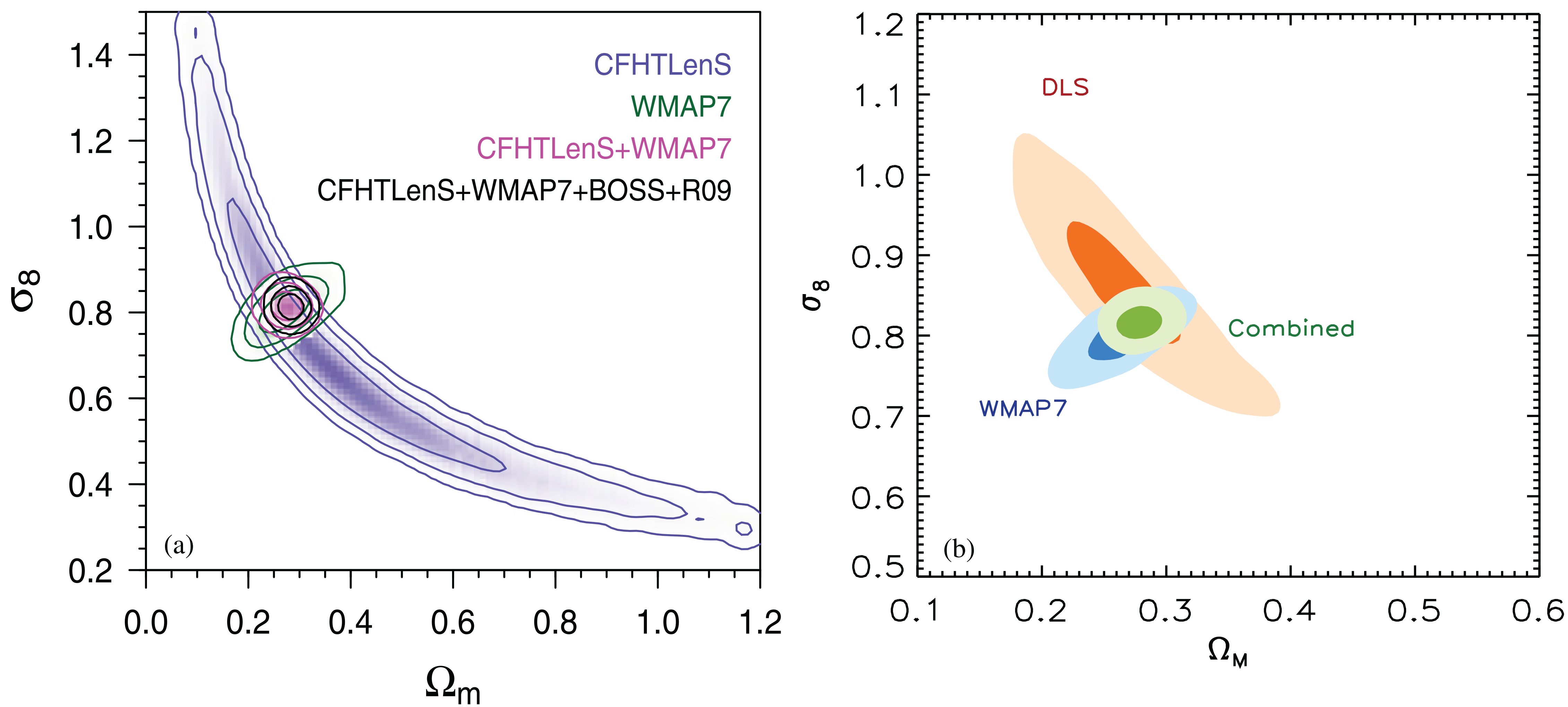
$$\xi_-(\theta) = \mathcal{R}\left[\langle \gamma\gamma \rangle(\theta) e^{-4i\phi}\right]$$

