Gravitational lensing

Abell 2218 (Hubble Space Telescope image)

Brief history of gravitational lensing

- Newton, in "Opticks" (1704):
  Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action (cæteris paribus [all else being equal]) strongest at the least distance?
- First calculated in detail in 1804 by J. Soldner: Newtonian calculation predicts a deflection of 0.84" for a light ray close to the edge of the Sun
- General Relativity (Einstein 1915): Deflection twice as large as Newtonian prediction
- Effect first measured by Eddington (1919) during Solar eclipse; measurements agreed with GR prediction
- Gravitational lensing for galaxies discussed by Zwicky (1937)
- First measured in extragalactic context (binary quasar) by Walsh et al. (1979)

The Schwarzschild solution

The *Schwarzschild metric* is the spherically symmetric solution to the vacuum field equations:

$$\mathrm{d}s^2 = \left(1 - \frac{r_s}{r}\right)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 - r_s/r} - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

$r_s = \dfrac{2GM}{c^2}$ = Schwarzschild radius

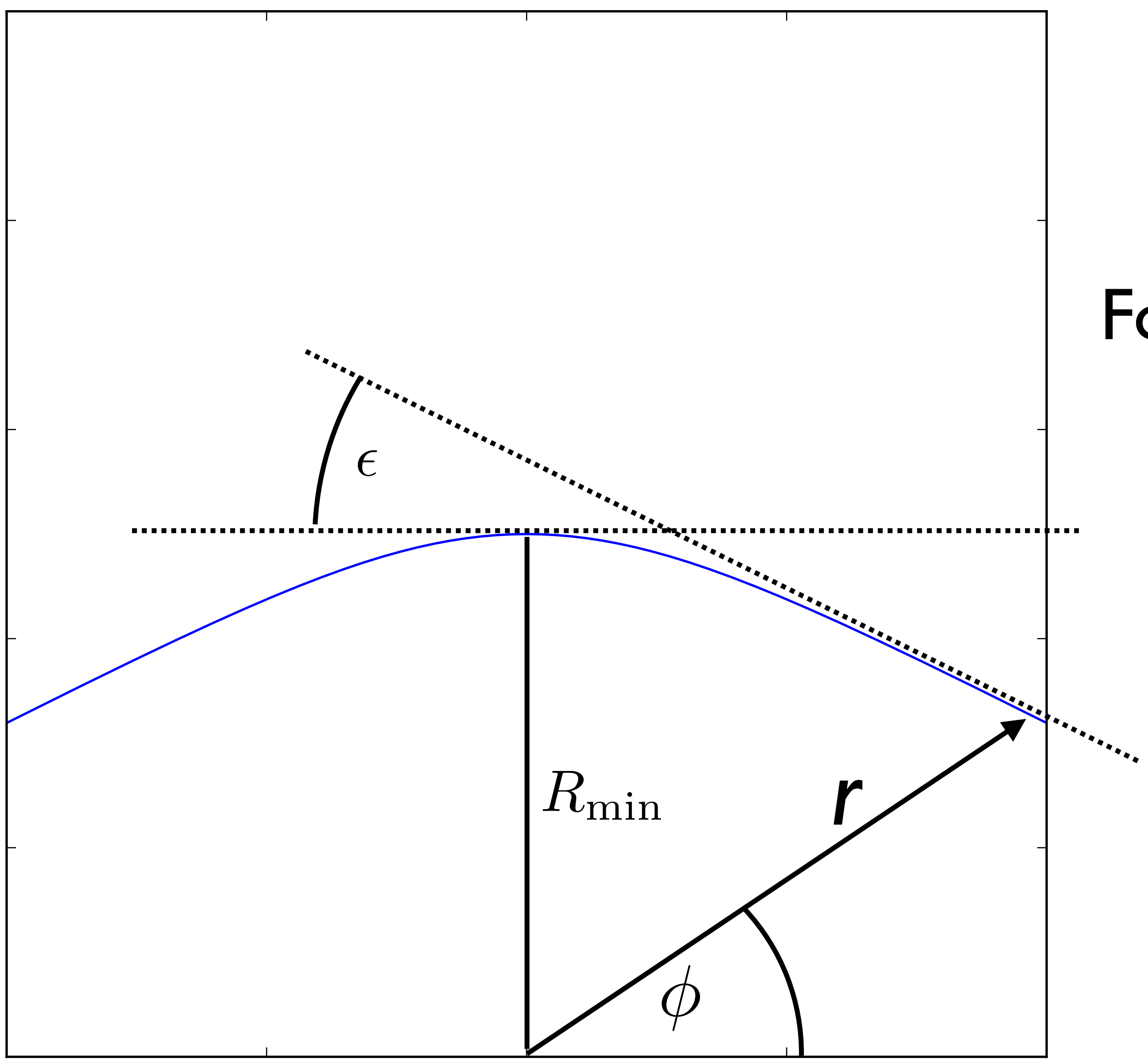
For a path in the "(*x,y*)" plane, we have $\theta=\pi/2$ and only (r,ϕ) vary.

Defining $u \equiv r^{-1}$, projection of null geodesic ($\mathrm{d}s^2=0$) onto (r,ϕ) plane is given by

$$\frac{\mathrm{d}^2u}{\mathrm{d}\phi^2} + u = \frac{3GM}{c^2}u^2$$

(see, e.g., *Introducing Einstein's Relativity*, R. D'Inverno, Oxford Univ. Press, 1992)

Bending of light



Null geodesics:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u = \frac{3GM}{c^2} u^2$$

For $R_{\min} \gg r_s$,

$$u \sim \frac{\sin\phi}{R_{\min}} + \frac{r_s}{2} \frac{(1+\cos^2\phi)}{R_{\min}^2}$$

$$\epsilon = r_s / R_{\min}$$

So the full deflection, $\alpha=2\varepsilon$, is

$$\boxed{\alpha = \frac{4GM}{c^2} \frac{1}{R_{\min}}}$$

(see, e.g., *Introducing Einstein's Relativity*, R. D'Inverno, Oxford Univ. Press, 1992)

Lensing terminology

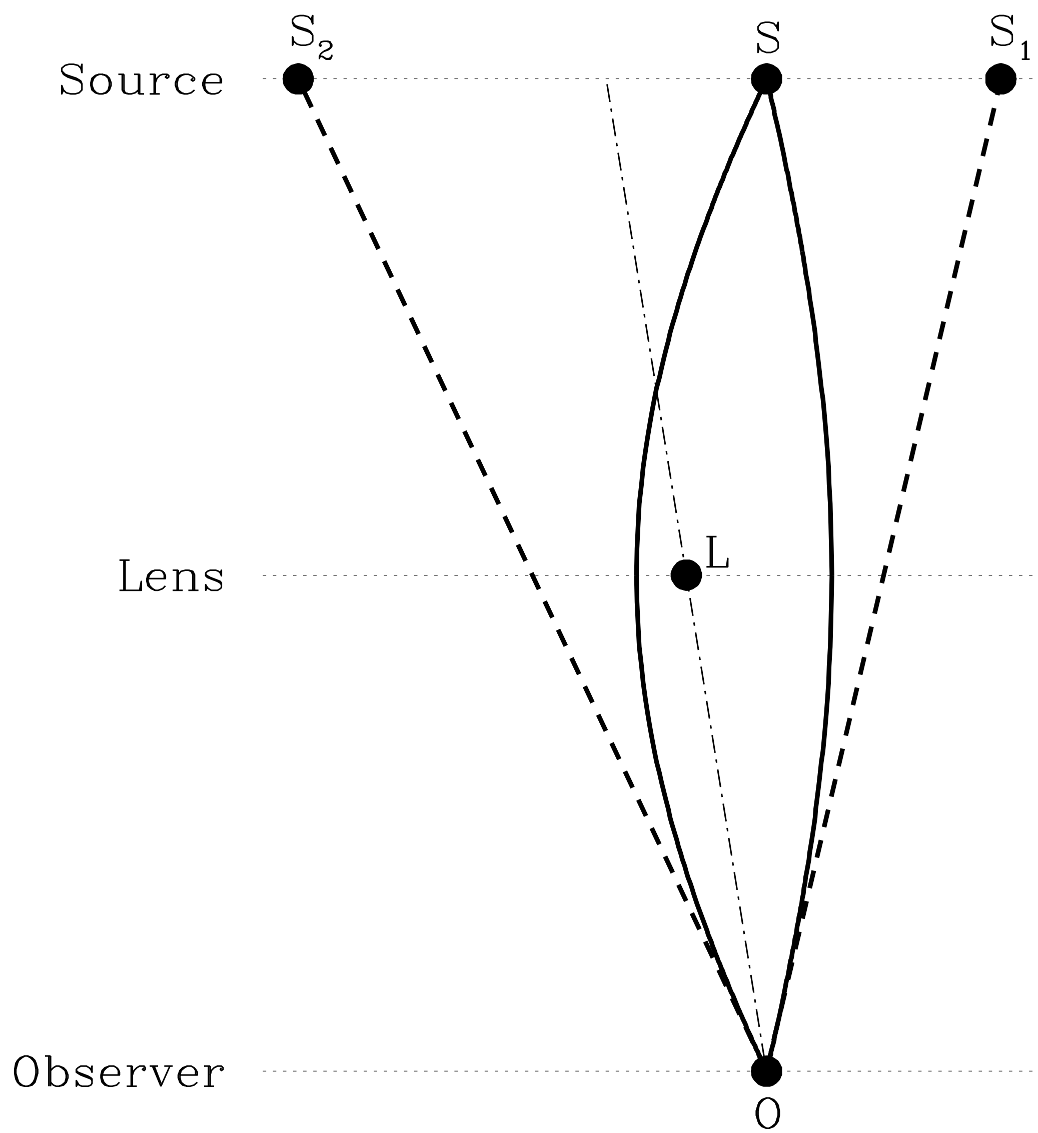
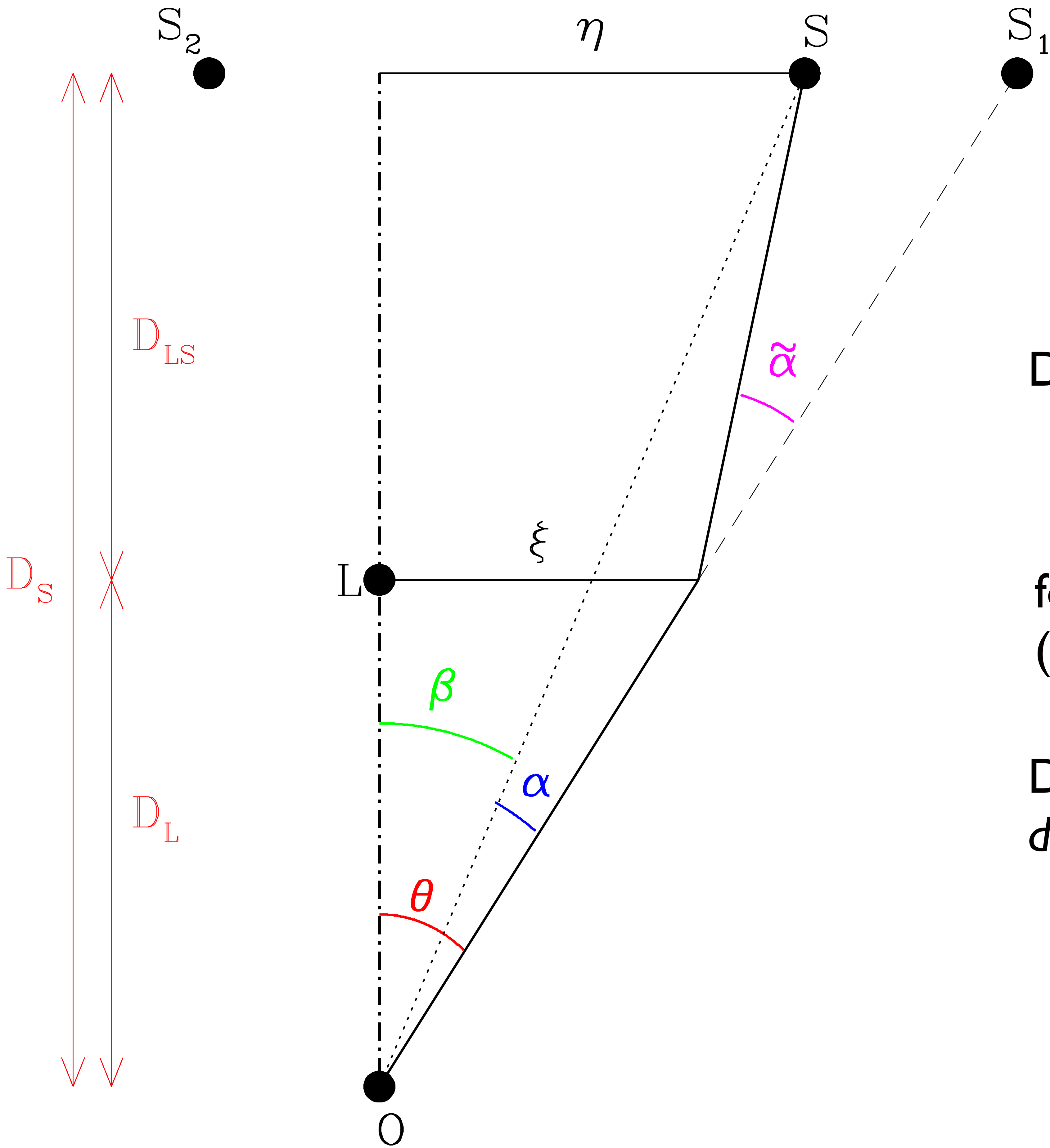


Figure 2: *Setup of a gravitational lens situation: The lens L located between source S and observer O produces two images S_1 and S_2 of the background source.*

Deflection depends on mass of lens (L)

Note: always (at least) two images (for point-like lens!)

