Acoustic oscillations

So much for the general picture. Now let's look at the details!

The tightly coupled limit:

Before recombination (η^*), mean free path for a photon was much smaller than horizon.

Define *optical depth* as integral of $n_e \sigma_T a$ over (conformal) time:

$$\tau(\eta) \equiv \int_{\eta}^{\eta_0} \mathrm{d}\eta' n_e \sigma_T a$$

with derivative:

$$\frac{\mathrm{d}\tau}{\mathrm{d}\eta} \equiv \dot{\tau} = -n_e \sigma_T a$$

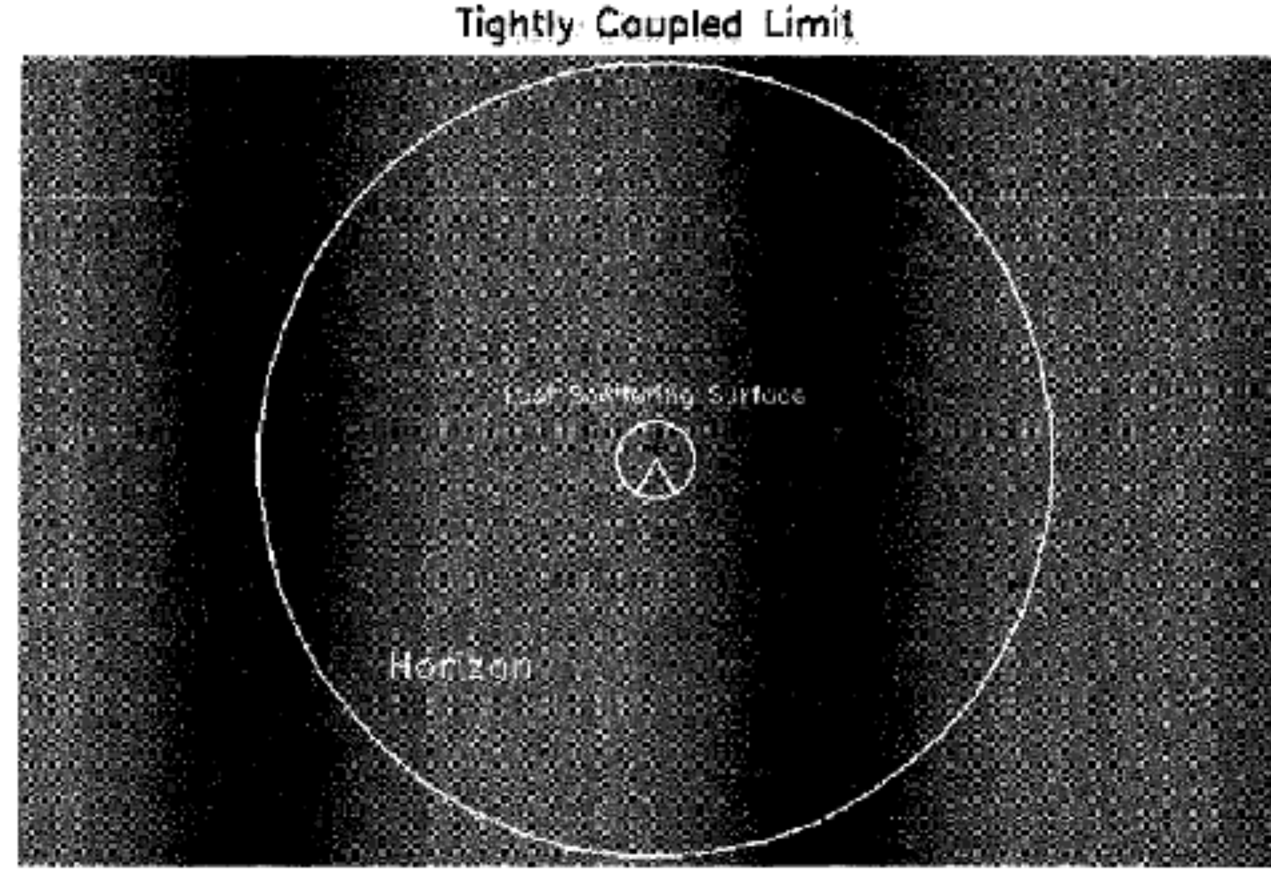
Tightly coupled limit corresponds to $\tau \gg 1$.

Acoustic oscillations

The tightly coupled limit:

$$\tau(\eta) \equiv \int_{\eta}^{\eta_0} \mathrm{d}\eta' n_e \sigma_T a$$

Tightly coupled limit corresponds to $\tau >> 1$ (last scattering surface much smaller than horizon)



Higher-order moments of radiation field are then negligible: Θ “looks the same in every direction”, apart from spatial and velocity dependencies. We only need to consider
[Θ_0(**x**, t)] - Monopole
[Θ_1(**x**, t)] - Dipole

(see Sect. 8.3.1 for formal derivation).

Multipole moments

Define the lth multipole moment of the temperature perturbations Θ as

$$\Theta_l \equiv \frac{1}{(-i)^l} \int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \mathcal{P}_l(\mu) \Theta(\mu)$$

The first three Legendre polynomials are defined as:

$$\mathcal{P}_0(\mu) = 1 \longrightarrow \Theta_0 = \int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \Theta(\mu)$$

$$\mathcal{P}_1(\mu) = \mu \longrightarrow \Theta_1 = \frac{i}{2} \int_{-1}^{1} \mathrm{d}\mu \, \mu \, \Theta(\mu)$$

$$\mathcal{P}_2(\mu) = \frac{3\mu^2 - 1}{2}$$

Acoustic oscillations

$$\Theta_l \equiv \frac{1}{(-i)^l} \int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \mathcal{P}_l(\mu) \Theta(\mu)$$

$$\mathcal{P}_0(\mu) = 1 \qquad \mathcal{P}_1(\mu) = \mu$$

The Boltzmann equation for photons in the tightly coupled limit:

General version:

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi + n_e \sigma_T a \left[\Theta_0 - \Theta + \mu \mathbf{v}_b\right]$$

Next: obtain two new equations by multiplying by P_0 and P_1 and integrating over all μ, dropping higher-order moments.

P_0, left-hand side:

$$\int \mathrm{d}\mu \, \dot{\Theta} + ik\mu\Theta = 2\dot{\Theta}_0 + ik \int_{-1}^{1} \mathrm{d}\mu \, \mu\Theta(\mu)$$

$$= 2\dot{\Theta}_0 + ik(-2i)\Theta_1$$

$$= 2\dot{\Theta}_0 + 2k\Theta_1$$

Acoustic oscillations

$$\Theta_l \equiv \frac{1}{(-i)^l} \int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \mathcal{P}_l(\mu)\Theta(\mu)$$

$$\mathcal{P}_0(\mu) = 1 \qquad \mathcal{P}_1(\mu) = \mu$$

The Boltzmann equation for photons in the tightly coupled limit:

General version:

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi + n_e\sigma_T a\left[\Theta_0 - \Theta + \mu\mathbf{v}_b\right]$$

P_0, left-hand side: $= 2\dot{\Theta}_0 + 2k\Theta_1$

P_0, right-hand side:

$$\int_{-1}^{1} \mathrm{d}\mu \left(-\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu\mathbf{v}_b\right]\right) = -\int_{-1}^{1} \mathrm{d}\mu\dot{\Phi} - ik\int_{-1}^{1} \mathrm{d}\mu\mu\Psi - \dot{\tau}\left[\int_{-1}^{1} \mathrm{d}\mu(\Theta_0 - \Theta) + v_b\int_{-1}^{1} \mathrm{d}\mu\mu\right]$$

$$-2\dot{\Phi} \qquad 0 \qquad -2\dot{\tau}\Theta_0 \qquad 2\dot{\tau}\Theta_0 \qquad 0$$

Equating l.h. and r.h. sides:

$$\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}$$

Acoustic oscillations

$$\Theta_l \equiv \frac{1}{(-i)^l}\int_{-1}^{1}\frac{\mathrm{d}\mu}{2}\mathcal{P}_l(\mu)\Theta(\mu)$$

$$\mathcal{P}_0(\mu) = 1 \qquad \mathcal{P}_1(\mu) = \mu$$

The Boltzmann equation for photons in the tightly coupled limit:

General version:

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi + n_e\sigma_T a\left[\Theta_0 - \Theta + \mu\mathbf{v}_b\right]$$

P_1, left-hand side:

$$\int_{-1}^{1}\mathrm{d}\mu\,\mu\left(\dot{\Theta} + ik\mu\Theta\right) = -2i\dot{\Theta}_1 + ik\int_{-1}^{1}\mathrm{d}\mu\mu^2\Theta$$

$$= -2i\dot{\Theta}_1 + 2ik\left(\frac{1}{3}\Theta_0 - \frac{2}{3}\Theta_2\right)$$

Exercise

Acoustic oscillations

$$\Theta_l \equiv \frac{1}{(-i)^l} \int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \mathcal{P}_l(\mu)\Theta(\mu)$$

$$\mathcal{P}_0(\mu) = 1 \qquad \mathcal{P}_1(\mu) = \mu$$

The Boltzmann equation for photons in the tightly coupled limit:

General version:

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi + n_e \sigma_T a \left[\Theta_0 - \Theta + \mu \mathbf{v}_b\right]$$

P_1, right-hand side:

$$\int_{-1}^{1} \mathrm{d}\mu\, \mu \left(-\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu \mathbf{v}_b\right]\right) =$$

$$- \int_{-1}^{1} \mathrm{d}\mu\mu\dot{\Phi} - ik \int_{-1}^{1} \mathrm{d}\mu\mu^2\Psi - \dot{\tau}\left[\int_{-1}^{1} \mathrm{d}\mu\mu(\Theta_0 - \Theta) + v_b \int_{-1}^{1} \mathrm{d}\mu\mu^2\right]$$

$$0 \qquad -\frac{2}{3}ik\Psi \qquad 0 \qquad -2\dot{\tau}i\Theta_1 \qquad -\frac{2}{3}\dot{\tau}v_b$$

$$= -\frac{2}{3}ik\Psi - \dot{\tau}\left[2i\Theta_1 + \frac{2}{3}v_b\right]$$

Acoustic oscillations

The Boltzmann equation for photons in the tightly coupled limit:

General version:

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi + n_e\sigma_T a\left[\Theta_0 - \Theta + \mu\mathbf{v}_b\right]$$

Equating l.h. and r.h. sides:

$$-2i\dot{\Theta}_1 + 2ik\left(\frac{1}{3}\Theta_0 - \frac{2}{3}\Theta_2\right) = -\frac{2}{3}ik\Psi - \dot{\tau}\left[2i\Theta_1 + \frac{2}{3}v_b\right]$$

Dividing by 2*i* and dropping the Θ_2 term:

$$\boxed{\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{iv_b}{3}\right]}$$

Acoustic oscillations

Moments of the Boltzmann equation for photons:

$$\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}$$

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{iv_b}{3}\right]$$

Two equations for Θ_0 and Θ_1 and their derivatives.
We would like to have a single equation for each multipole (and eliminate v_b).