for galaxy pairs separated by an angle θ at orientation ϕ

Kilbinger 2015, Rep. Prog. Phys. 78

Results - Ω_M and σ_8

Flat ΛCDM



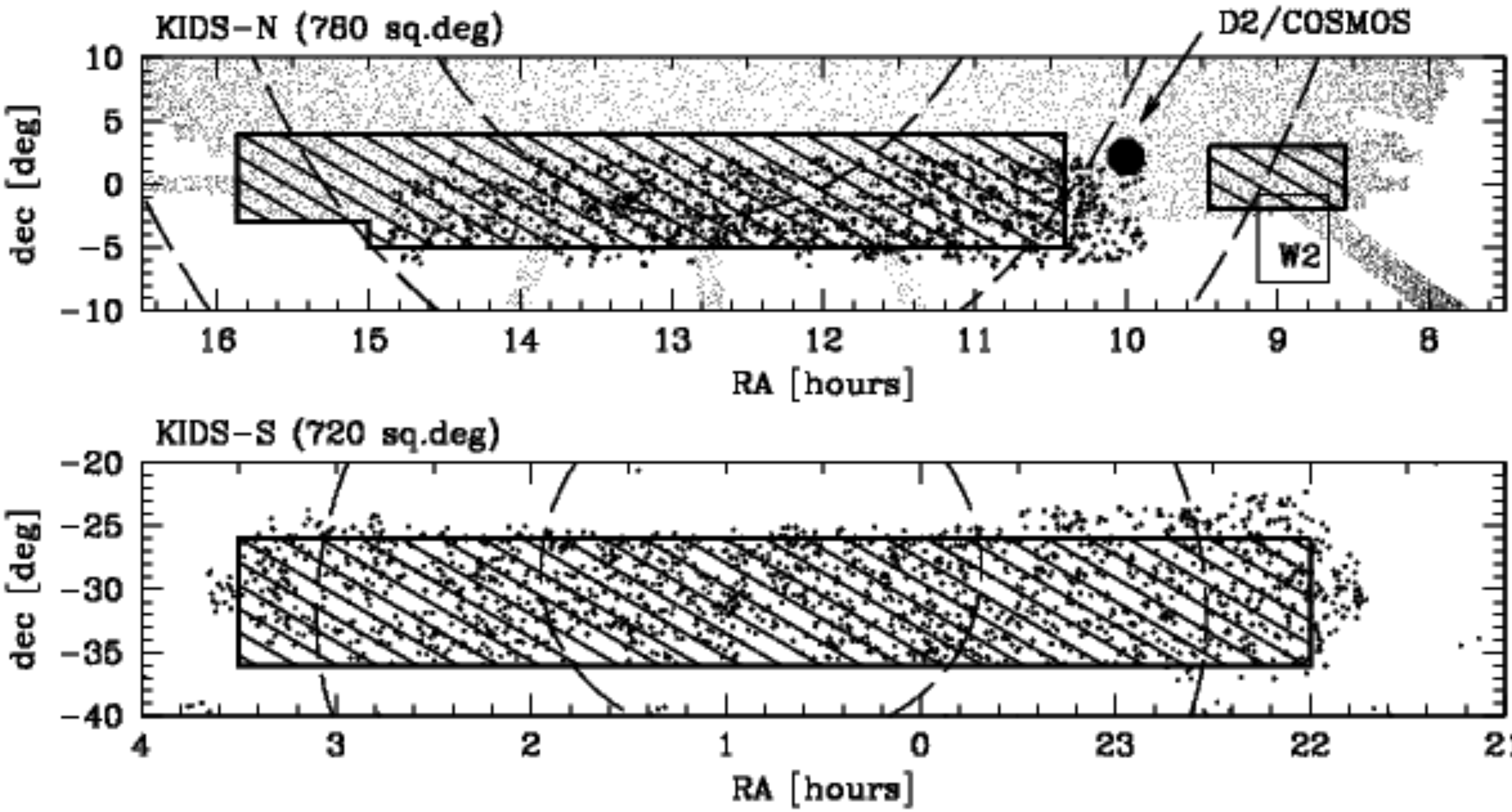
σ_8 = amplitude of density perturbations at scale of 8 Mpc
Ω_M = Matter density