Wambsganss (1998)

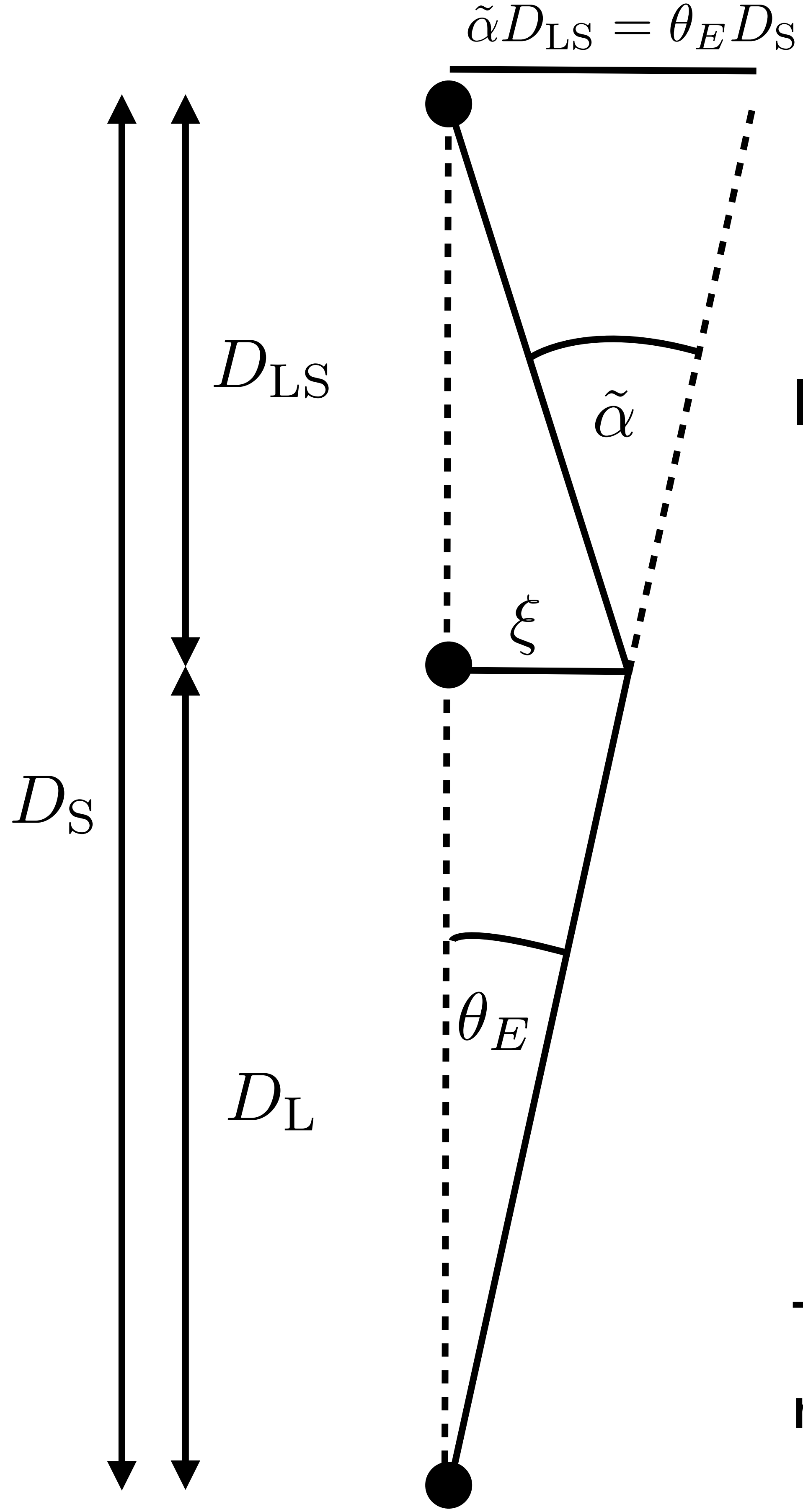


Deflection angle:

$$\tilde{\alpha}(\xi) = \frac{4GM(\xi)}{c^2}\frac{1}{\xi}$$

for mass M(ξ) within ξ
(Einstein 1915)

Distances are *angular diameter* distances.



$$\tilde{\alpha}(\xi) = \frac{4GM(\xi)}{c^2}\frac{1}{\xi}$$

If β=0 (source, lens and observer perfectly aligned):

$$\theta_E = \tilde{\alpha}\left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\right) = \frac{4GM}{c^2\xi}\left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\right)$$

$$= \frac{4GM}{c^2\theta_E D_{\mathrm{L}}}\left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\right)$$

$$\boxed{\theta_E^2 = \frac{4GM}{c^2}\left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}} D_{\mathrm{L}}}\right)}$$

The image of the background source is an “Einstein ring” with radius θ_E.

Einstein ring

$$\theta_E^2 = \frac{4GM}{c^2}\left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}D_{\mathrm{L}}}\right)$$

For stars in Milky Way:
$M \sim M_\odot$, $D \sim 1$ kpc $\rightarrow$ $\theta_E \sim 3\times10^{-3}$ arcsec

For galaxy clusters at cosmological distances ($z\sim1$)
$M \sim 10^{15}\ M_\odot$, $D \sim 2$ Gpc $\rightarrow$ $\theta_E \sim 60$ arcsec

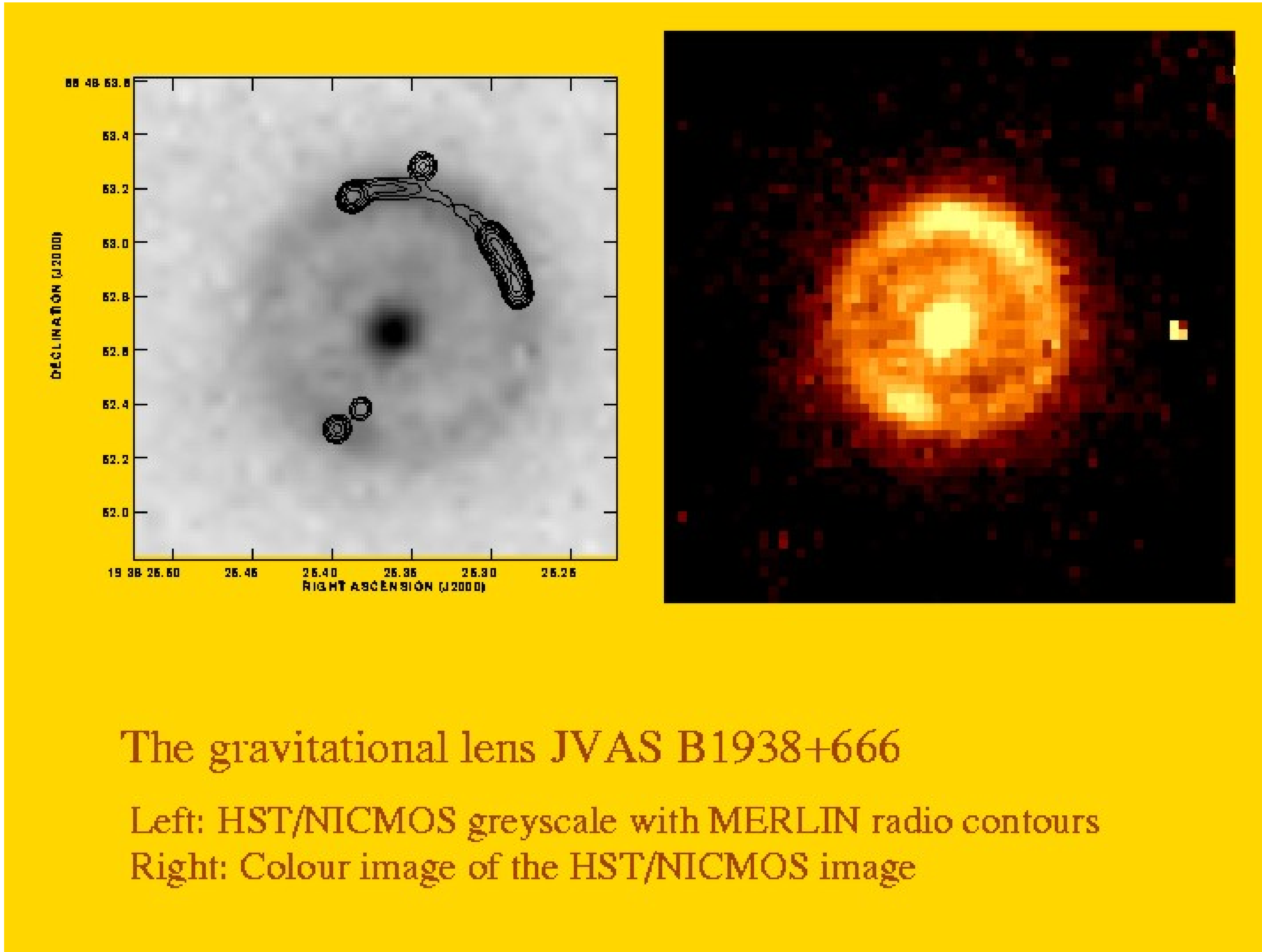
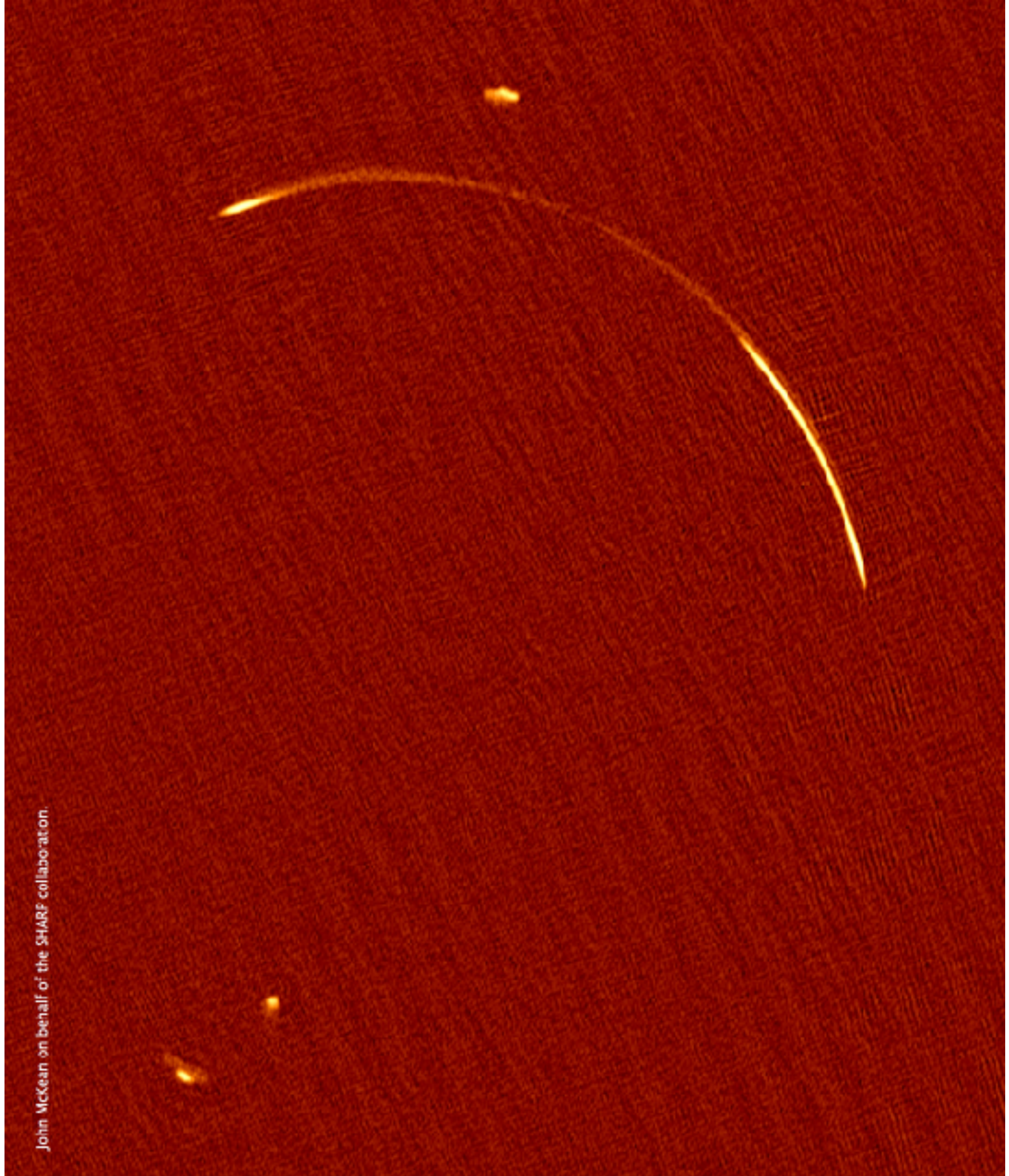


Figure 15: *Einstein ring 1938+666 (from [72]): The left panel shows the radio map as contour superimposed on the grey scale HST/NiCMOS image; the right panel is a color depiction of the infrard HST/NICMOS image. The diameter of the ring is about 0.95 arcseconds. (Credits: Neal Jackson.)*