For v_b, invoke the B.E. for baryons:

The Boltzmann equations

- **Photons:**

$$\dot{\tilde{\Theta}} + ik\mu\tilde{\Theta} + \dot{\tilde{\Phi}} + ik\mu\tilde{\Psi} = n_e\sigma_T a\left[\tilde{\Theta}_0 - \tilde{\Theta} + \mu\mathbf{v}_b\right]$$

- **(Cold) dark matter**: no collision terms; particles are non-relativistic.

Density fluctuations:

$$\dot{\tilde{\delta}} + ik\tilde{v} + 3\dot{\tilde{\Phi}} = 0$$

Velocity field:

$$\dot{\tilde{v}} + \frac{\dot{a}}{a}\tilde{v} + ik\tilde{\Psi} = 0$$

- **Baryons**: Collision terms from Coulomb scattering;

$$\dot{\tilde{\delta}}_b + ik\tilde{v}_b + 3\dot{\tilde{\Phi}} = 0$$

$$\boxed{\dot{\tilde{v}}_b + \frac{\dot{a}}{a}\tilde{v}_b + ik\tilde{\Psi} = n_e\sigma_T a\frac{4\rho_\gamma}{3\rho_b}\left[3i\tilde{\Theta}_1 + \tilde{v}_b\right]}$$

- **Neutrinos**: similar to photons, but no collision terms

Acoustic oscillations

$$\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}$$

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{i v_b}{3}\right]$$

For v_b, invoke the B.E. for baryons:

$$\dot{v}_b + \frac{\dot{a}}{a}v_b + ik\Psi = n_e\sigma_T a\frac{4\rho_\gamma}{3\rho_b}\left[3i\Theta_1 + v_b\right] \qquad n_e\sigma_T a = -\dot{\tau}$$

or

$$v_b = -3i\Theta_1 + \frac{R}{\dot{\tau}}\left(\dot{v}_b + \frac{\dot{a}}{a}v_b + ik\Psi\right)$$

where $\frac{1}{R} \equiv \frac{4\rho_\gamma^{(0)}}{3\rho_b^{(0)}}$

Second term proportional to $1/\dot{\tau}$, i.e. small in tightly coupled regime. We thus expand using $v_b \sim -3i\Theta_1$:

$$v_b = -3i\Theta_1 + \frac{R}{\dot{\tau}}\left(-3i\dot{\Theta}_1 - 3i\frac{\dot{a}}{a}\Theta_1 + ik\Psi\right)$$

Insert in B.E. for photons (first moment):

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{i}{3}\left\{-3i\Theta_1 + \frac{R}{\dot{\tau}}\left(-3i\dot{\Theta}_1 - 3i\frac{\dot{a}}{a}\Theta_1 + ik\Psi\right)\right\}\right]$$

Acoustic oscillations

$$\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}$$

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{iv_b}{3}\right]$$

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{i}{3}\left\{-3i\Theta_1 + \frac{R}{\dot{\tau}}\left(-3i\dot{\Theta}_1 - 3i\frac{\dot{a}}{a}\Theta_1 + ik\Psi\right)\right\}\right]$$

$$= \frac{k\Psi}{3} - R\dot{\Theta}_1 - R\frac{\dot{a}}{a}\Theta_1 + R\frac{k}{3}\Psi$$

Collecting the terms proportional to Θ_0, Θ_1, $\dot{\Theta}_1$, we get

$$\dot{\Theta}_1 + \frac{R}{(1+R)}\frac{\dot{a}}{a}\Theta_1 - \frac{k}{3(1+R)}\Theta_0 = \frac{k\Psi}{3}$$

Differentiating $\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}$ we get $\ddot{\Theta}_0 + k\dot{\Theta}_1 = -\ddot{\Phi}$

And we can eliminate $\dot{\Theta}_1$:

$$\ddot{\Theta}_0 + k\left(\frac{k\Psi}{3} - \frac{R}{(1+R)}\frac{\dot{a}}{a}\Theta_1 + \frac{k}{3(1+R)}\Theta_0\right) = -\ddot{\Phi}$$

Acoustic oscillations

$$\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}$$

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{iv_b}{3}\right]$$

And we can eliminate $\dot{\Theta}_1$:

$$\ddot{\Theta}_0 + k\left(\frac{k\Psi}{3} - \frac{R}{(1+R)}\frac{\dot{a}}{a}\Theta_1 + \frac{k}{3(1+R)}\Theta_0\right) = -\ddot{\Phi}$$

Use $\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}$ once again to eliminate $\Theta_1 = -(\dot{\Phi} + \dot{\Theta}_0)/k$

$$\ddot{\Theta}_0 + k\left(\frac{k\Psi}{3} + \frac{R}{(1+R)}\frac{\dot{a}}{a}\frac{(\dot{\Phi} + \dot{\Theta}_0)}{k} + \frac{k}{3(1+R)}\Theta_0\right) = -\ddot{\Phi}$$

Collecting the $\mathbf{\Phi}$ and $\boldsymbol{\psi}$ terms in *F(k, η),* introducing the sound speed $c_s = \sqrt{1/3(1+R)}$ and re-arranging, we finally get an equation for the *monopole* oscillations:

$$\ddot{\Theta}_0 + \frac{R}{(1+R)}\frac{\dot{a}}{a}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = F(k,\eta)$$

Acoustic oscillations

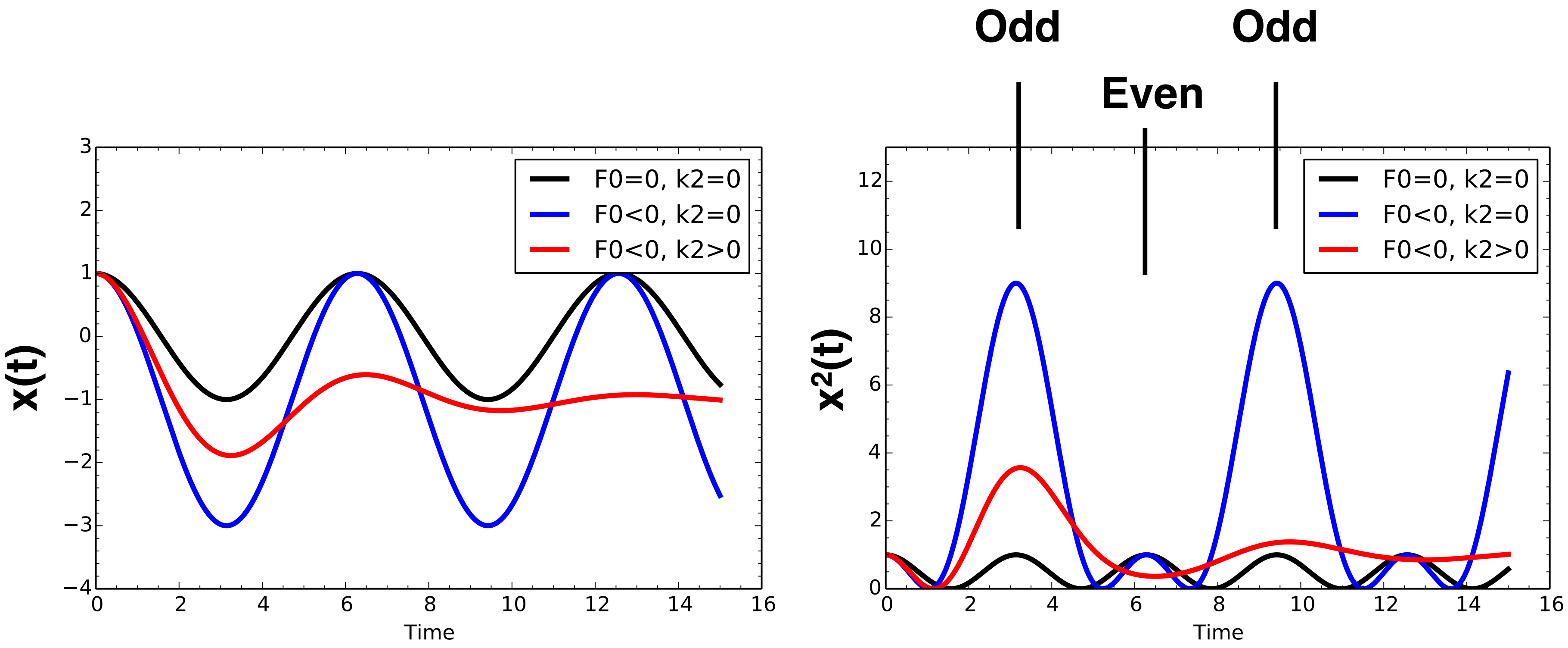
Monopole oscillations:

$$\ddot{\Theta}_0 + \frac{R}{(1+R)}\frac{\dot{a}}{a}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = F(k,\eta)$$

Note similarity to classical (damped, forced) harmonic oscillator!
This justifies the "spring analogies" that we discussed earlier.