Kilbinger 2015, Rep. Prog. Phys. 78

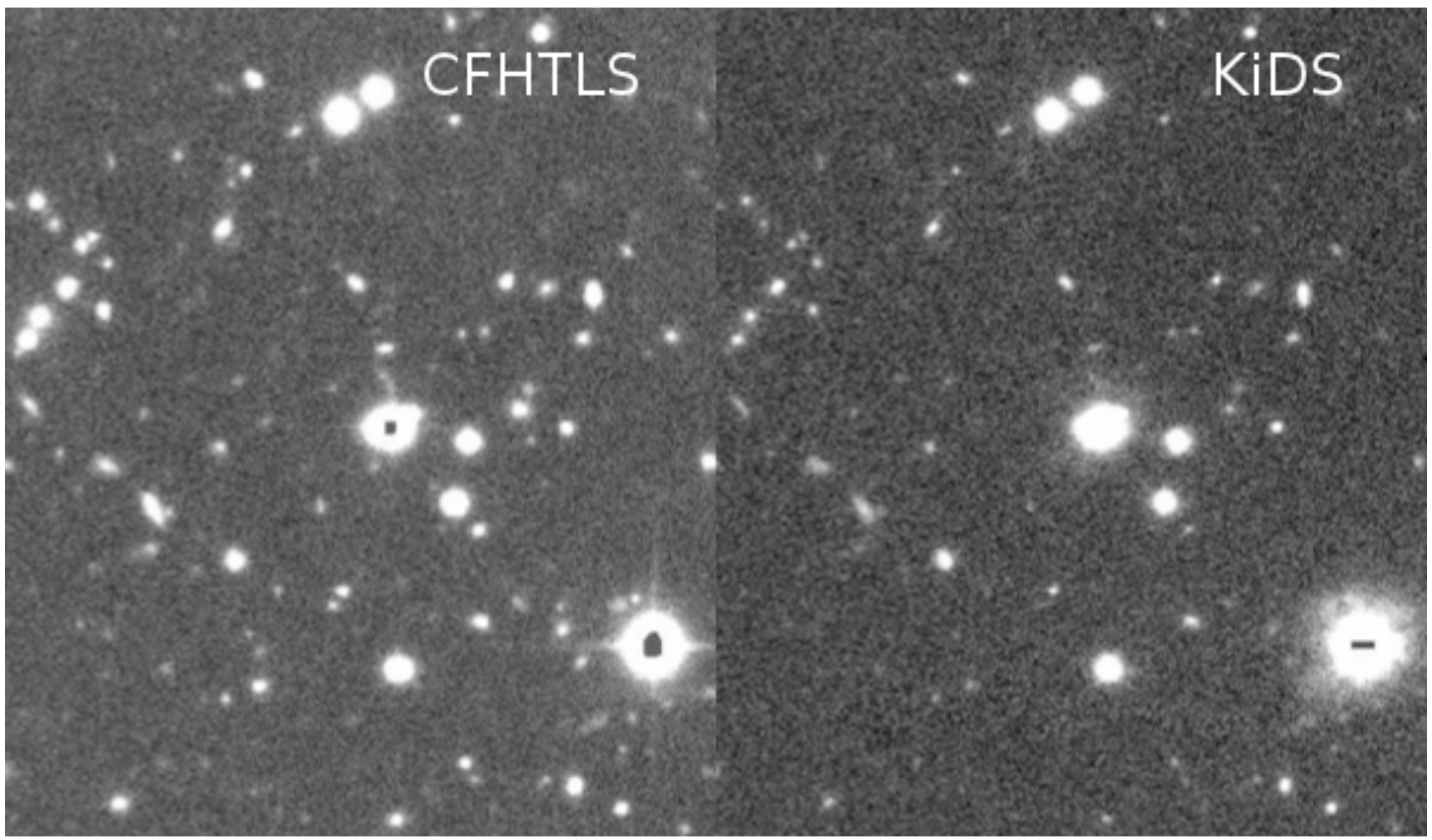
KiDS

Kilo-Degree Survey:

- Survey of 1500 deg^2 to measure weak lensing
- OmegaCam on *VLT Survey Telescope* (VST)