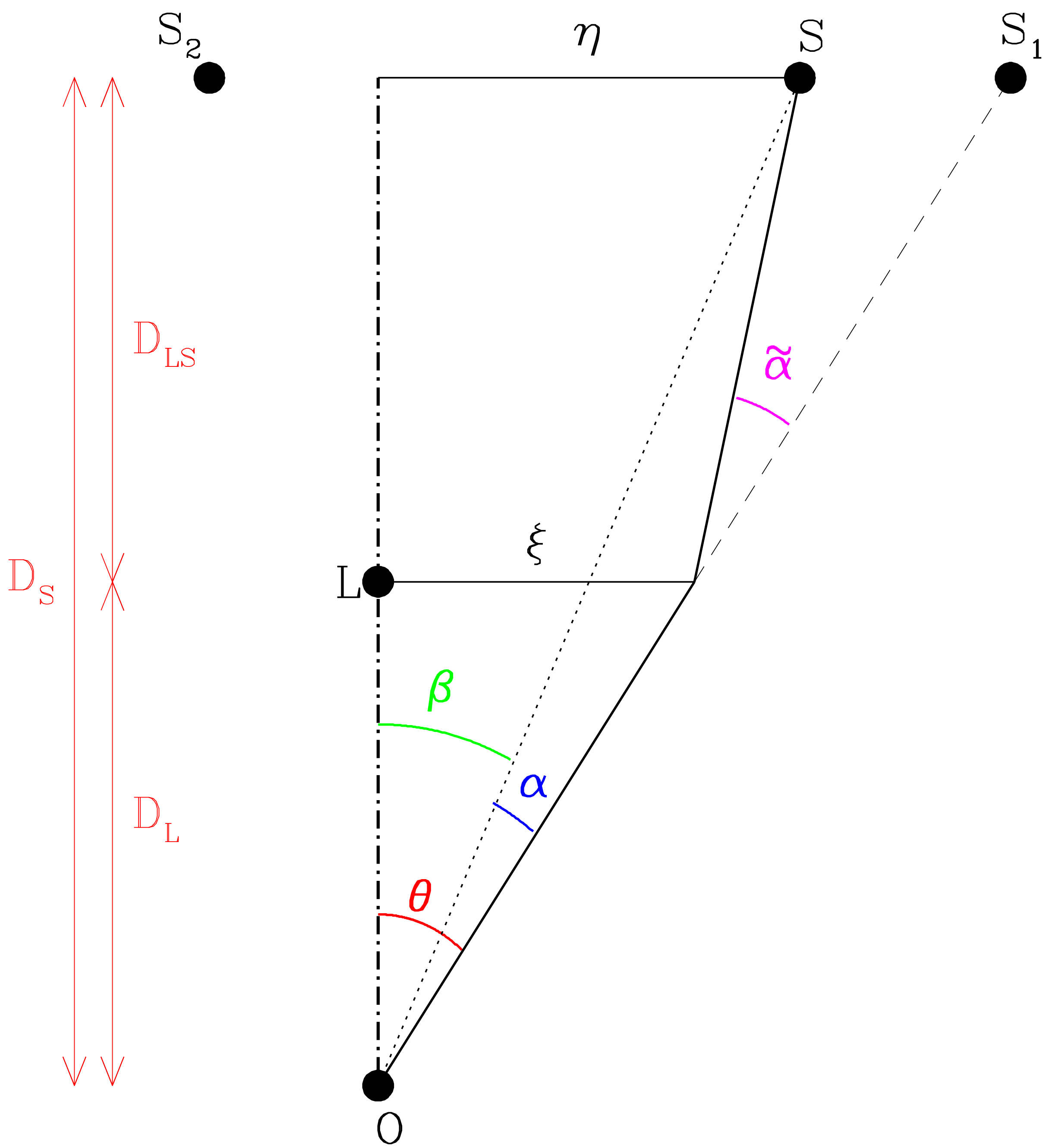


Global VLBI imaging of the gravitational lens JVAS B1938+666.

ASTRON

Netherlands Institute for Radio Astronomy

The lens equation



Reduced deflection angle, α (seen from O):

$$\beta = \theta - \alpha(\theta)$$

For small angles (as before)

$$\alpha(\theta) = \tilde{\alpha}\left(\frac{D_{\rm LS}}{D_{\rm S}}\right)$$

$$= \frac{4GM}{c^2\xi}\frac{D_{\rm LS}}{D_{\rm S}} = \frac{4GM}{c^2\theta}\left(\frac{D_{\rm LS}}{D_{\rm S}D_{\rm L}}\right)$$

Recall Einstein radius

$$\theta_E^2 = \frac{4GM}{c^2}\left(\frac{D_{\rm LS}}{D_{\rm S}D_{\rm L}}\right)$$

Thus, $\alpha(\theta) = \dfrac{\theta_E^2}{\theta}$ $\quad\quad \beta = \theta - \theta_E^2/\theta$

Which has the two solutions:

$$\boxed{\theta = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2}}$$

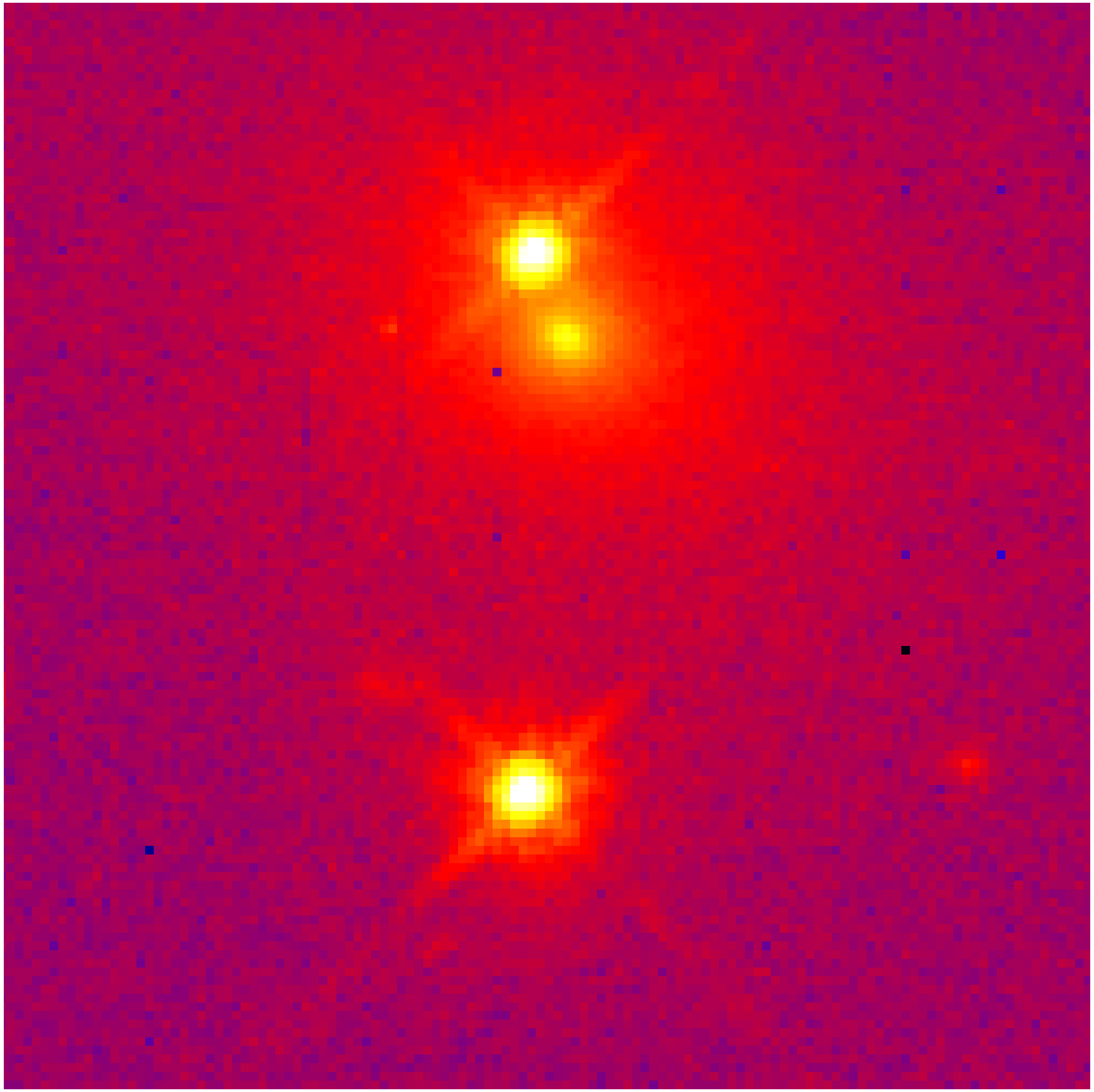


Figure 6: *In this false color Hubble Space Telescope image of the double quasar Q0957+561A,B. The two images A (bottom) and B (top) are separated by 6.1 arcseconds. Image B is about 1 arcsecond away from the core of the galaxy, and hence seen "through" the halo of the galaxy. (Credits: E.E. Falco et al. (CASTLE collaboration [47]) and NASA.)*

Wambsganss 1998

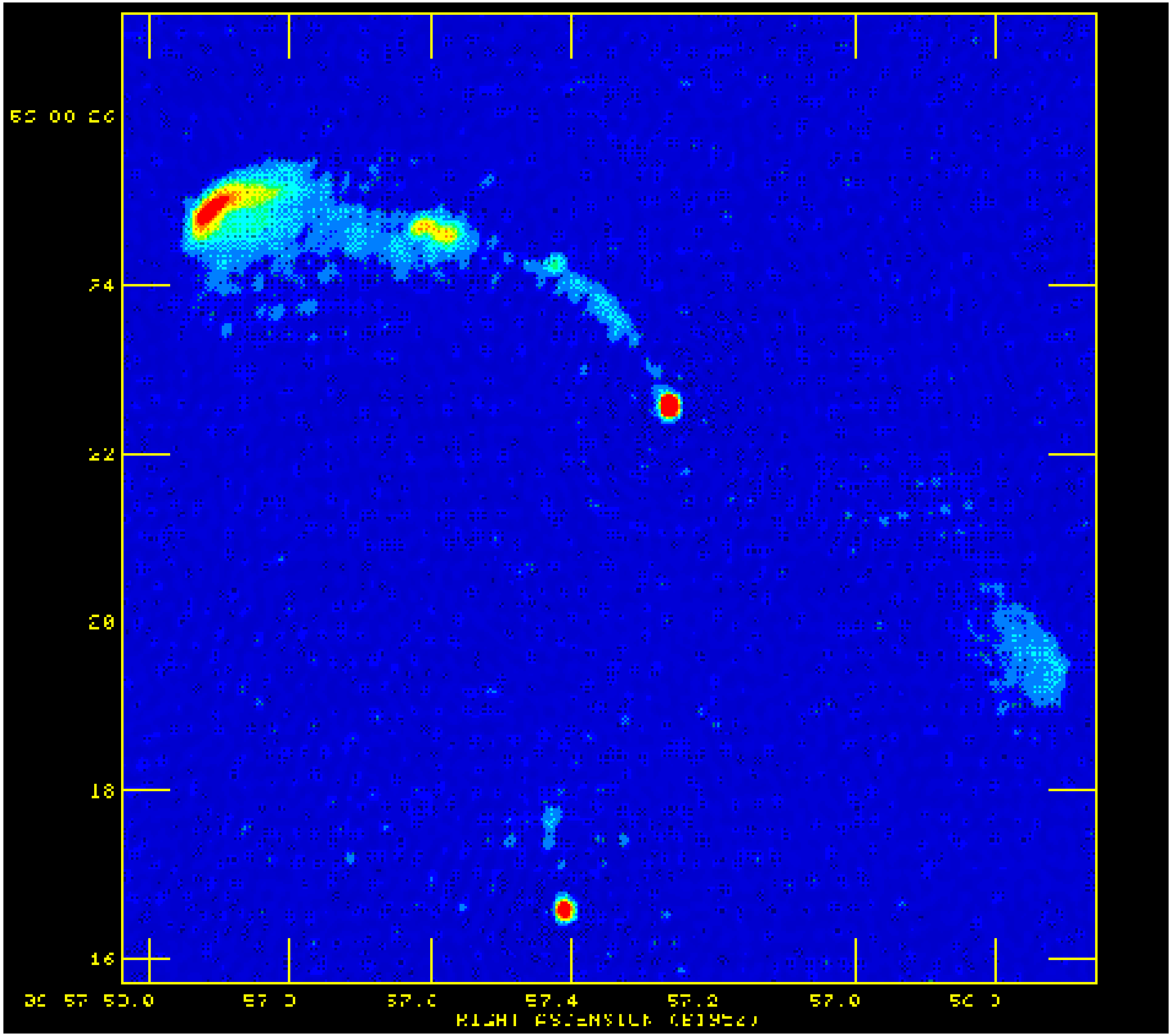
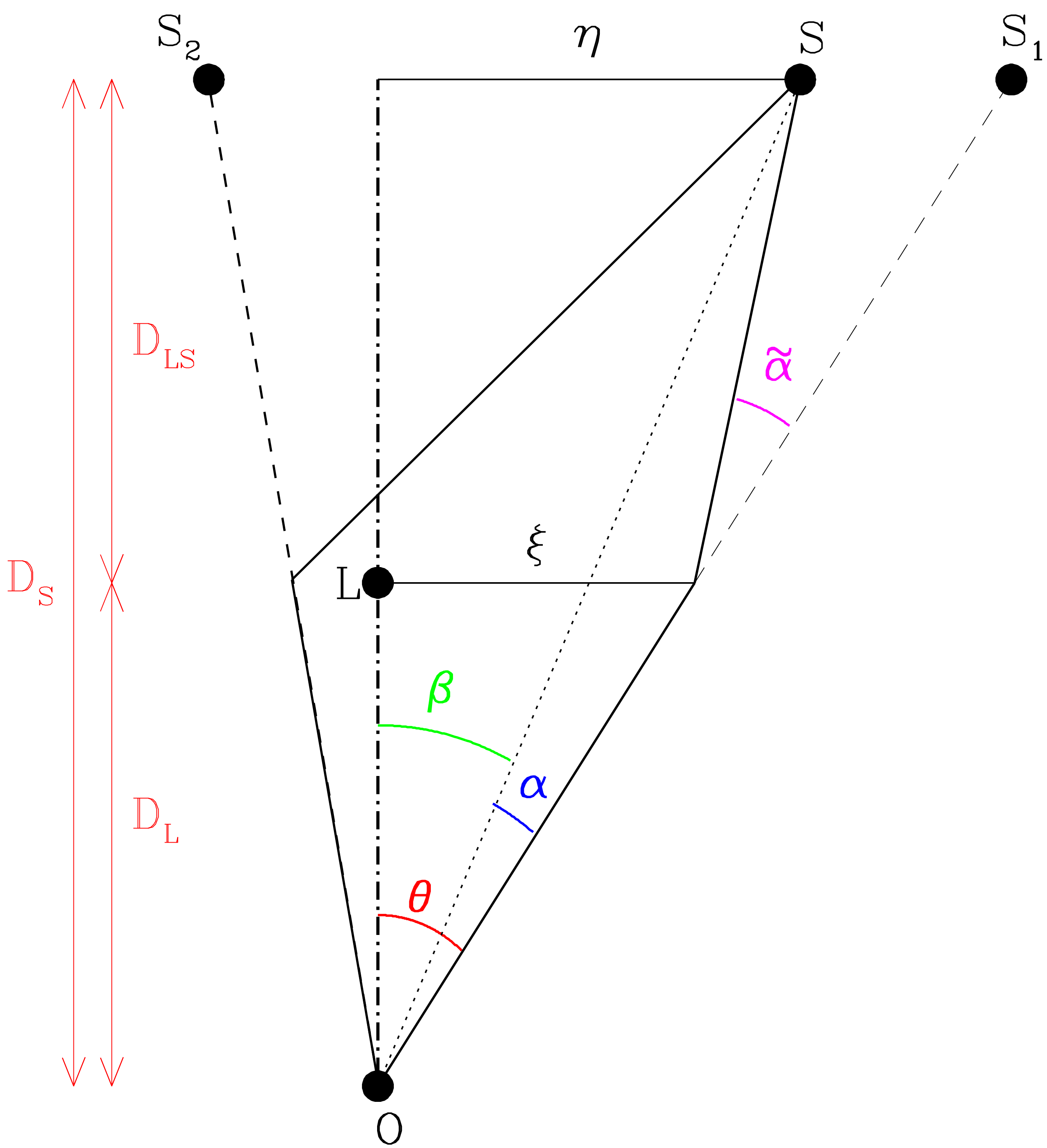