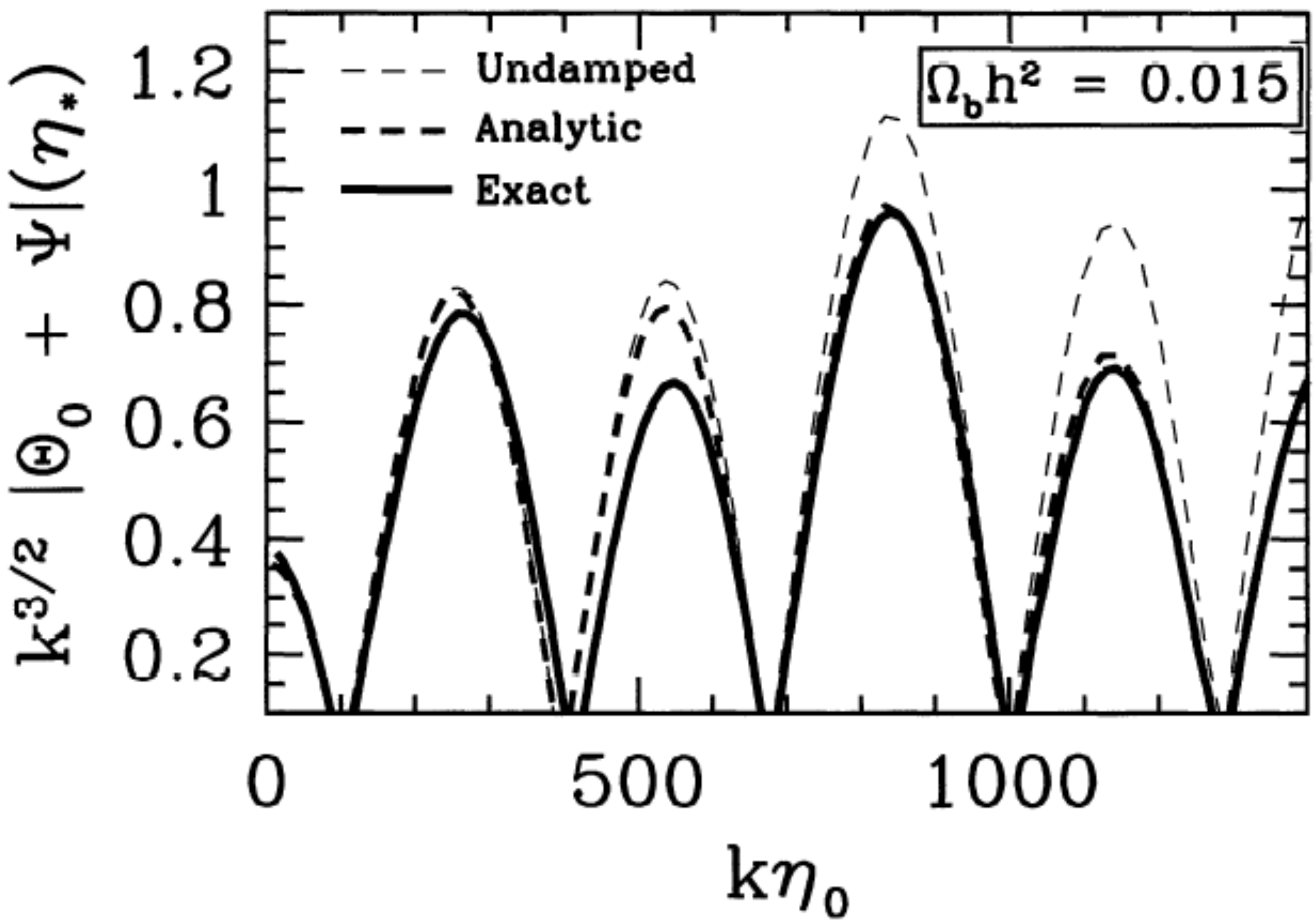
$$m\ddot{x} = F_0 - k_1 x - k_2 \dot{x}$$

$$\ddot{x} + \frac{k_2}{m}\dot{x} + \frac{k_1}{m}x = F_0/m$$

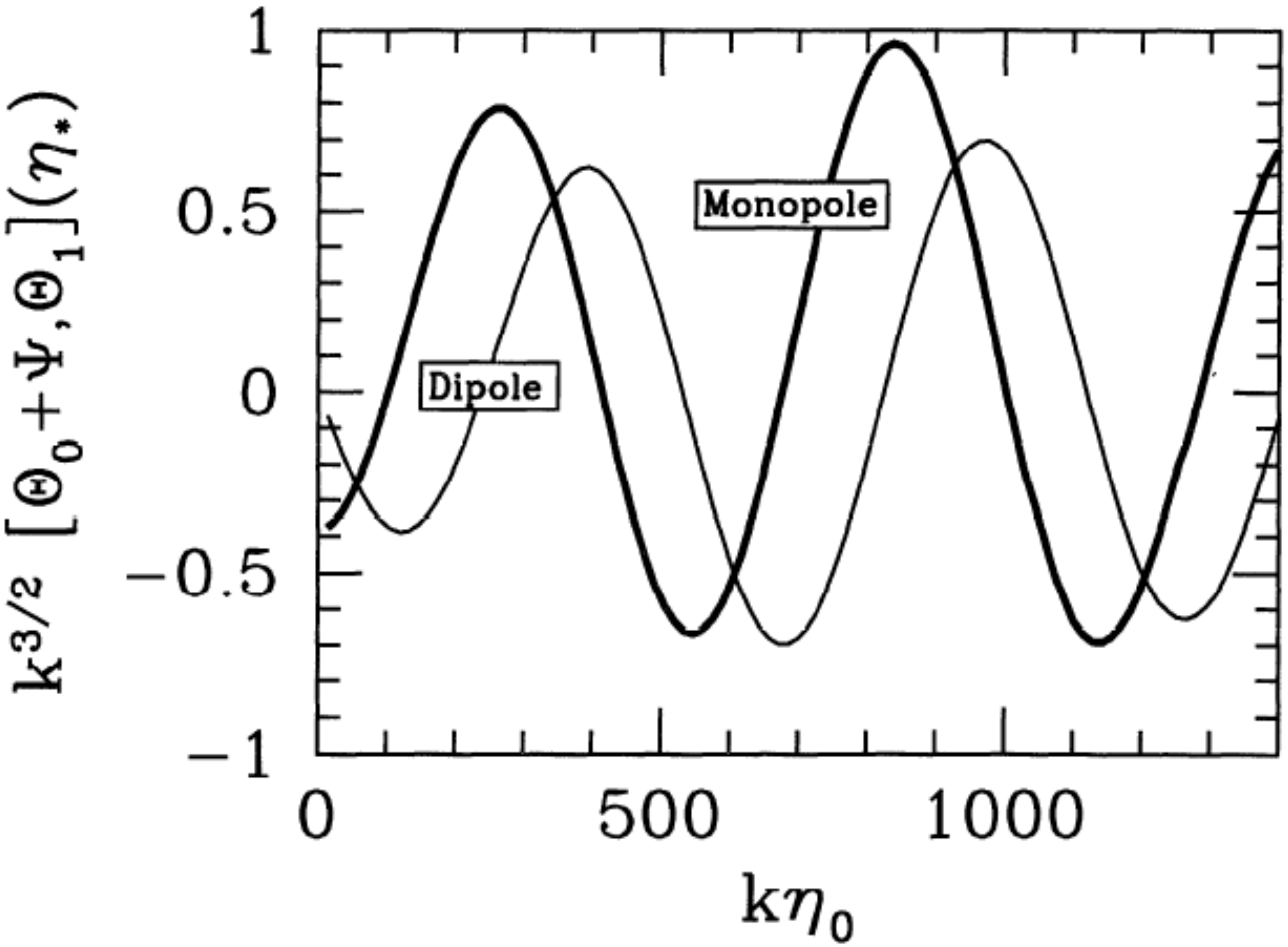


Solutions

Monopole at recombination in a standard CDM Universe



Dipole perturbations

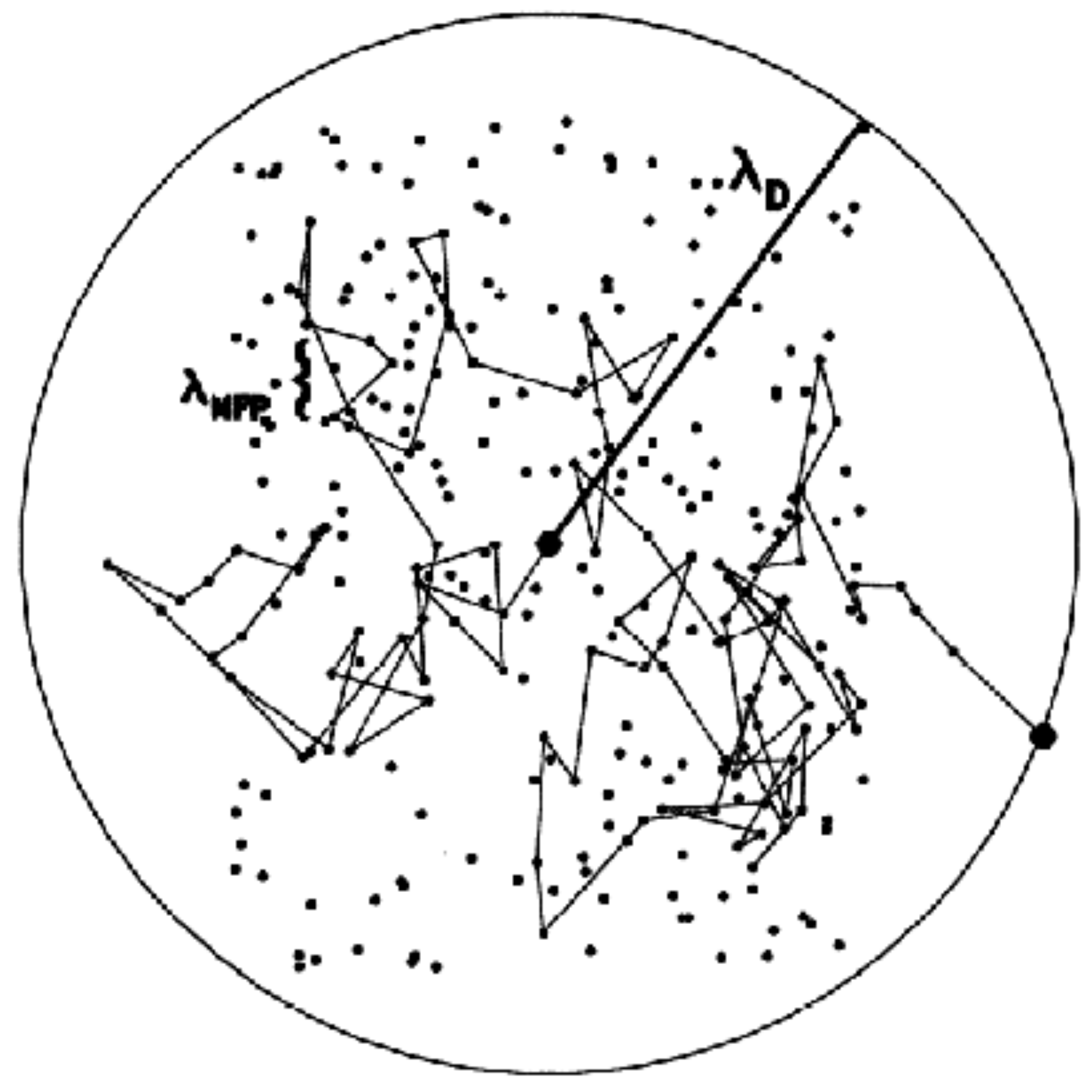


Θ_1 is out of phase with Θ_0:

The dipole ("velocity") is small at maximum compression/rarefaction of the monopole, and vice versa.