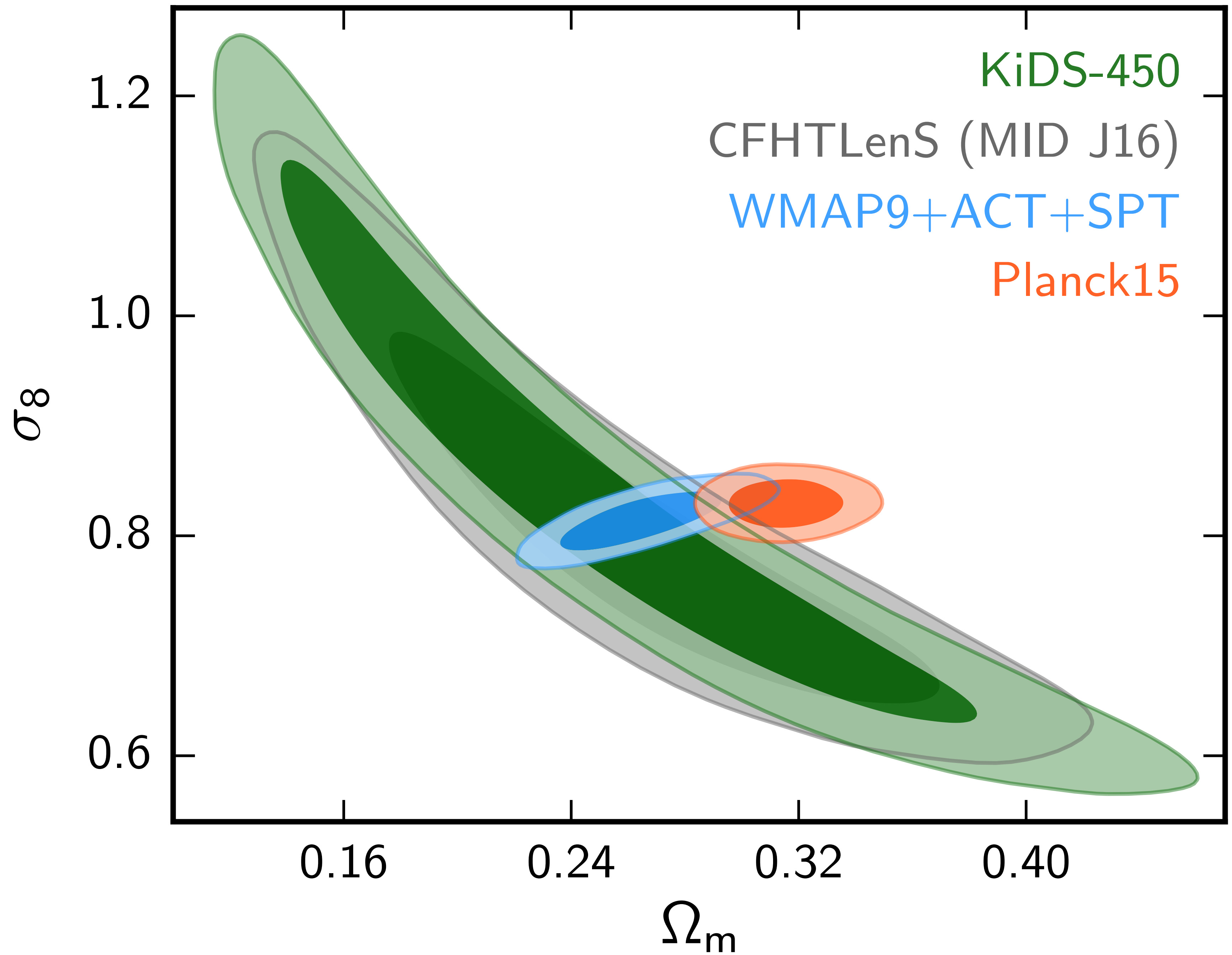

KIDS-N (760 sq.deg)
D2/COSMOS
W2
dec [deg]
RA [hours]
KIDS-S (720 sq.deg)
dec [deg]
RA [hours]

KiDS image quality



Better than 1”