Figure 7: *Radio image of Q0957+561 from MERLIN telescope. It clearly shows the two point like images of the quasar core and the jet emanating only from the Northern part. (Credits: N. Jackson, Jodrell Bank.)*

Time delay



Light travel times differ for S_1 and S_2:

Time delay:

$$\tau(\theta,\beta) = \frac{1+z_{\rm L}}{c}\frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}\left(\frac{1}{2}(\theta-\beta)^2 - \psi(\theta)\right)$$

Geometry "Shapiro delay"

Distances depend on H_0:
$D_L = f(z_L, H_0)$
$D_S = f(z_S, H_0)$

Gravitational lensing can constrain H_0.

Wambsganss (1998)

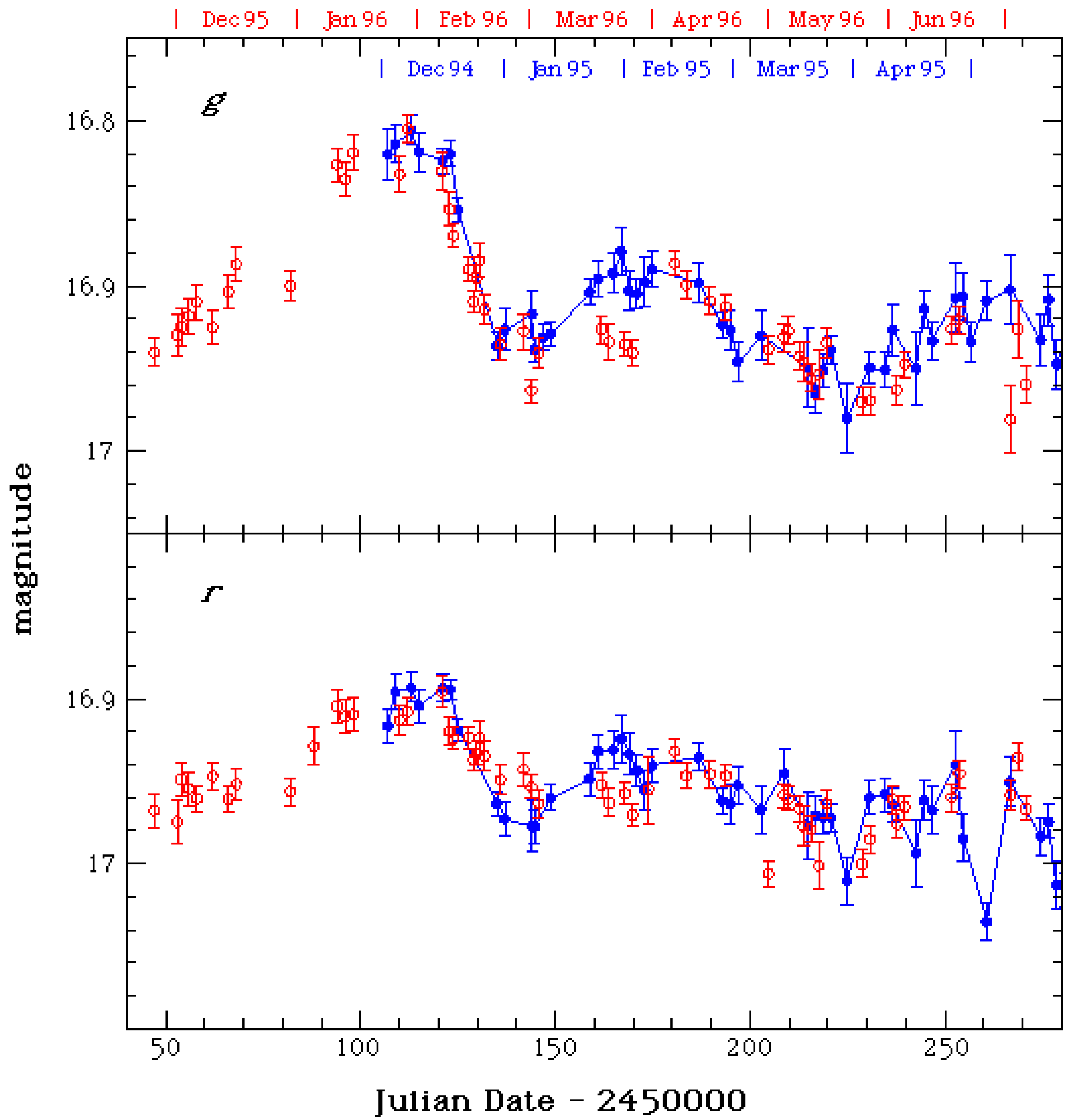
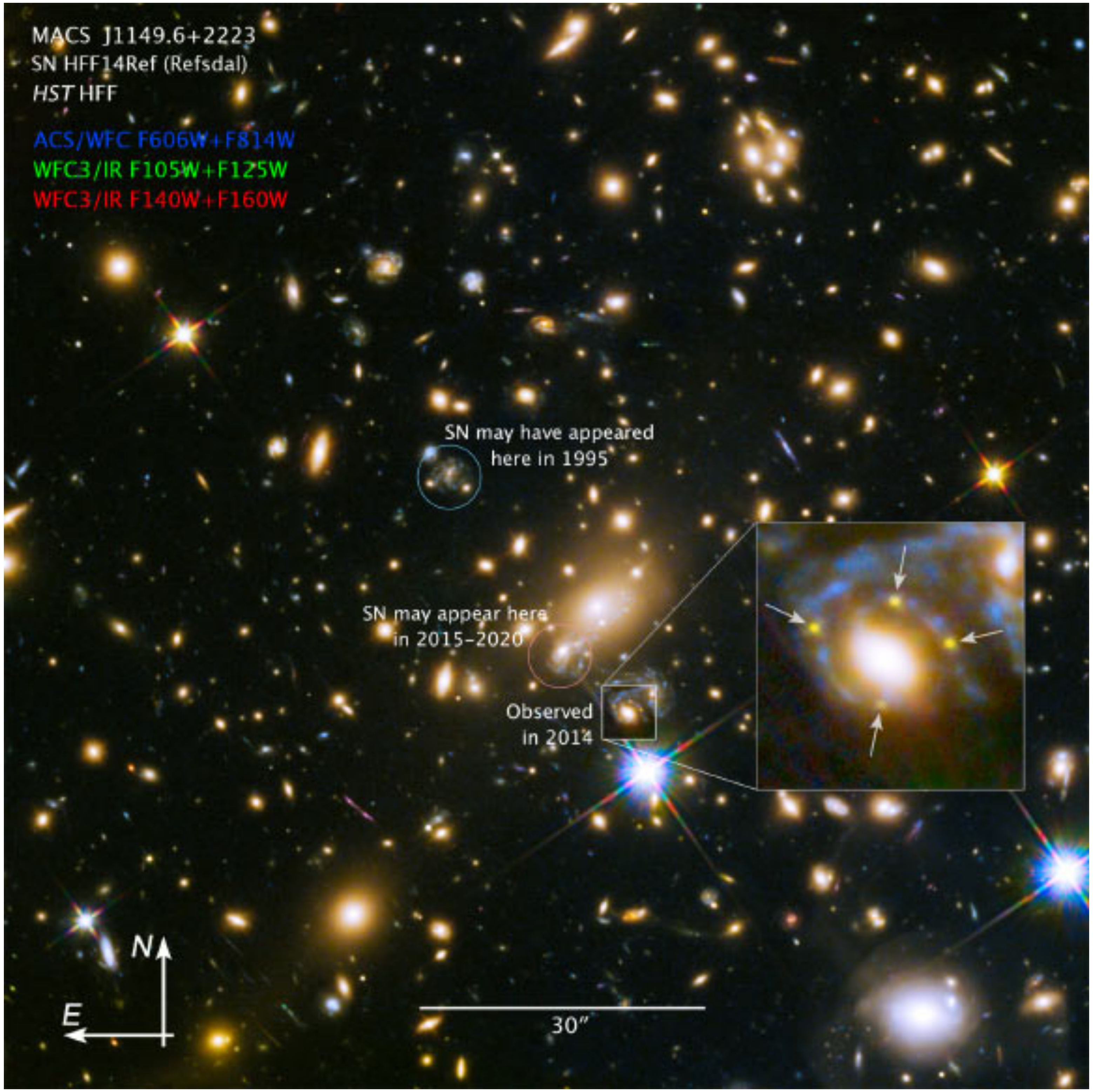
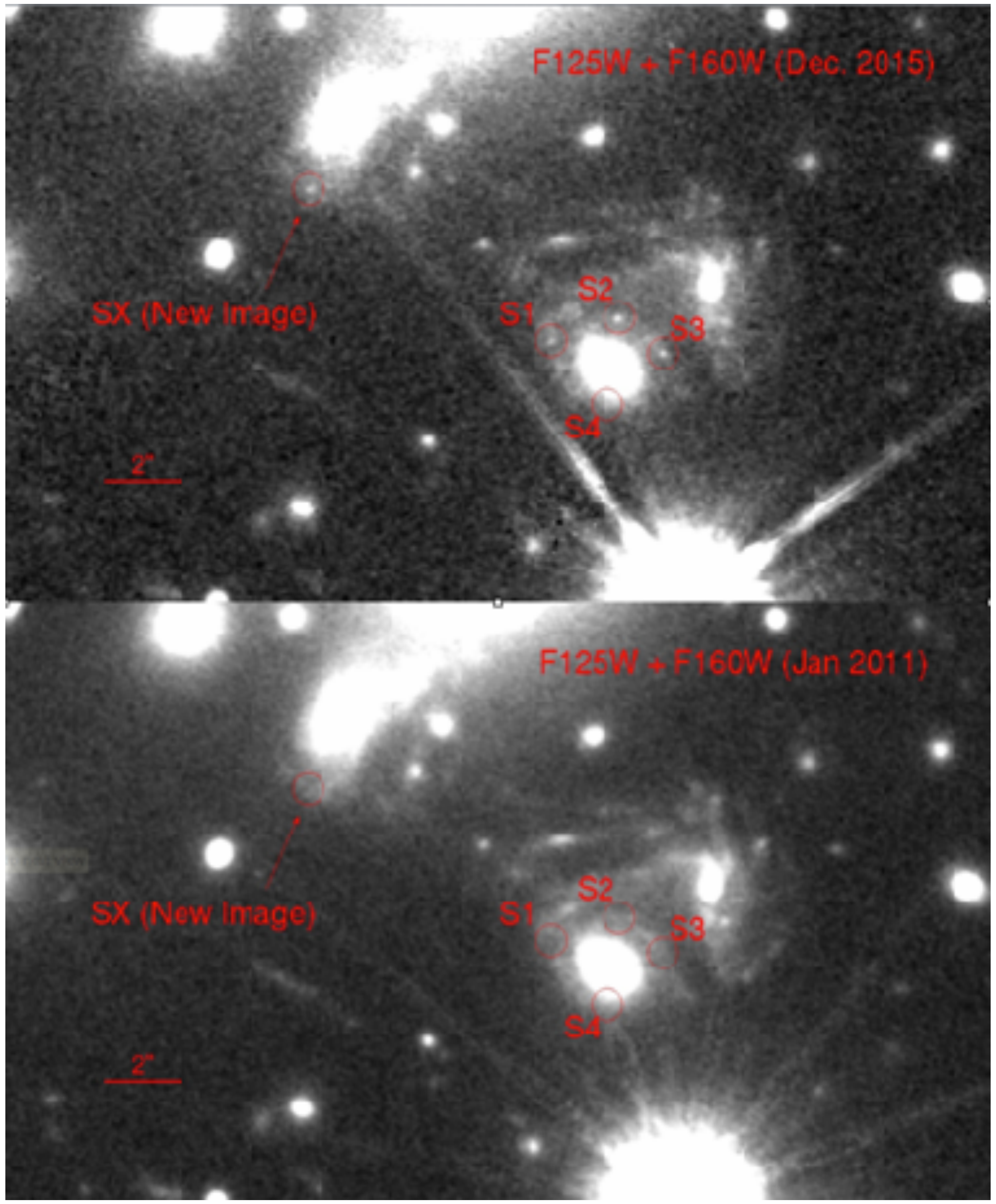
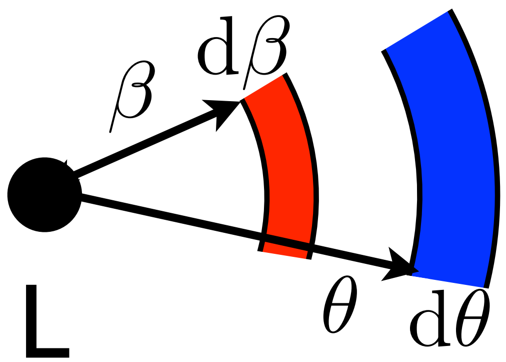