Damping

Perturbations on small scales (large *k*) are "washed out" by scattering of photons off of free electrons:



Mean free path:

$$\lambda_{\rm MFP} \approx (n_e \sigma_T)^{-1}$$

Number of scatterings in a Hubble time ~

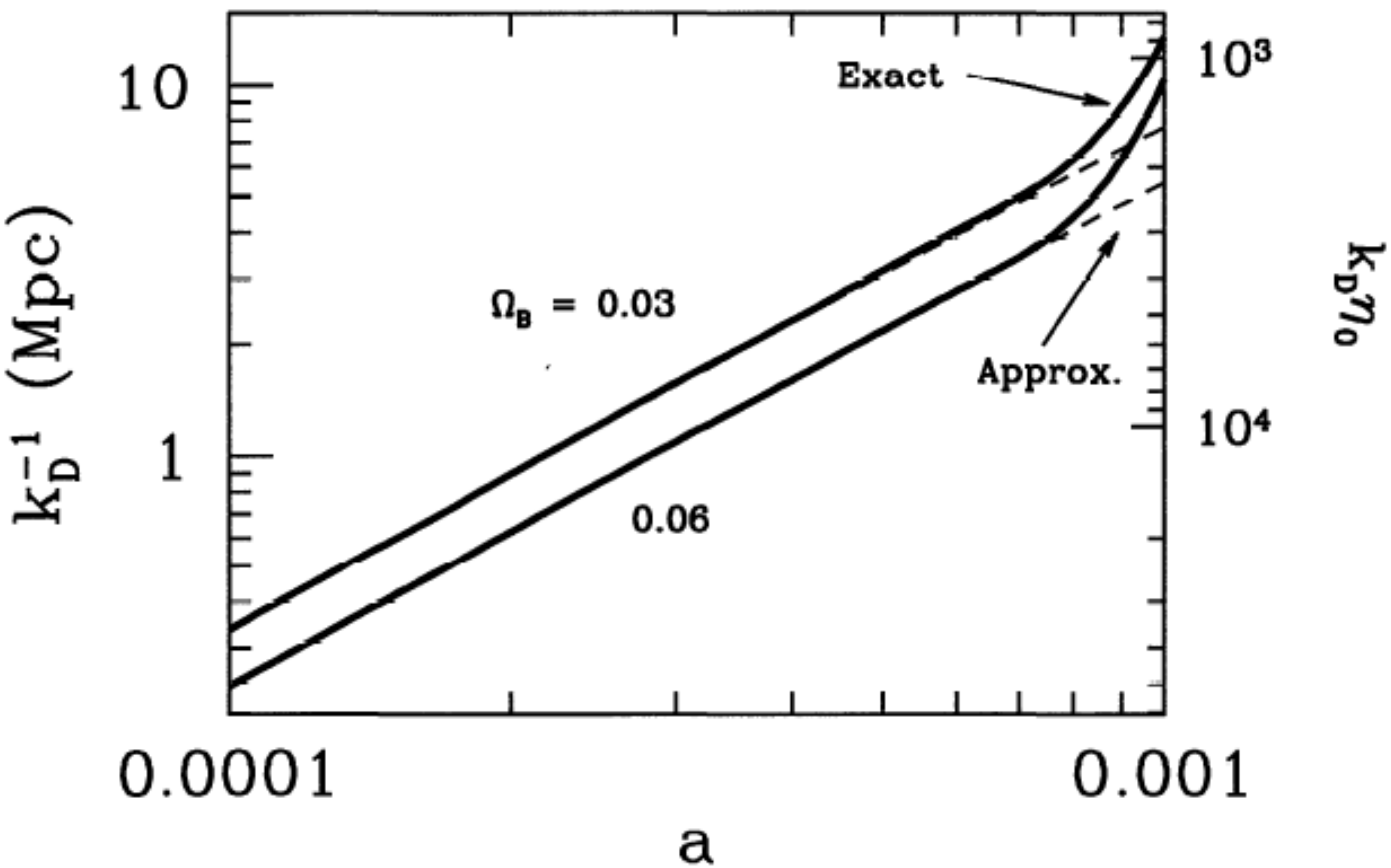
$$n_e \sigma_T H^{-1}$$

Mean distance travelled by photon (random walk)

$$\lambda_D \sim \lambda_{\rm MFP} \sqrt{n_e \sigma_T H^{-1}} \sim (\sqrt{n_e \sigma_t / H})^{-1}$$

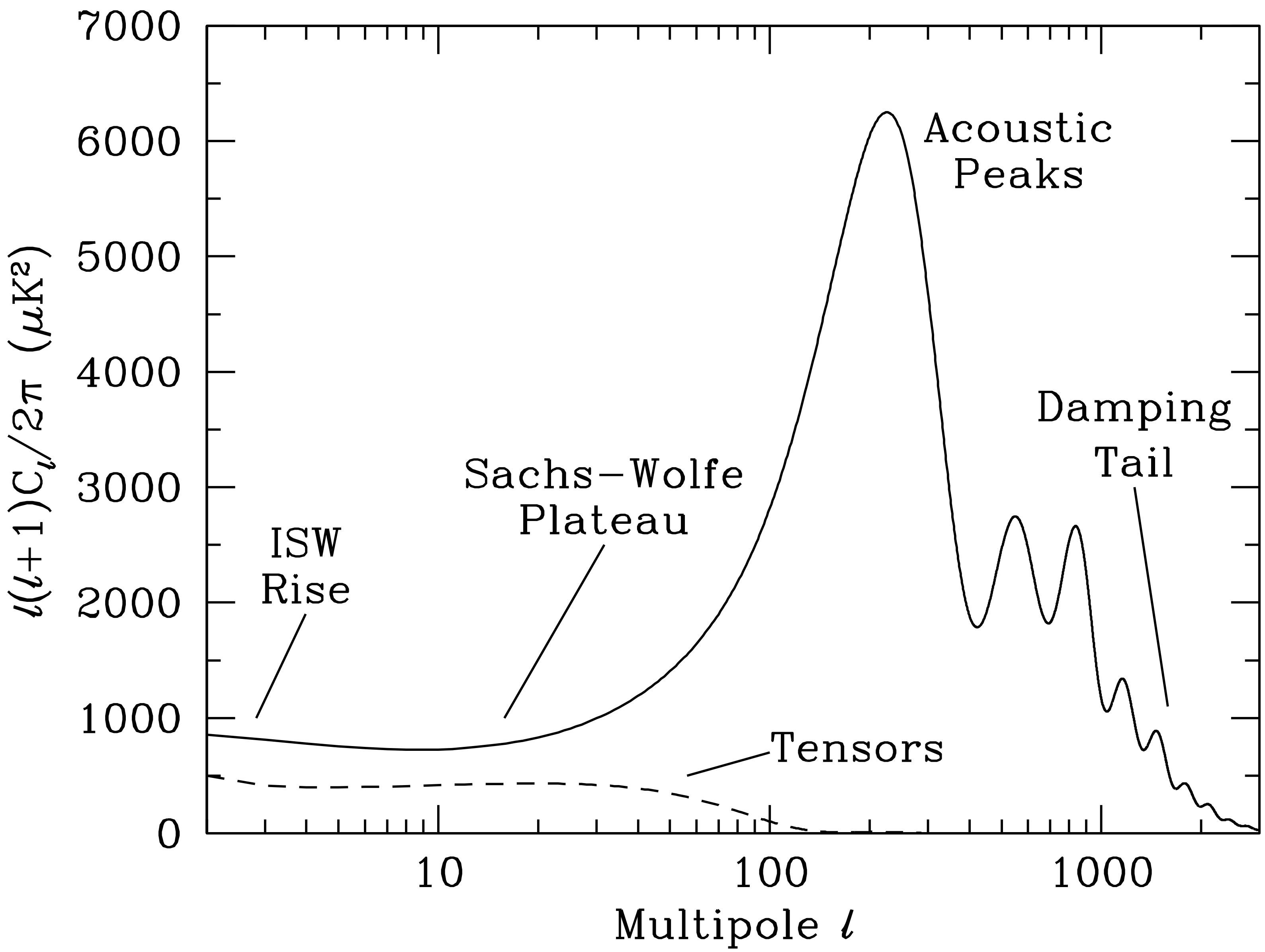
- Scale where damping becomes important depends (inversely) on baryon density

Damping



Late epochs, close to recombination:

λ_{MFP} increases as n_e decreases.



Scott & Smoot 2004

Inhomogeneities to anisotropies

We have learned how to evolve an initial spectrum of perturbations forward in time, obtaining equations for:

- **Mono- and dipoles of photon perturbations (Θ_0,Θ_1)**
- **Density perturbations of dark and baryonic matter (δ, δ_b)**
- **The potential and curvature terms (Ψ, Φ)**

We have analysed the behaviour of Θ_0, Θ_1 in the tightly coupled limit, i.e., until recombination.

But how do the *inhomogeneities* at recombination translate into the *anisotropies* in the CMB observed today?

Inhomogeneities to anisotropies

How do the *inhomogeneities* at recombination translate into the *anisotropies* in the CMB observed today?

Start again from the B.E. for photons:

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu \mathbf{v}_b\right]$$

We are now interested in the high order moments of Θ that are visible as anisotropies in the CMB *today*.

So we need to

(1) Evolve Θ forward until today

(2) Compute the multipole moments, Θ_l.