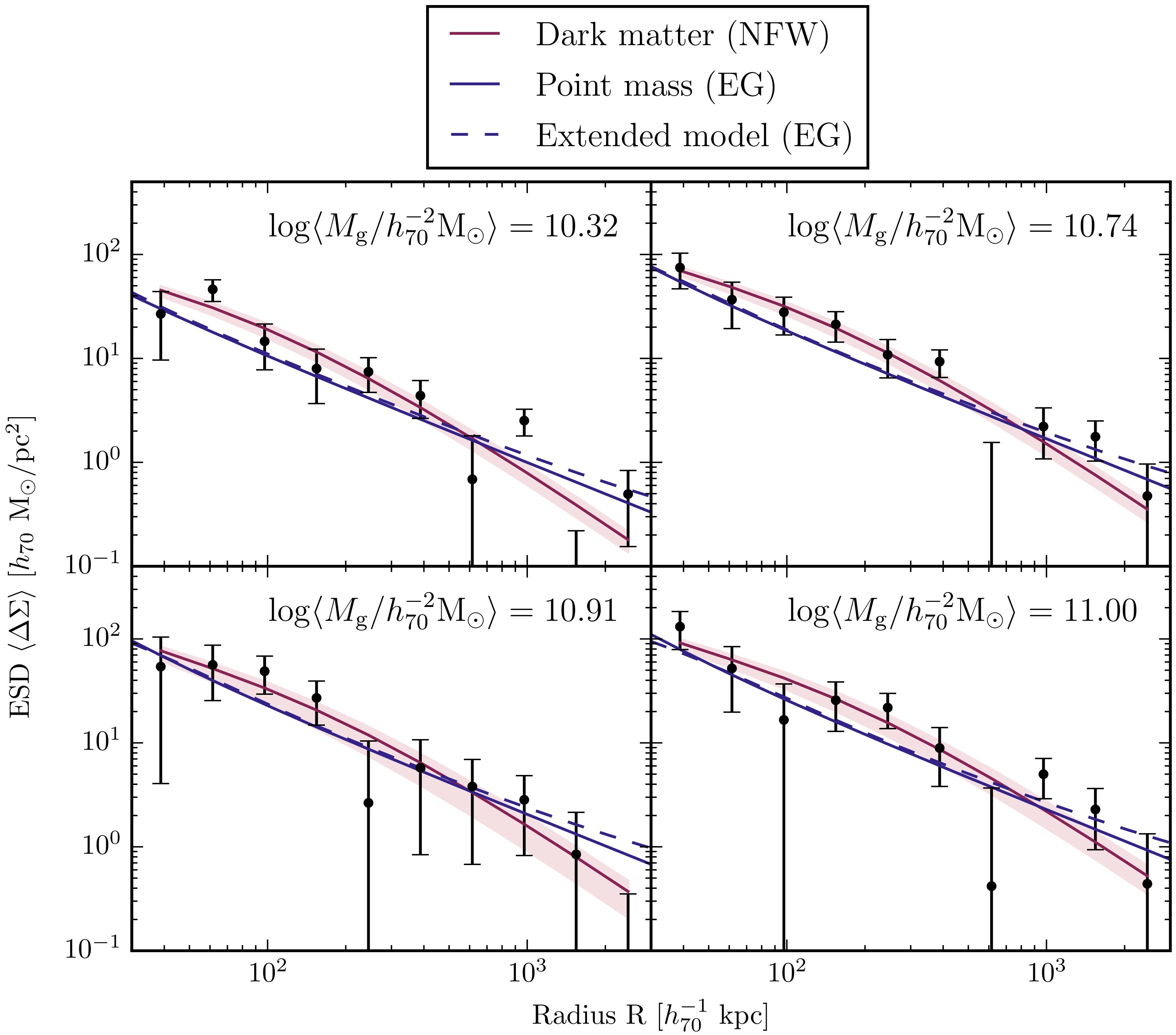
KiDS vs. other surveys



Tension w. Planck results

Hildebrandt et al. (2015)