Figure 8: *Optical Lightcurves of images Q0957+561 A and B (top panel: g-band; bottom panel: r-band). The blue curve is the one of leading image A, the red one the trailing image B. Note the steep drop that occured in December 1994 in image A and was seen in February 1996 in image B. The light curves are shifted in time by about 417 days relative to each other. (Credits: Tomislav Kundić; see also [79])*


MACS J1149.6+2223
SN HFF14Ref (Refsdal)
HST HFF
ACS/WFC F606W+F814W
WFC3/IR F105W+F125W
WFC3/IR F140W+F160W
SN may have appeared here in 1995
SN may appear here in 2015–2020
Observed in 2014
N
E
30″


F125W + F160W (Dec. 2015)
SX (New image)
S1
S2
S3
S4
2"
F125W + F160W (Jan 2011)
SX (New image)
S1
S2
S3
S4
2"

Magnification by lensing



Area of unlensed image: $A_U \propto \beta \, \mathrm{d}\beta$

Area of lensed image: $A_L \propto \theta \, \mathrm{d}\theta$

Magnification: $$\mu = \frac{\theta \, \mathrm{d}\theta}{\beta \, \mathrm{d}\beta}$$

$$\beta = \theta - \theta_E^2/\theta$$

$$\mu^{-1} = \frac{\beta \, \mathrm{d}\beta}{\theta \, \mathrm{d}\theta} = \left(1 - \left[\frac{\theta_E}{\theta}\right]^2\right)\left(1 + \left[\frac{\theta_E}{\theta}\right]^2\right) = \left(1 - \left[\frac{\theta_E}{\theta}\right]^4\right)$$

Using

$$\theta = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2}$$

this reduces to

$$\mu = \frac{u^2 + 2}{2u\sqrt{u^2 + 4}} \pm \frac{1}{2} \qquad u = \beta/\theta_E$$

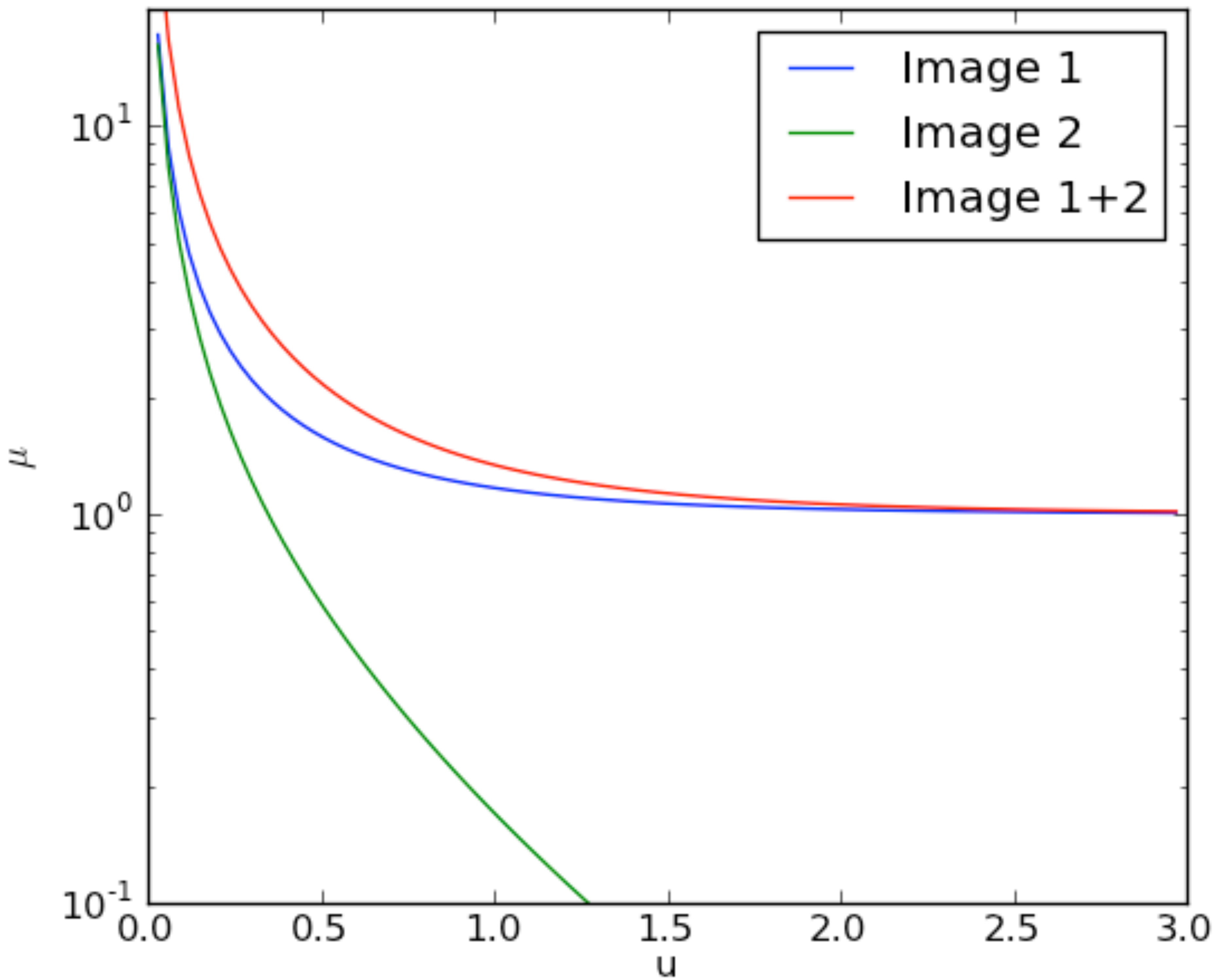
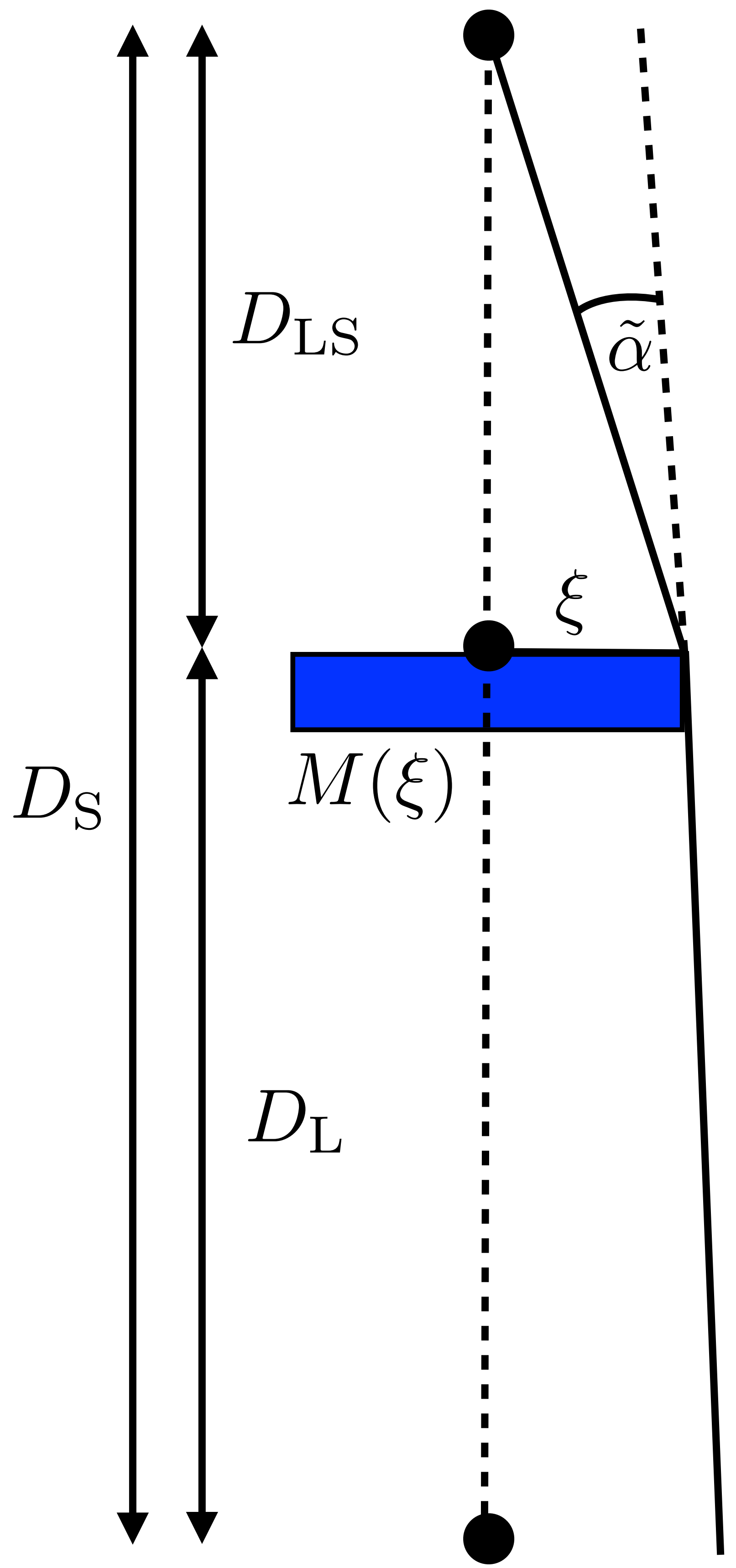


Image 1
Image 2
Image 1+2
10^1
10^0
10^{-1}
0.0
0.5
1.0
1.5
2.0
2.5
3.0
u
μ

Extended deflectors

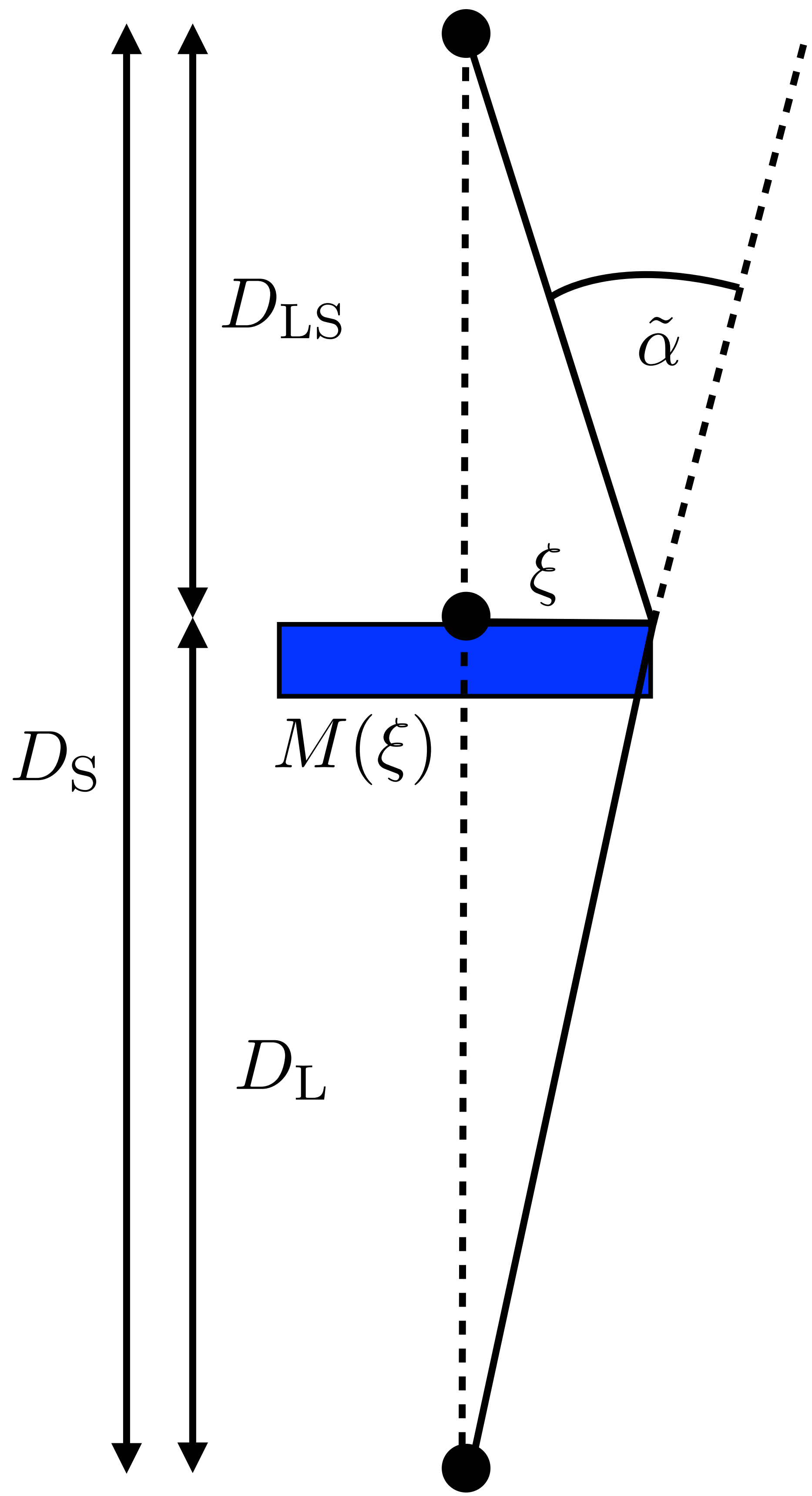


Why don't we *always* get multiple lensed images?

Consider a light ray grazing the edge of the mass distribution at radius ξ

If $M(\xi)$ is too small, then the deflected light ray does not reach the observer and no multiple images are seen.