Inhomogeneities to anisotropies

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu \mathbf{v}_b\right]$$

Start by subtracting $\dot{\tau}\Theta$ from both sides:

$$\dot{\Theta} + (ik\mu - \dot{\tau})\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 + \mu \mathbf{v}_b\right]$$

Then the **left-hand side** can be written

$$e^{-ik\mu\eta+\tau}\frac{\mathrm{d}}{\mathrm{d}\eta}\left[\Theta e^{ik\mu\eta-\tau}\right]$$

On the **right-hand side** we define the *source function*:

$$\tilde{S} \equiv -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 + \mu \mathbf{v}_b\right]$$

Multiply both sides by $e^{ik\mu\eta-\tau}$, then integrate over η

Inhomogeneities to anisotropies

Integrate over conformal time from $\eta_{init} = 0$ to η_0 (today):

$$\Theta(\eta_0) \simeq \int_0^{\eta_0} \mathrm{d}\eta \tilde{S} e^{ik\mu(\eta-\eta_0)-\tau(\eta)}$$

To calculate the moments, we then need to evaluate the usual integral,

$$\Theta_l = \frac{1}{(-i)^l} \int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \mathcal{P}_l(\mu)\Theta(\mu)$$

That is,

$$\Theta_l = \frac{1}{(-i)^l} \int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \mathcal{P}_l(\mu) \int_0^{\eta_0} \mathrm{d}\eta \left\{-\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 + \mu \mathbf{v}_b\right]\right\} e^{ik\mu(\eta-\eta_0)-\tau(\eta)}$$

Inhomogeneities to anisotropies

After performing the integral over angles, the source function can be rewritten as

$$S(k,\eta) = g(\eta)[\Theta_0 + \Psi] + \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{i v_b g(\eta)}{k}\right) + e^{-\tau}\left[\dot{\Psi}(k,\eta) - \dot{\Phi}(k,\eta)\right]$$

where *g* is the *visibility* function:

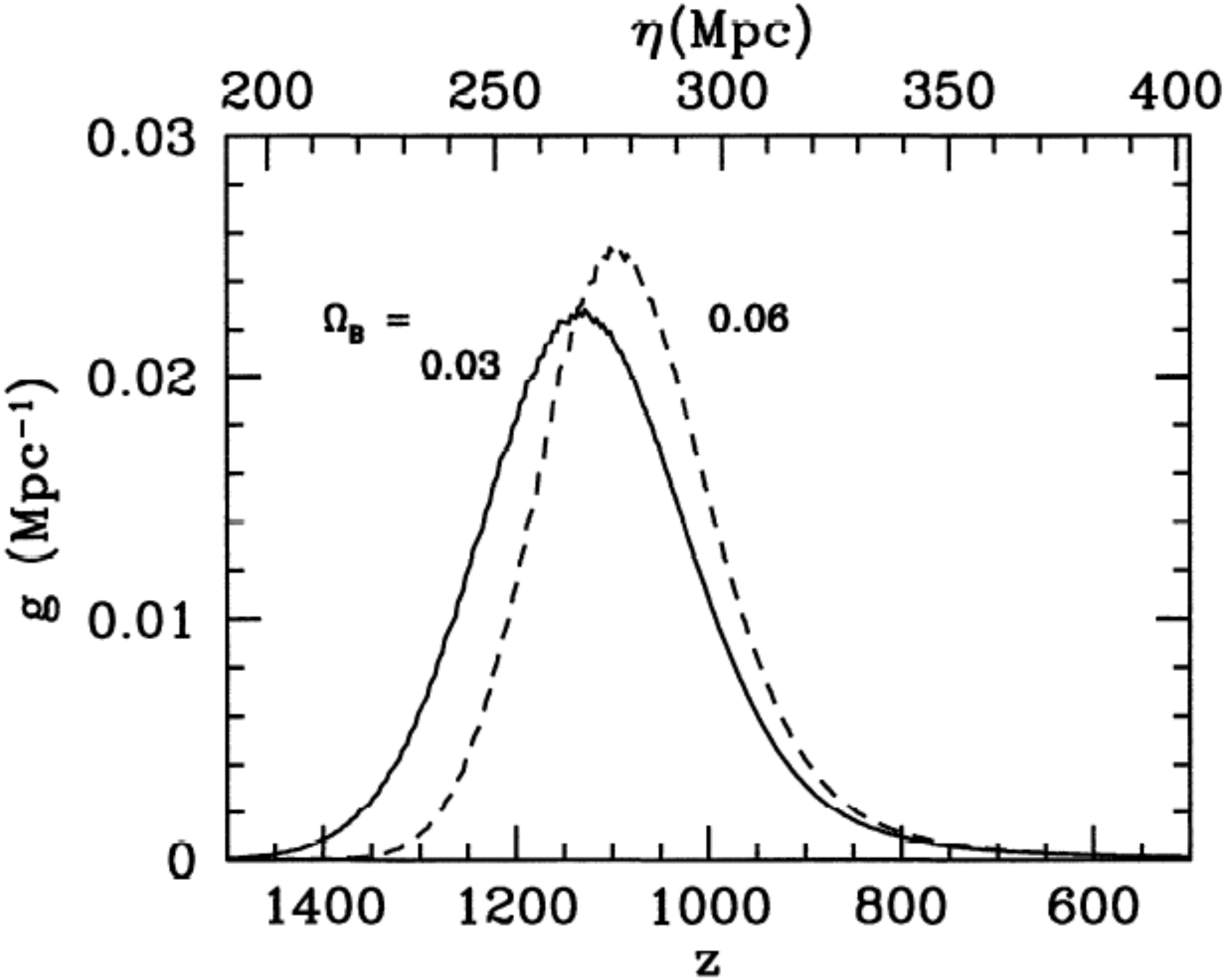
$$g(\eta) \equiv -\dot{\tau} e^{-\tau}$$

Then we get

$$\Theta_l = \int_0^{\eta_0} S(k,\eta) j_l[k(\eta_0 - \eta)]\mathrm{d}\eta$$

where $j_l(x)$ is the *spherical Bessel function* of order l.

The visibility function



Dodelson, *Modern Cosmology*.

$$g(\eta) = \dot{\tau} e^{-\tau}$$

$$\dot{\tau} = -n_e \sigma_T a$$

Scattering rate, large at high z (large τ)

$$e^{-\tau}$$

Probability that a photon escapes - small for large τ

g(*η*) is strongly peaked around recombination.

Multipoles

$$S(k,\eta) = g(\eta)[\Theta_0 + \Psi] + \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{iv_b g(\eta)}{k}\right) + e^{-\tau}\left[\dot{\Psi}(k,\eta) - \dot{\Phi}(k,\eta)\right]$$

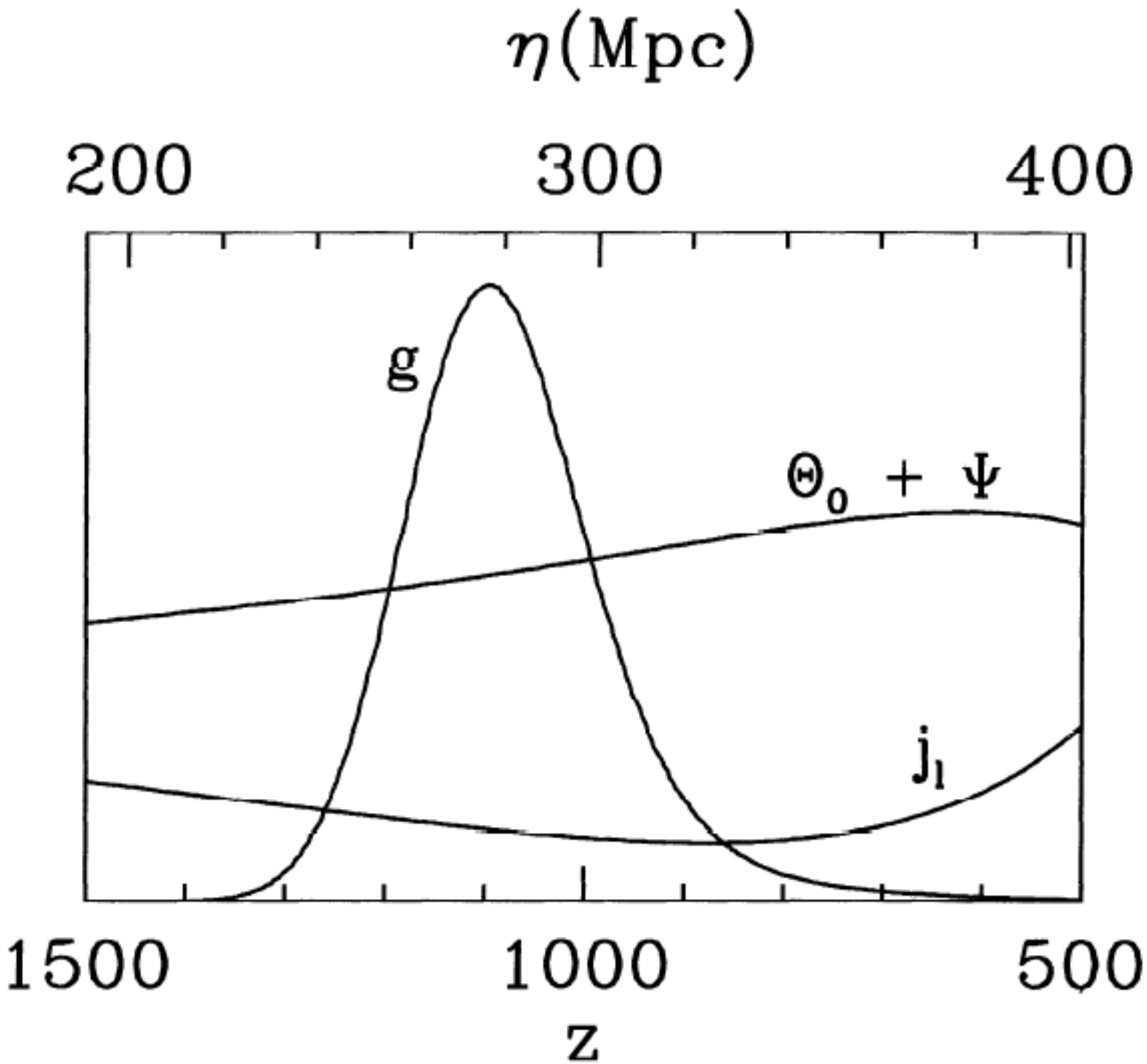
Expanding *S* and integrating the middle term by parts,

$$\begin{aligned}\Theta_l = &\int_0^{\eta_0} \mathrm{d}\eta\, g(\eta)[\Theta_0(k,\eta) + \Psi(k,\eta)] j_l[k(\eta_0 - \eta)] \\ &- \int_0^{\eta_0} \mathrm{d}\eta\, g(\eta)\left(\frac{iv_b(k,\eta)}{k}\right)\frac{\mathrm{d}}{\mathrm{d}\eta} j_l[k(\eta_0 - \eta)] \\ &+ \int_0^{\eta_0} \mathrm{d}\eta e^{-\tau}\left[\dot{\Psi}(k,\eta) - \dot{\Phi}(k,\eta)\right] j_l[k(\eta_0 - \eta)]\end{aligned}$$

First two terms: Terms weighted by *g(η)* - sharply peaked near recombination

Third term: Late-time contributions from changing potentials: *Sachs-Wolfe* effect

The visibility function and integrands



g(η) is sharply peaked near recombination, at *η** corresponding to *z*~1100.