KiDS & Emergent Gravity



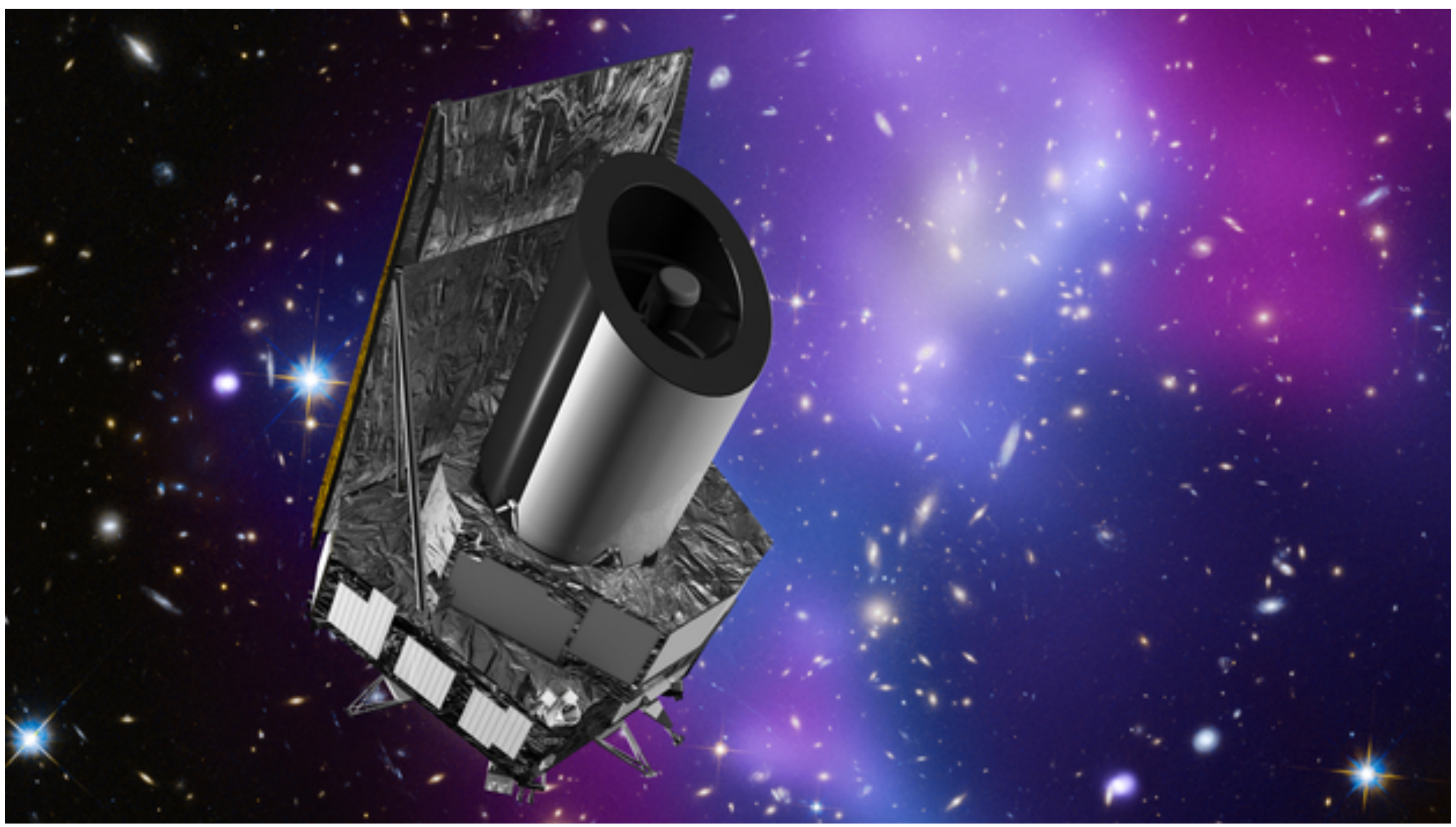
Mass profiles for early-type galaxies apparently (also) consistent with Verlinde's emergent gravity theory.

Brouwer et al. (2016)

Euclid

Science goals:

- Use *weak lensing* to map distribution of dark matter and constrain nature of dark energy (e.g. *w*, equation of state), test GR vs. alternative theories of gravity, etc.
- BAOs (baryonic acoustic oscillations) - "wiggles" in the power-spectrum in the distribution of baryonic matter



- Launch ~Q4 2020
- 1.2 m primary mirror → 0.2" resolution
- Image ~15000 deg^2 (~1/3 of the sky)
- ~ 10^9 galaxies to z~2