Extended deflectors



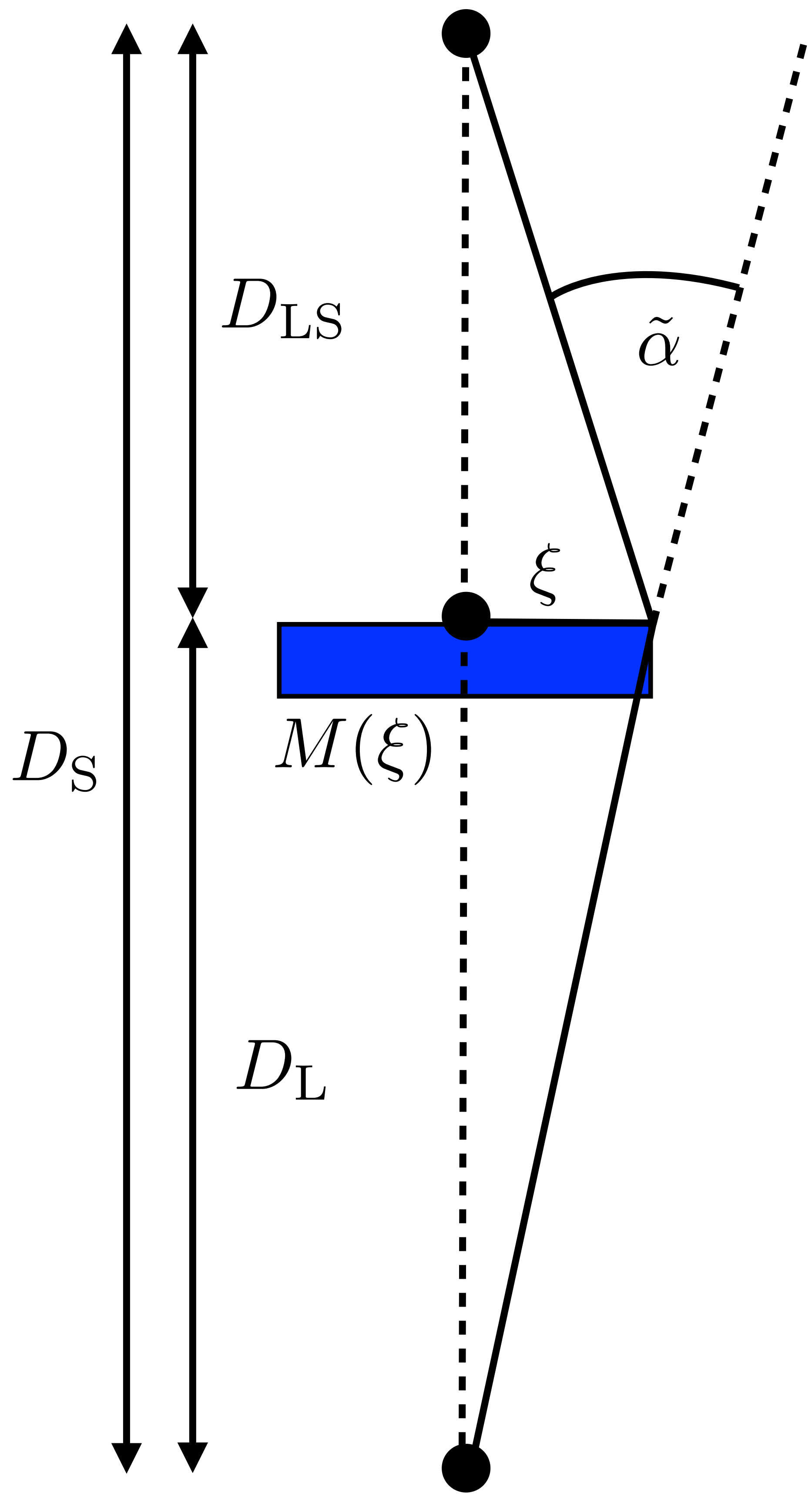
Why don't we *always* get multiple lensed images?

Consider a light ray grazing the edge of the mass distribution at radius ξ

If $M(\xi)$ is too small, then the deflected light ray does not reach the observer and no multiple images are seen.

The enclosed mass $M(\xi)$ must be sufficient to produce an Einstein ring with radius ξ/D_L

Extended deflectors



Einstein radius:

$$\theta_E^2 = \frac{4GM}{c^2}\left(\frac{D_{\rm LS}}{D_{\rm S} D_{\rm L}}\right)$$

Define *critical mass surface density*

$$\Sigma_{\rm crit} = \frac{M}{\pi R_E^2} = \frac{M}{\pi \theta_E^2 D_L^2}$$

$$\boxed{\Sigma_{\rm crit} = \frac{c^2}{4\pi G}\left(\frac{D_{\rm S}}{D_{\rm L} D_{\rm LS}}\right)}$$

For cosmological distances,

$$\Sigma_{\rm crit} \approx 10^{15} M_\odot \, {\rm Mpc}^{-2}$$

Comparable to that of rich galaxy clusters

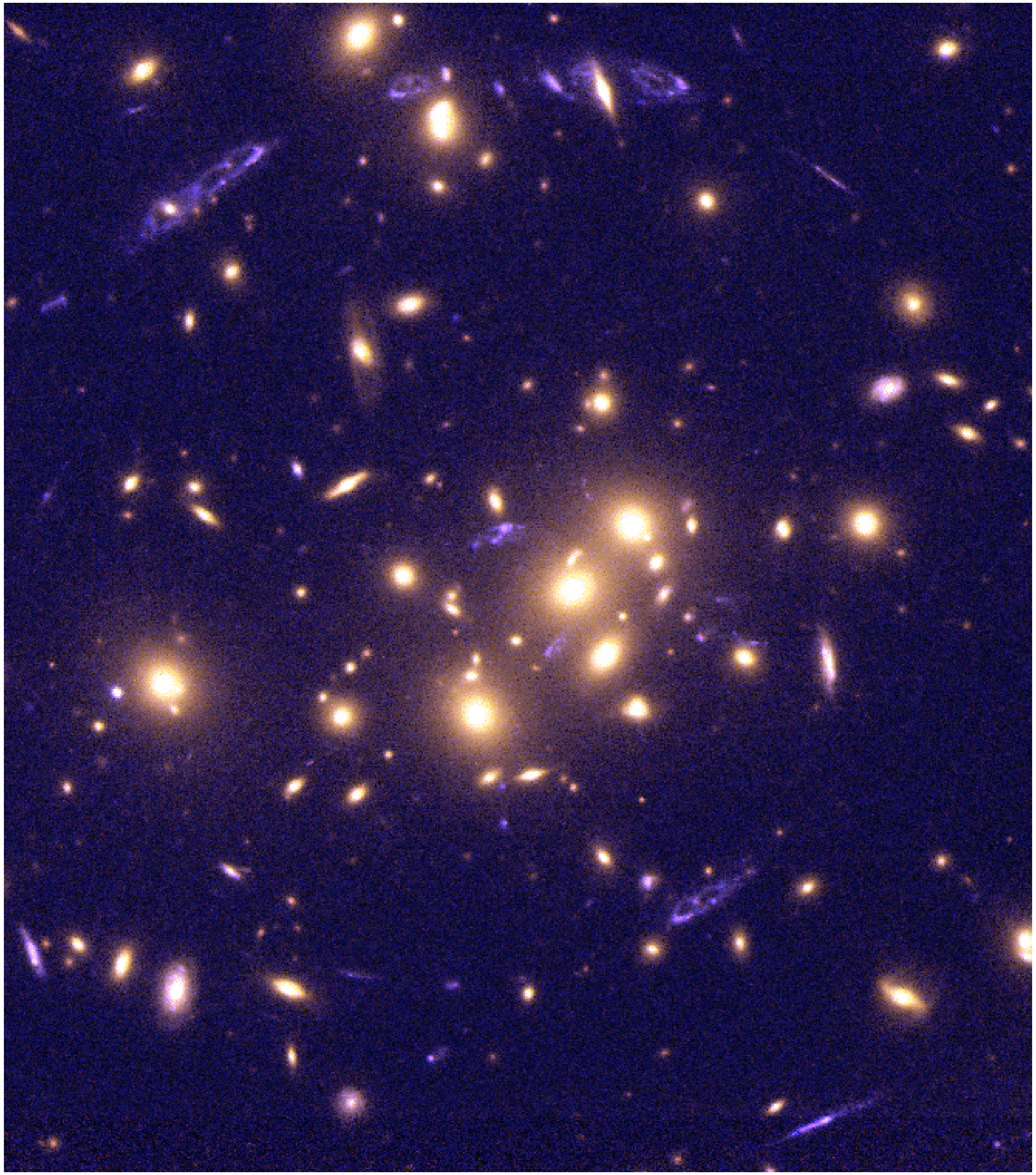
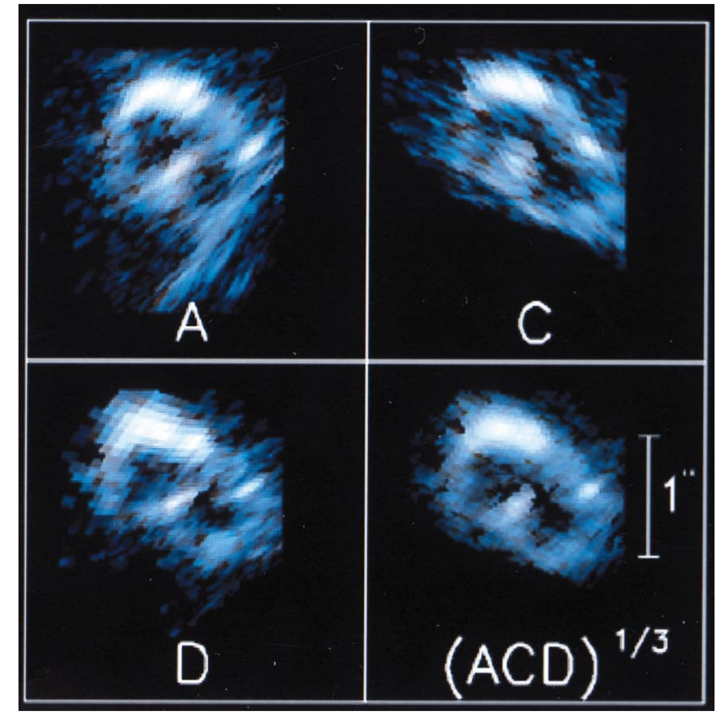
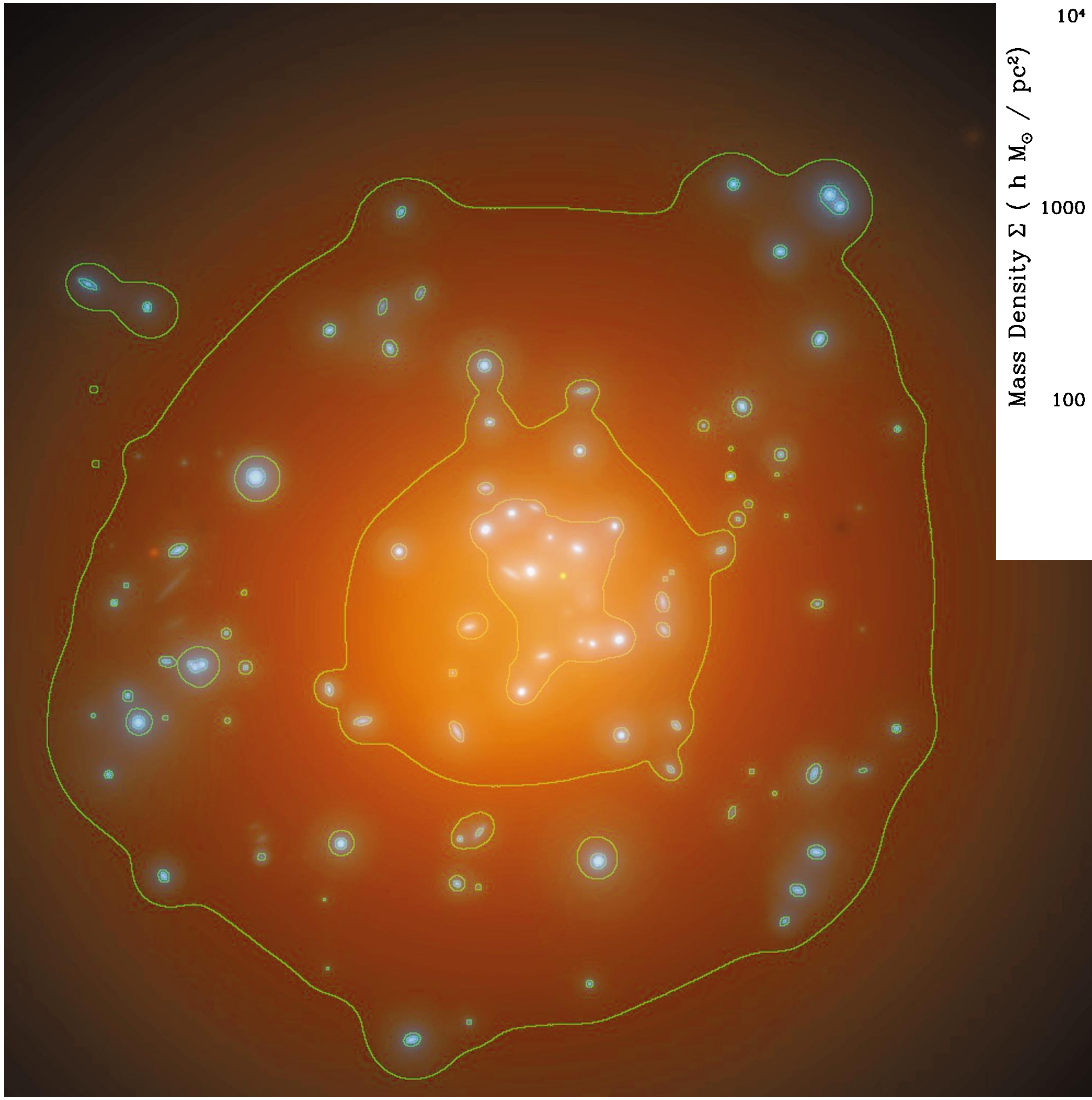