Other factors vary much more slowly.

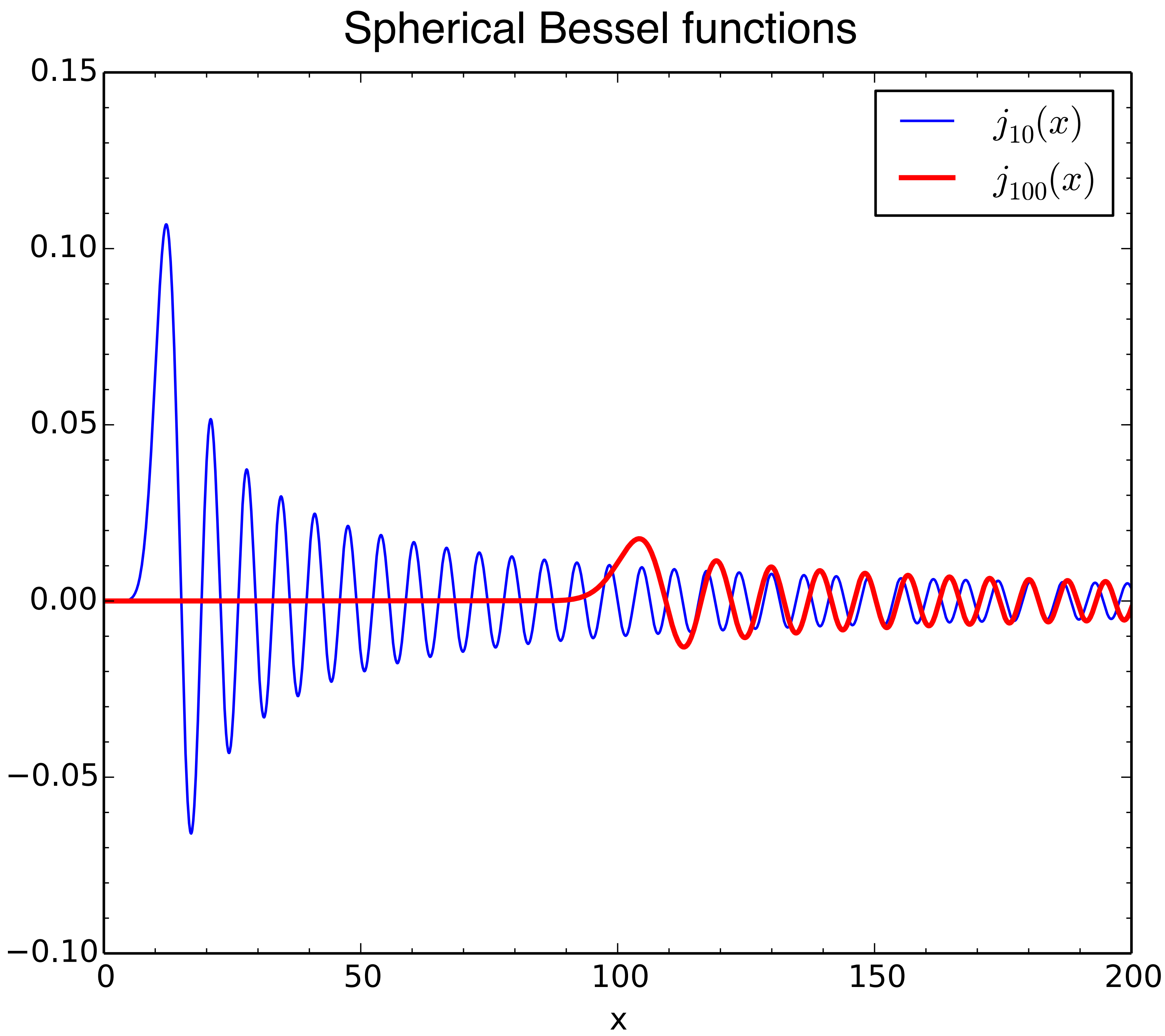
Inhomogeneities to anisotropies

Approximating *g(η) ~ δ(η-η*)*, we get:

$$\Theta_l(k,\eta_0) = [\Theta_0(k,\eta^*) + \Psi(k,\eta^*)]j_l[k(\eta_0-\eta^*)]$$
$$+3\Theta_1(k,\eta^*)\left(j_{l-1}[k(\eta_0-\eta^*)] - \frac{l+1}{k(\eta_0-\eta^\star)}j_l[k(\eta_0-\eta^*)]\right)$$
$$+\int_0^{\eta_0} \mathrm{d}\eta e^{-\tau}\left[\dot{\Psi}(k,\eta) - \dot{\Phi}(k,\eta)\right]j_l[k(\eta_0-\eta)]$$

Note: Θ_l has three types of terms:

- Contributions from the *monopole* of the photon perturbations around η^*. Note that the *sum* ($\Theta_0 + \psi$) enters - photons have to “climb out” of the potentials.
- Contributions from the dipole - material moving into/out of perturbations
- Late-type modifications of Θ_l by decaying potentials
- The Bessel functions “select” modes with $k\eta > l$.



Spherical Bessel functions
0.15
0.10
0.05
0.00
−0.05
−0.10
0
50
100
150
200
x
$j_{10}(x)$
$j_{100}(x)$

One last thing..

We have derived expressions for the *multipoles* Θ_l of the photon perturbations today.

However, observers typically measure the C_l's, i.e. the (squared) coefficients of the spherical harmonic expansion of the temperature fluctuations.

Luckily, the transformation is relatively straight forward:

$$C_l = \frac{2}{\pi} \int_0^\infty \mathrm{d}k \, k^2 P(k) \left| \frac{\Theta_l(k)}{\delta_{\mathrm{DM}}(k)} \right|^2$$

P(k) is the power spectrum of the DM perturbations and $\delta_{\mathrm{DM}}(k)$ are the corresponding Fourier coefficients.

(see chapter 8.5.2 in “Modern Cosmology” for the details).

CAMB Web Interface

Supports the January 2011 Release

Most of the configuration documentation is provided in the sample parameter file provided with the application.

This form uses JavaScript to enable certain layout features, and it uses Cascading Style Sheets to control the layout of all the form components. If either of these features are not supported or enabled by your browser, this form will NOT display correctly.

Actions to Perform

- [x] Scalar C_l's
- [ ] Vector C_l's
- [ ] Tensor C_l's
- [x] Do Lensing
- [ ] Transfer Functions
- (•) Linear
- ( ) Non-linear Matter Power (HALOFIT)
- ( ) Non-linear CMB Lensing (HALOFIT)

Sky Map Output: None

Vector C_l's are incompatible with Scalar and Tensor C_l's. The Transfer functions require Scalar and/or Tensor C_l's.
The HEALpix synfast program is used to generate maps from the resultant spectra. The random number seed governs the phase of the a_{lm}'s generated by synfast. The default of zero causes synfast to generate a new seed from the system time with each run. Specifying a fixed nonzero value will return fixed phases with successive runs.

*Maximum Multipoles and k*eta*

Scalar

2000 l_{max}
4000 $k*eta_{max}$

Tensor

1500 l_{max}
3000 $k*eta_{max}$

Tensor limits should be less than or equal to the scalar limits.

Cosmological Parameters

Use Physical Parameters? Yes
70 Hubble Constant
2.725 T_{cmb}

0.0226 $\Omega_b h^2$
0.228 $\Omega_c h^2$
0 $\Omega_\nu h^2$
0 Ω_k

Neutrino mass splittings

1 # Eigenstates
0 Degeneracies
1 Mass Fractions

0.24 Helium Fraction
3.04 Massless Neutrinos
0 Massive Neutrinos
-1 Eqn. of State
1 Comoving Sound Speed

The Equation of State entry is the effective equation of state parameter for dark energy and is assumed constant. The Comoving Sound Speed parameter is the constant comoving sound speed of the dark energy; 1=quintessence.
Setting Degeneracies to zero sets the mass degeneracies parameter to massive neutrinos. Otherwise this should be a space separated list of values, one per eigenstate.
Fractions should be a space separated list indicating the fraction of $\Omega_\nu h^2$ accounted for by each eigenstate.

Reionization

Include Reionization? Yes
Use Optical Depth? No
11 Redshift
0.5 Width of Transition
1 Ionization Fraction

Power Spectrum

1 Number
2.46e-9 Scalar Amplitude
0.96 Scalar Spectral Index
0 Scalar Run Count
0 Tensor Spectral Index
1 Initial Ratio

The ratio is that of the initial tensor/scalar power spectrum amplitudes. The vector modes use the scalar settings.
Supply 'Number' values in each after the first, separated by spaces.