Figure 17: *Galaxy Cluster CL0024+1654 with multiple images of a blue background galaxy. The original picture and more information can be obtained at [144]. A scientific analysis which includes a reconstruction of the source galaxy can be found in [33]. (Credits: W.N. Colley, E. Turner, J.A. Tyson and NASA.)*



Reconstructed images of the source galaxy (Colley et al. 1996)



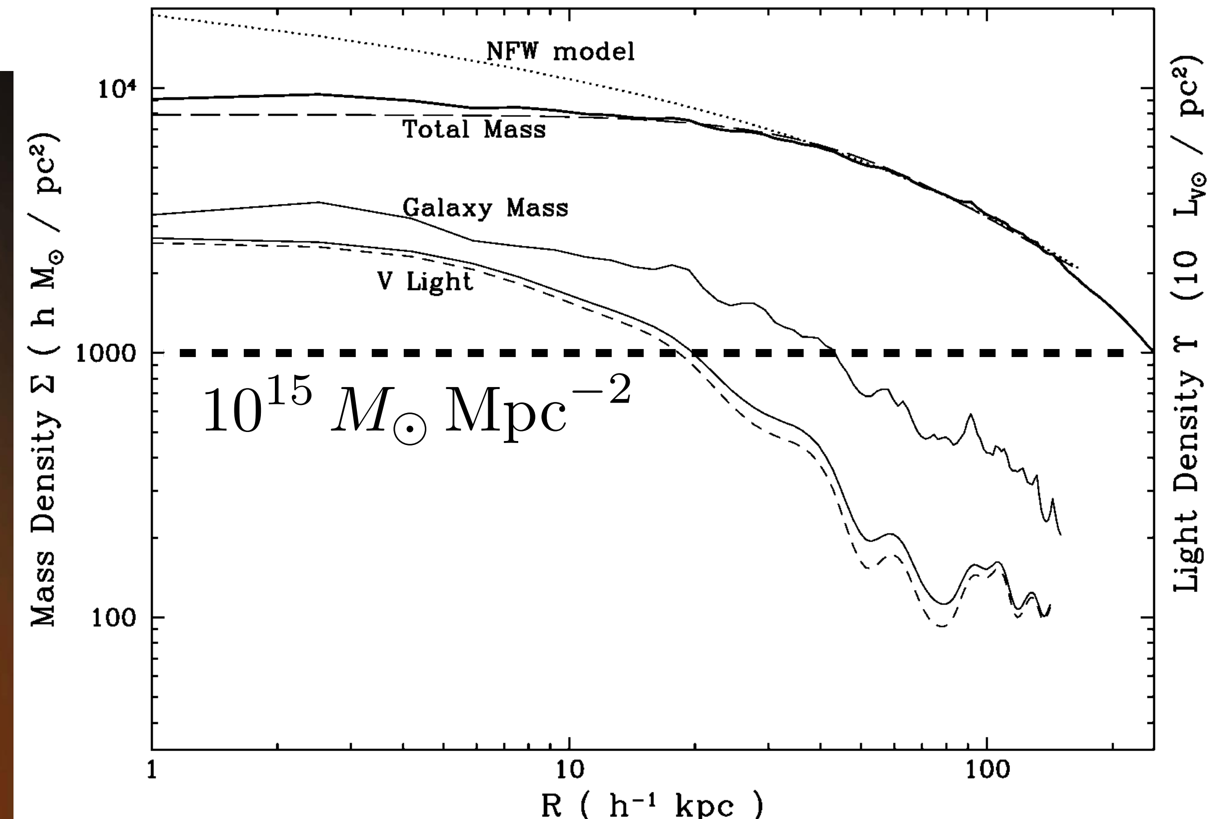


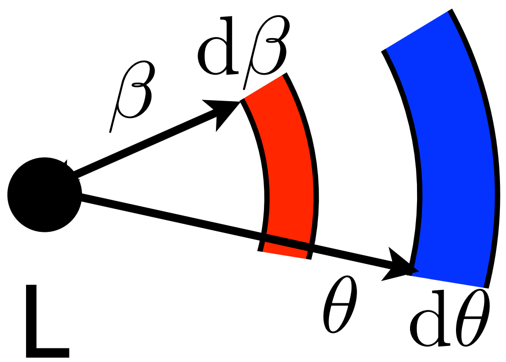
FIG. 1.—The reconstructed total mass density in CL 0024 is shown as a color-coded mass image. The DM is shown in orange. The mass associated with visible galaxies is shown in blue. The contours are at 0.5, 1, and 1.5 times the critical lensing density ($4497\,h\,d_{0.57}^{-1}\,M_\odot\,\mathrm{pc}^{-2}$), with heavier contour at the critical lensing density. This image is 336 h^{-1} kpc across. North is up, and east is left.

TYSON, KOCHANSKI, & DELL'ANTONIO (see 496, L108)

Weak lensing

- In principle, any line of sight in the Universe is affected by gravitational lensing at some level
- This modifies the brightness, shape and position of distant galaxies
- Effect generally too weak to measure for individual galaxies
- Measurable by averaging over large samples of galaxy images.

Distortion by lensing



Area of unlensed image: $A_U \propto \beta \,\mathrm{d}\beta$

Area of lensed image: $A_L \propto \theta \,\mathrm{d}\theta$

Magnification: $\mu = \dfrac{\theta \,\mathrm{d}\theta}{\beta \,\mathrm{d}\beta}$

$$\beta = \theta - \theta_E^2/\theta$$

$$\mu^{-1} = \frac{\beta \,\mathrm{d}\beta}{\theta \,\mathrm{d}\theta} = \left(1 - \left[\frac{\theta_E}{\theta}\right]^2\right)\left(1 + \left[\frac{\theta_E}{\theta}\right]^2\right) = \left(1 - \left[\frac{\theta_E}{\theta}\right]^4\right)$$

Distortion of lensed images, for $\theta > \theta_E$ (weak lensing):

$\dfrac{\mathrm{d}\theta}{\mathrm{d}\beta} < 1$ - images get *compressed* in radial direction

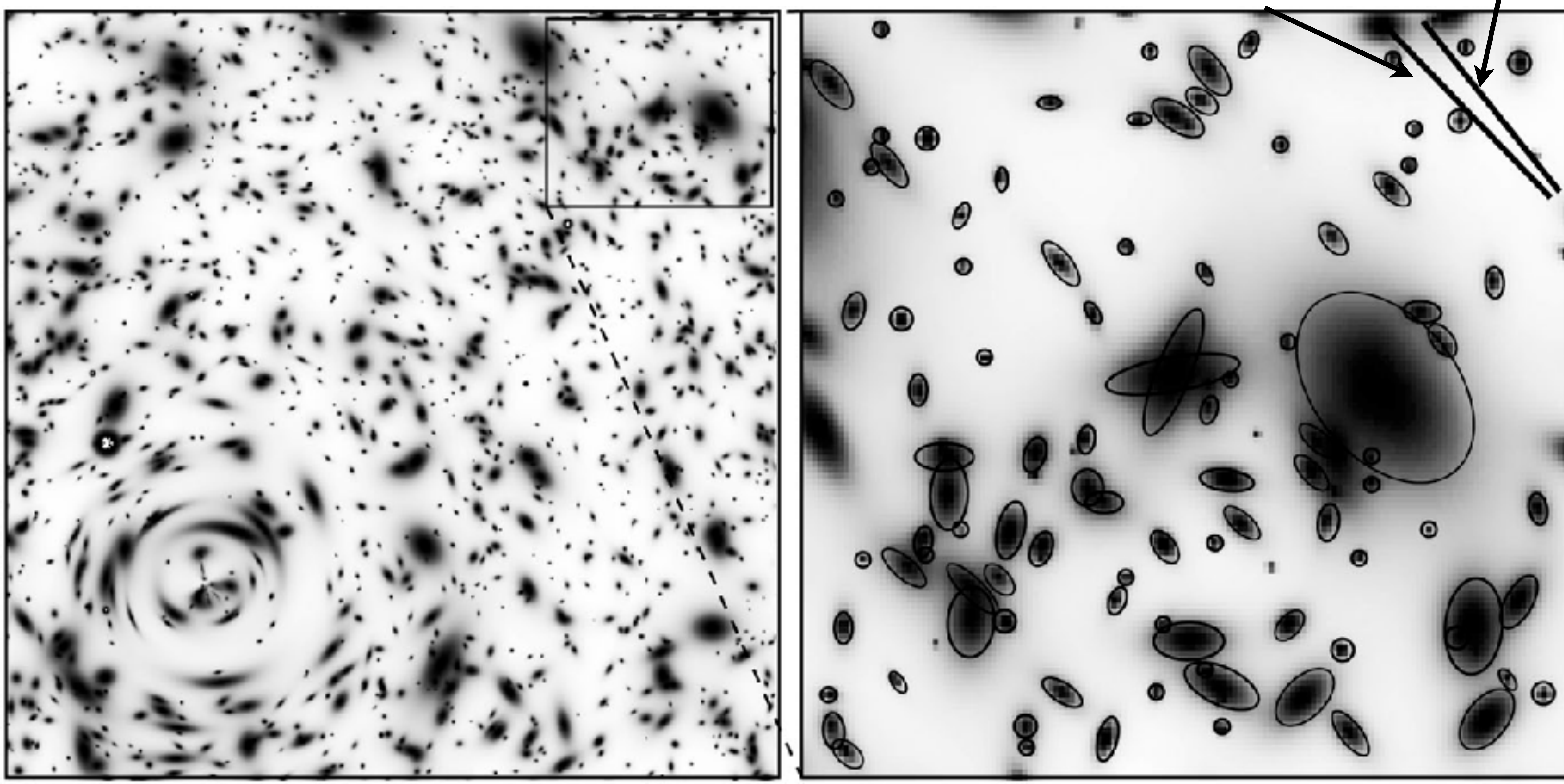
$\dfrac{\theta}{\beta} > 1$ - images get *enlarged* in tangential direction

Strong and weak lensing

Average measured orientation

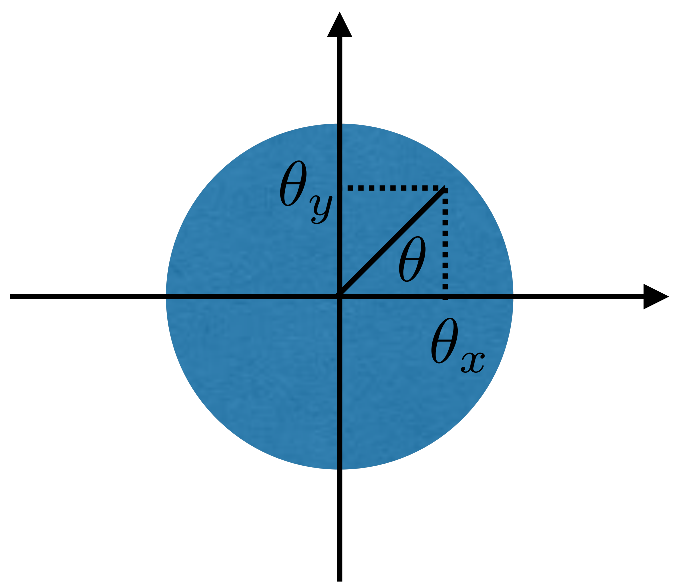
Mellier 1999, ARA&A 37, 127

True orientation of shear



Simulation of galaxy cluster at z=0.15 and background galaxies at <z>=1

Quantifying the shear



Define quadrupole moments of light distribution:

$$q_{ij} \equiv \int I_{\rm obs}(\theta)\theta_i\theta_j {\rm d}^2\theta$$

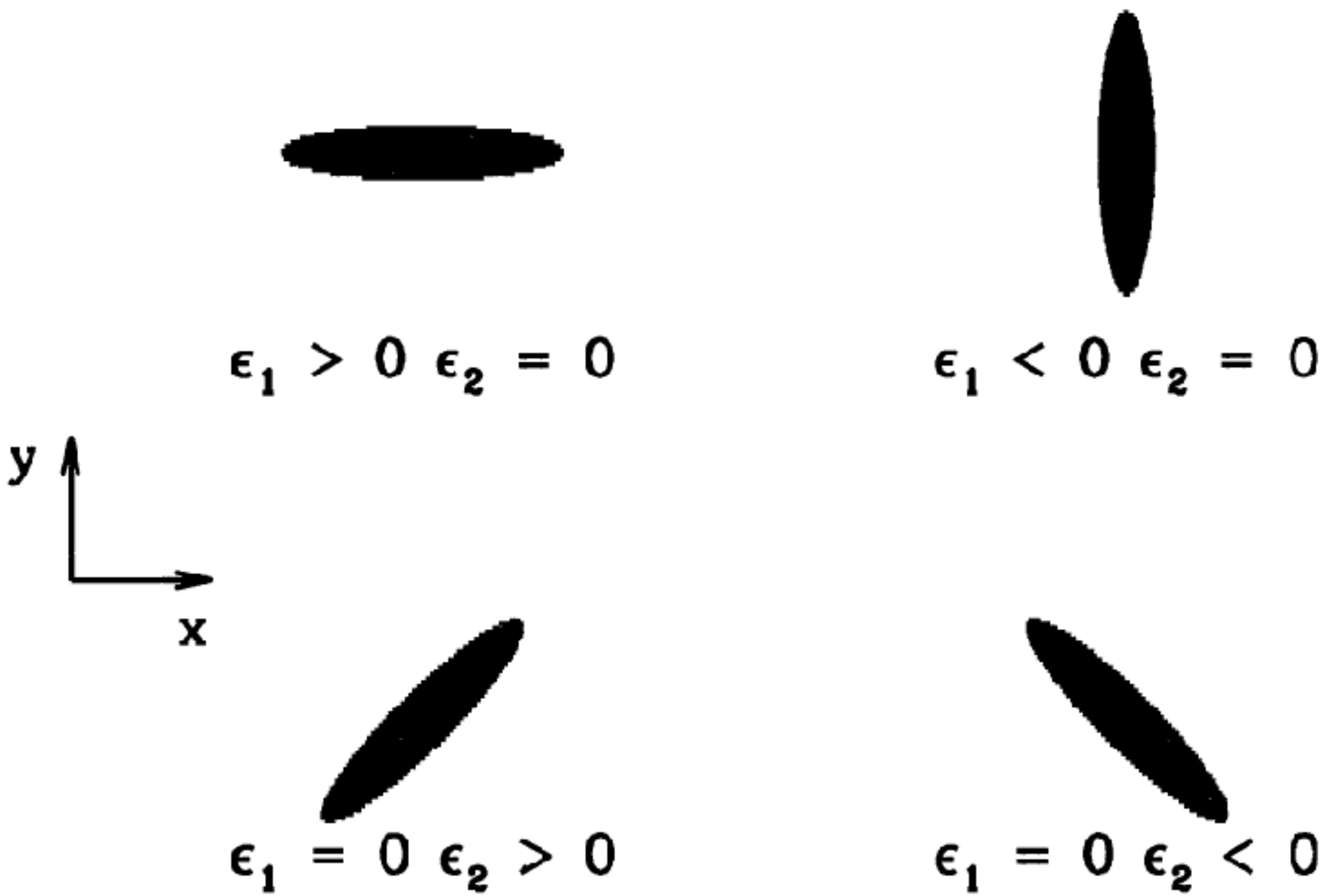
For a circularly symmetric source, we have

$$q_{xx} = q_{yy} \quad \text{and} \quad q_{xy} = 0$$

Define parameters ε_1 and ε_2:

$$\epsilon_1 \equiv \frac{q_{xx} - q_{yy}}{q_{xx} + q_{yy}} \qquad \epsilon_2 \equiv \frac{2q_{xy}}{q_{xx} + q_{yy}}$$