Initial Scalar Perturbation Mode

Adiabatic

For Vector Modes:

Regular (Neutrino Vorticity Mode)

Or.. download the (Fortran 90) code yourself: http://camb.info

CAMB — vi params.ini — 80×32

```
l_max_scalar      = 2200
#k_eta_max_scalar  = 4000

#  Tensor settings should be less than or equal to the above
l_max_tensor      = 1500
k_eta_max_tensor  = 3000

#Main cosmological parameters, neutrino masses are assumed degenerate
# If use_phyical set physical densities in baryons, CDM and neutrinos + Omega_k
use_physical    = T
ombh2           = 0.0226
omch2           = 0.112
omnuh2          = 0.00064
omk             = 0
hubble          = 70

#effective equation of state parameter for dark energy
w               = -1
#constant comoving sound speed of the dark energy (1=quintessence)
cs2_lam         = 1

#varying w is not supported by default, compile with EQUATIONS=equations_ppf to
use crossing PPF w-wa model:
#wa              = 0
##if use_tabulated_w read (a,w) from the following user-supplied file instead of
 above
#use_tabulated_w = F
#wafile = wa.dat

#if use_physical = F set parameters as here
#omega_baryon    = 0.0462
```

Or.. download the (Fortran 90) code yourself: http://camb.info

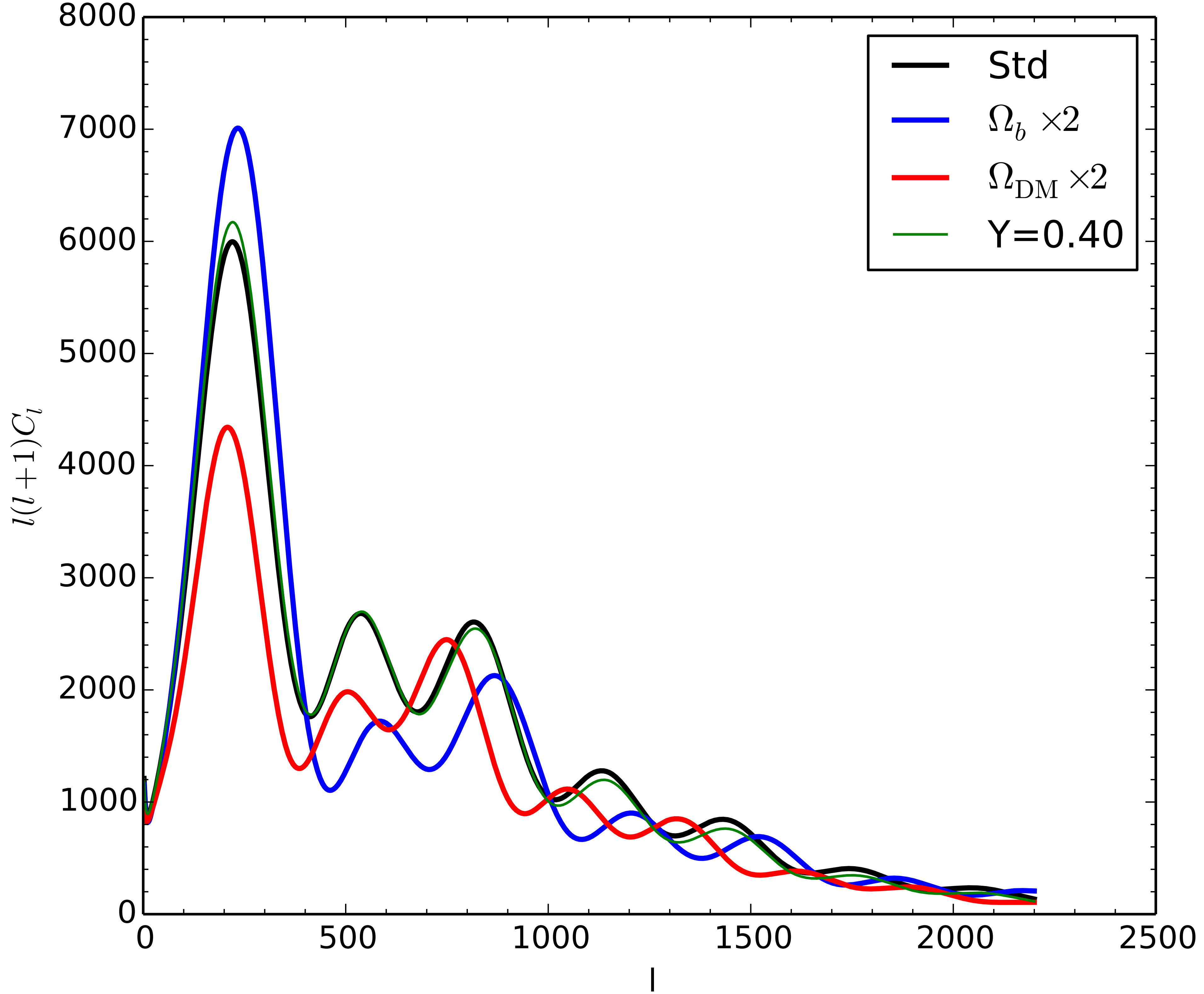
CAMB — vi params.ini — 80×32

```
l_max_scalar      = 2200
#k_eta_max_scalar  = 4000

#  Tensor settings should be less
l_max_tensor      = 1500
k_eta_max_tensor  = 3000

#Main cosmological parameters, ne
# If use_phyical set physical den
use_physical   = T
ombh2          = 0.0226
omch2          = 0.112
omnuh2         = 0.00064
omk            = 0
hubble         = 70

#effective equation of state para
w              = -1
#constant comoving sound speed of
cs2_lam        = 1

#varying w is not supported by de
use crossing PPF w-wa model:
#wa             = 0
##if use_tabulated_w read (a,w) f
 above
#use_tabulated_w = F
#wafile = wa.dat

#if use_physical = F set paramete
#omega_baryon    = 0.0462
```

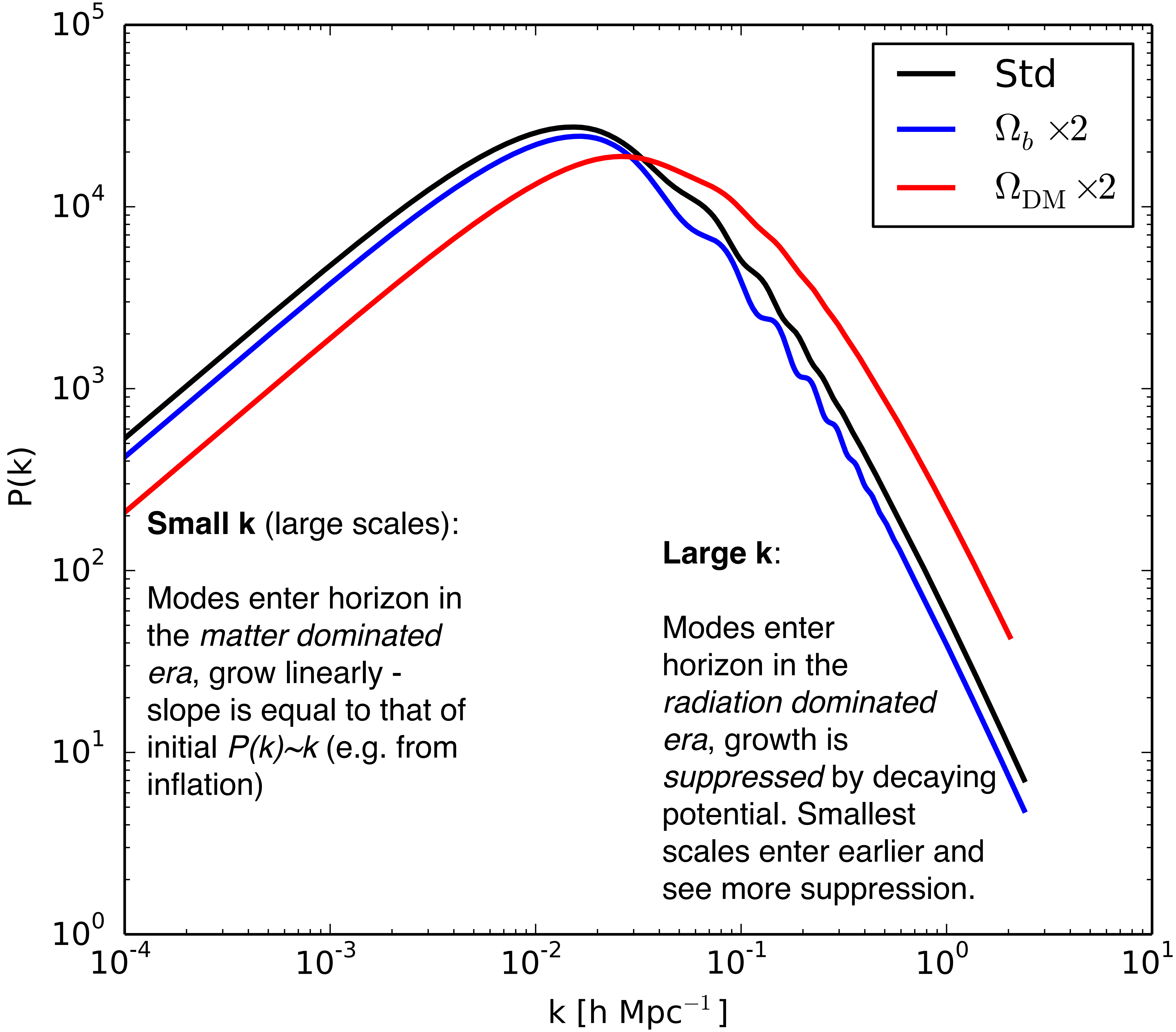
CAMB — -bash — 80×31

```
Air:Software/CAMB> ./camb params.ini
Reion redshift         =  10.713
Om_b h^2               =  0.022600
Om_c h^2               =  0.112000
Om_nu h^2              =  0.000640
Om_Lambda              =  0.724000
Om_K                   =  0.000000
Om_m (1-Om_K-Om_L)     =  0.276000
100 theta (CosmoMC)    =  1.039532
N_eff (total)          =  3.046000
 1 nu, g= 1.0153 m_nu*c^2/k_B/T_nu0=    353.71 (m_nu=  0.060 eV)
Reion opt depth        =  0.0900
Age of universe/GYr    =  13.777
zstar                  =  1088.72
r_s(zstar)/Mpc         =  146.38
100*theta              =  1.039840
DA(zstar)/Gpc          =  14.07762
zdrag                  =  1059.70
r_s(zdrag)/Mpc         =  149.01
k_D(zstar) Mpc         =  0.1392
100*theta_D            =  0.160271
z_EQ (if v_nu=1)       =  3216.47
k_EQ Mpc (if v_nu=1)   =  0.009817
100*theta_EQ           =  0.847737
100*theta_rs_EQ        =  0.467101
tau_recomb/Mpc         =  284.95  tau_now/Mpc =  14362.3
at z =  0.000 sigma8 (all matter) =  0.7781
at z =  0.000 sigma8^2_vd/sigma8  =  0.3812
Note: The following floating-point exceptions are signalling: IEEE_UNDERFLOW_FLA
G
Air:Software/CAMB>
```

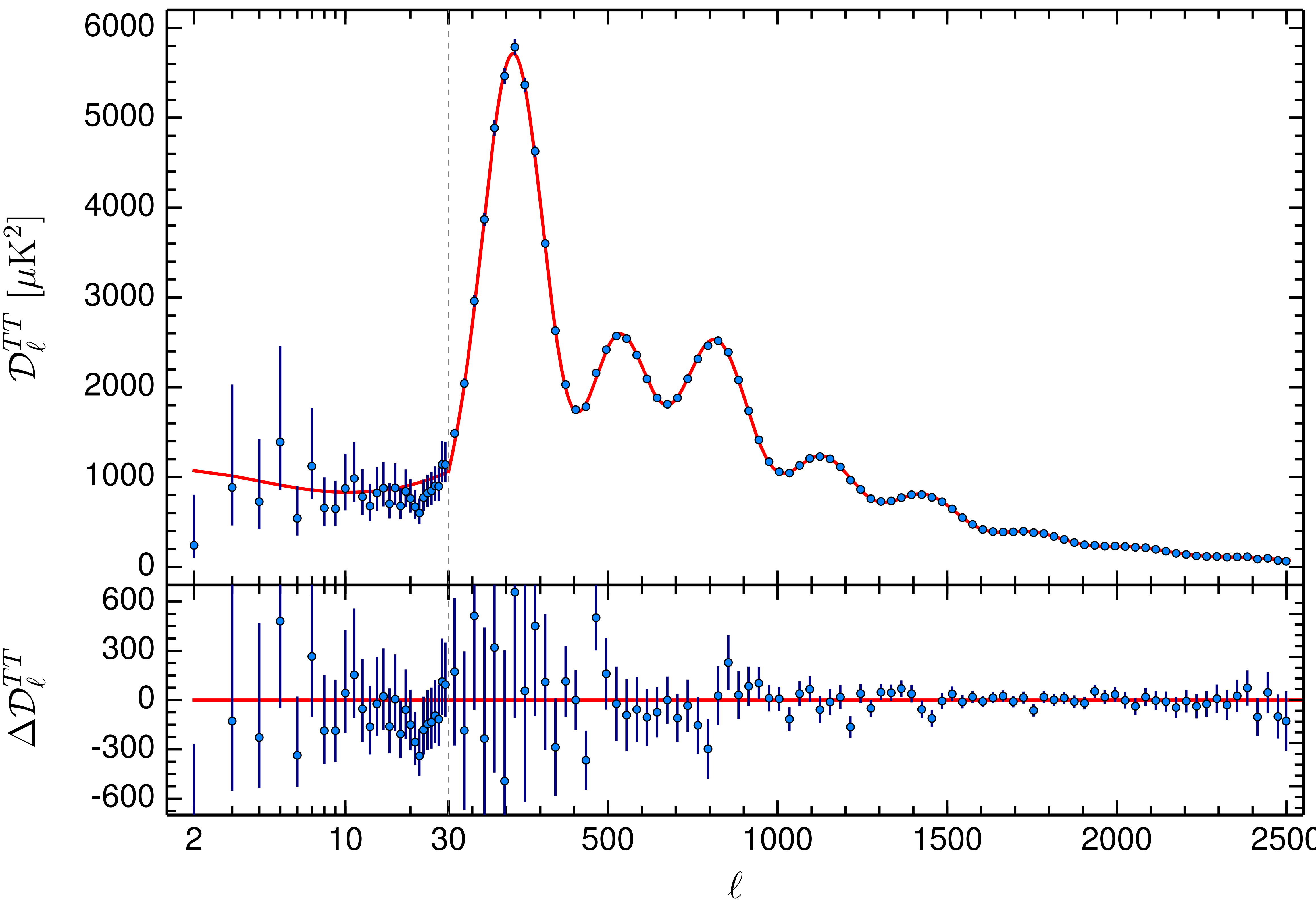
CAMB output - examples: the **CMB** power-spectrum



CAMB output - examples: the **matter** power-spectrum



The 2015 Planck temperature power-spectrum



Planck collaboration, 2015, http://arxiv.org/pdf/1502.01589v2.pdf

Table 4. Parameter 68 % confidence limits for the base ΛCDM model from *Planck* CMB power spectra, in combination with lensing reconstruction ("lensing") and external data ("ext," BAO+JLA+H_0). Nuisance parameters are not listed for brevity (they can be found in the *Planck Legacy Archive* tables), but the last three parameters give a summary measure of the total foreground amplitude (in μK^2) at $\ell = 2000$ for the three high-ℓ temperature spectra used by the likelihood. In all cases the helium mass fraction used is predicted by BBN (posterior mean $Y_P \approx 0.2453$, with theoretical uncertainties in the BBN predictions dominating over the *Planck* error on $\Omega_b h^2$).

Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	0.1413 ± 0.0011	0.1427 ± 0.0014	0.1422 ± 0.0013	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8 \Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8 \Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
z_{drag}	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
r_{drag}	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_D	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
z_{eq}	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{s,eq}$	0.4502 ± 0.0047	0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023
f_{2000}^{143}	29.9 ± 2.9	30.4 ± 2.9	30.3 ± 2.8	29.5 ± 2.7	30.2 ± 2.7	30.0 ± 2.7
$f_{2000}^{143\times217}$	32.4 ± 2.1	32.8 ± 2.1	32.7 ± 2.0	32.2 ± 1.9	32.8 ± 1.9	32.6 ± 1.9
f_{2000}^{217}	106.0 ± 2.0	106.3 ± 2.0	106.2 ± 2.0	105.8 ± 1.9	106.2 ± 1.9	106.1 ± 1.8

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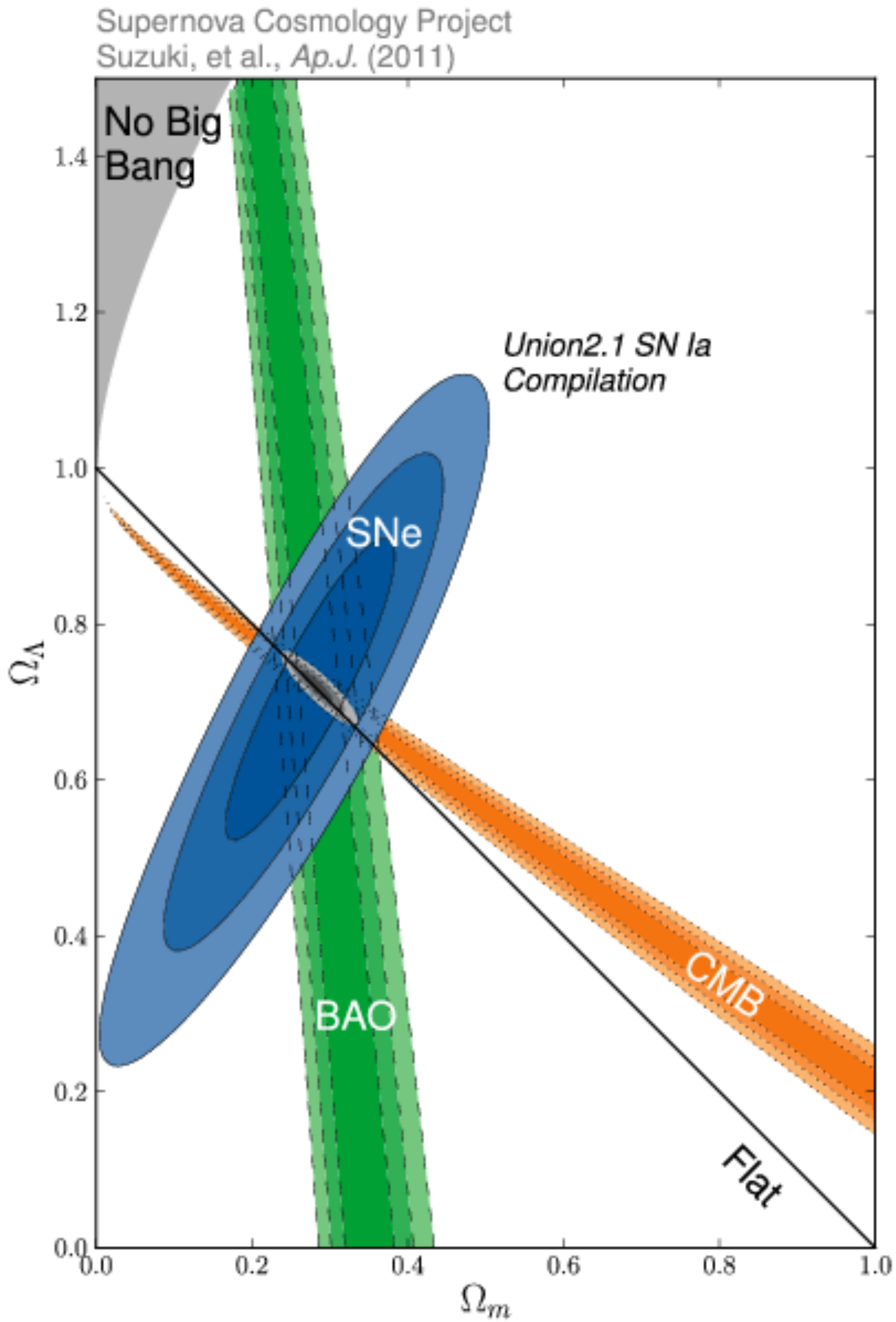
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



Constraints on cosmological parameters from different techniques.

More reading:

- Dodelson, S., 2003: *Modern Cosmology* (Academic Press)
- Hu, W. & Dodelson, S. 2002: *Cosmic Microwave Background Anisotropies*, ARA&A 40, 171:.
- Challinor, A, 2005: *Cosmic Microwave Background Anisotropies*, in: The Physics of the Early Universe (http://adsabs.harvard.edu/abs/2005LNP...653...71C)
- Hu, W. et al. 1997: *The Physics of Microwave Background Anisotropies*, Nature 386, 37 (http://adsabs.harvard.edu/abs/1997Natur.386...37H)
- Many recent Planck papers