These will both be zero for symmetric sources


$\epsilon_1 > 0\ \epsilon_2 = 0$
$\epsilon_1 < 0\ \epsilon_2 = 0$
y
x
$\epsilon_1 = 0\ \epsilon_2 > 0$
$\epsilon_1 = 0\ \epsilon_2 < 0$

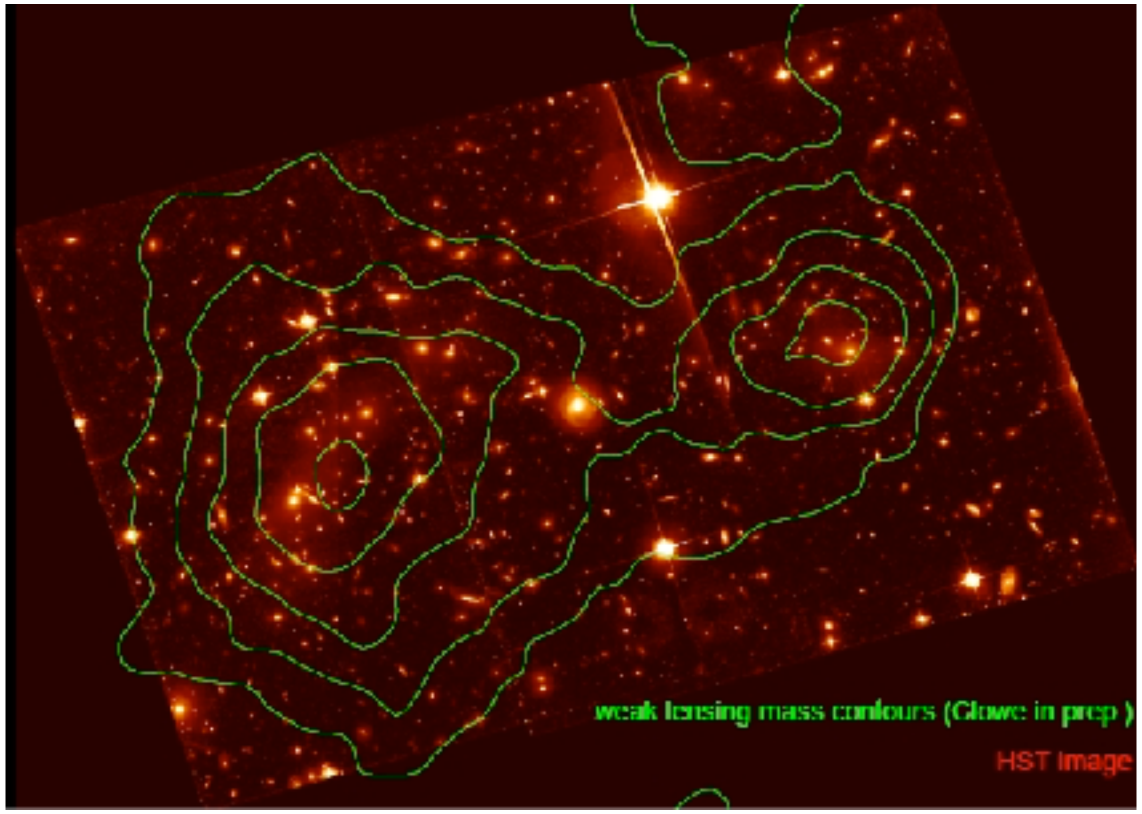
Weak lensing

- By measuring distortion of numerous background galaxies, one can work backwards and find the spatial distribution of lensing mass
- Statistical technique: distortion must be averaged over large number of background galaxies
- Technically challenging: distortion is small; need very good image quality

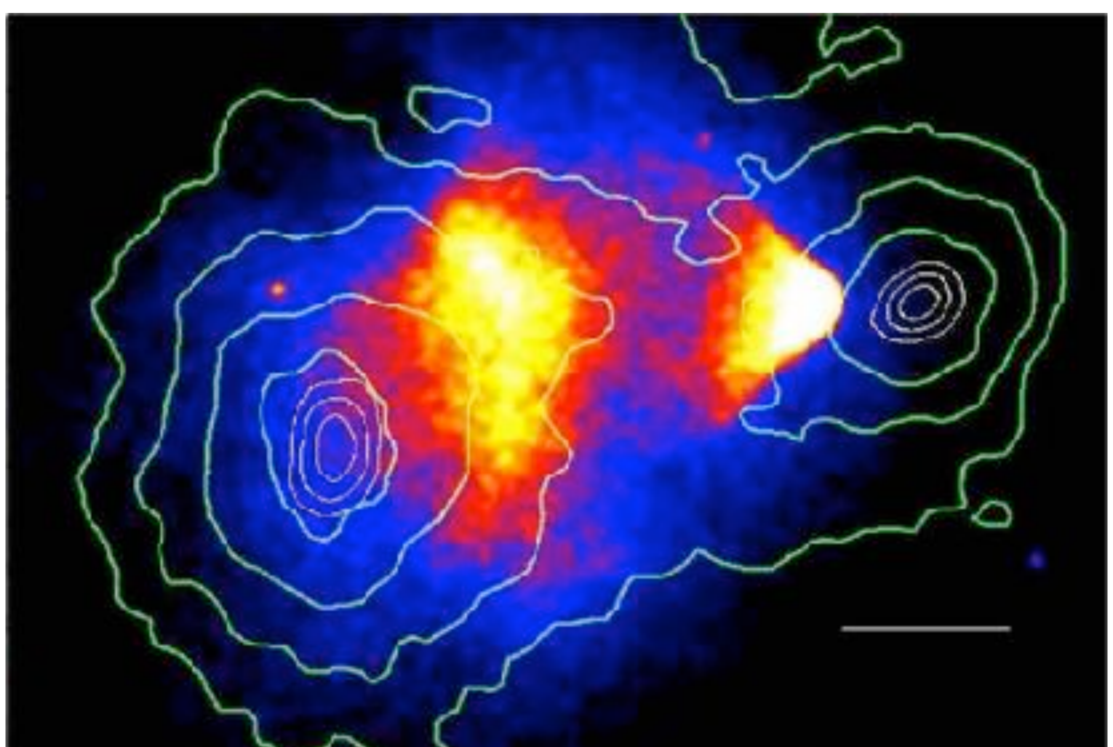
The Bullet Cluster

Colliding galaxy clusters at z=0.30

Visible and dark matter (non-collisional) separated from X-ray gas by collision.



Optical image and contours of mass distribution from Weak Lensing



X-Ray image and contours of mass distribution from Weak Lensing

Dark matter candidates

- The critical density, $\rho_{crit} = 10^{-26}$ kg m^{-3} (for H_0 = 72 km s^{-1} Mpc^{-1}) corresponds to about 6 H atoms m^{-3} (if DM were baryonic)
- MAssive Compact Halo Objects (MACHOs)?
- Weakly Interacting Massive Particles (WIMPs)?

Searching for MACHOs

- Paczynski (1986): "optical depth" of MW Halo to gravitational lensing is about 10^{-6}, independent of lens mass
- Out of 1 million stars, on average one will be magnified by gravitational lensing
- Monitoring of large numbers of stars in nearby galaxies might reveal MACHOs

Recap: gravitational lensing

Einstein ring:

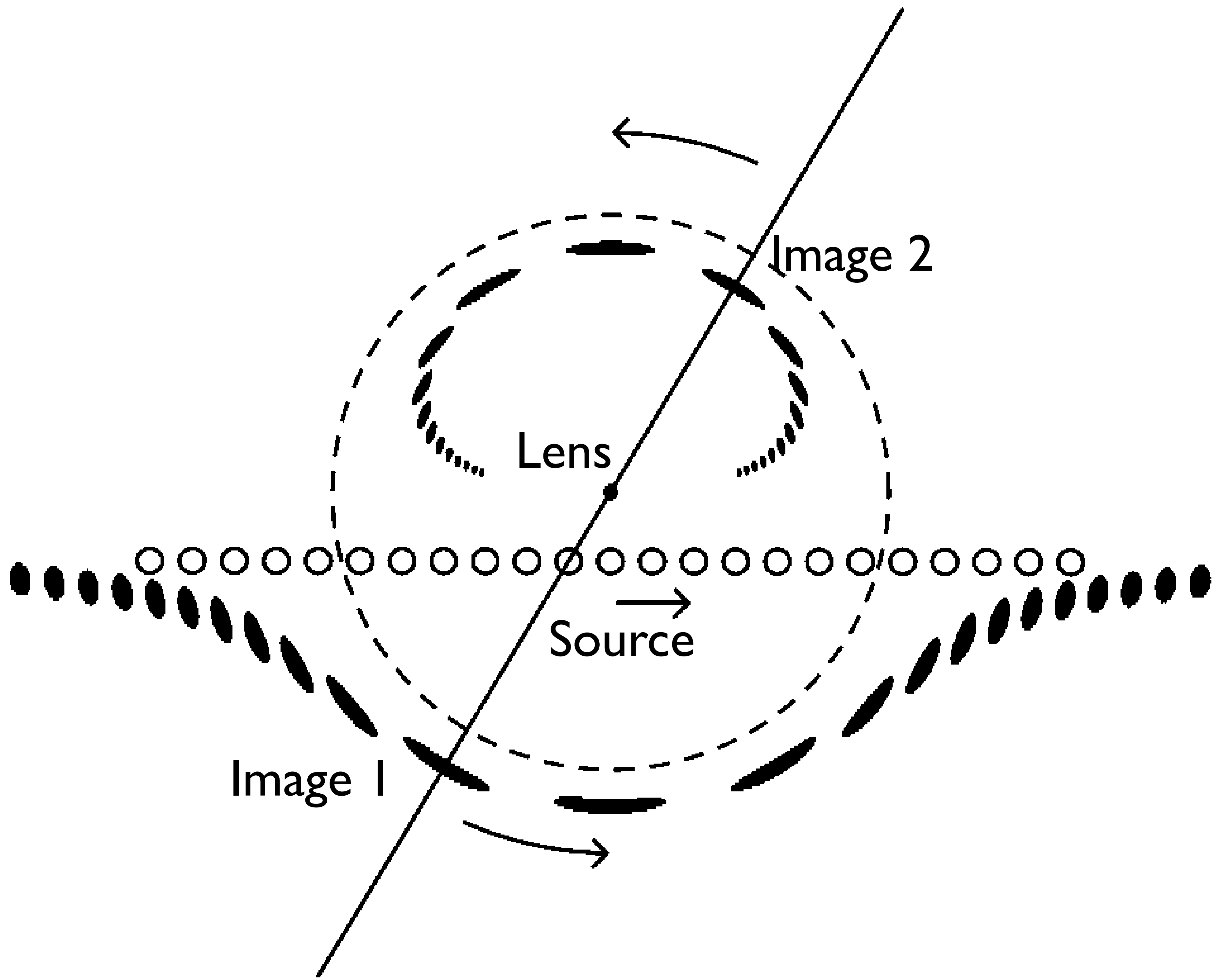
$$\theta_E^2 = \frac{4GM}{c^2}\left(\frac{D_{\rm LS}}{D_{\rm S}D_{\rm L}}\right)$$

Magnification:

$$\mu = \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2} \qquad\qquad u = \beta/\theta_E$$

$$\mu_{1+2} = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$$\mu_{1+2}(u=1) = 1.34$$



Paczynski (1996)

Gravitational microlensing

Optical depth to GL:

Probability *p(A)* that a star is magnified by a factor 1.34 or more

To first order, *p(A)* is *independent* of the mass of the deflectors:

For mass surface density Σ_M and lens mass *M*, the surface density of lenses Σ_L is

$$\Sigma_L = \Sigma_M / M$$

The Einstein ring of a single lens has area

$$A_E = \pi \theta_E^2 = \frac{4\pi G M}{c^2} \left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}} D_{\mathrm{L}}} \right)$$

Total optical depth is then

$$\tau_L = A_E \Sigma_L = \Sigma_M \frac{4\pi G}{c^2} \left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}} D_{\mathrm{L}}} \right)$$

Optical depth:

$$\tau_L = A_E \Sigma_L = \Sigma_M \frac{4\pi G}{c^2} \left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}} D_{\mathrm{L}}} \right)$$

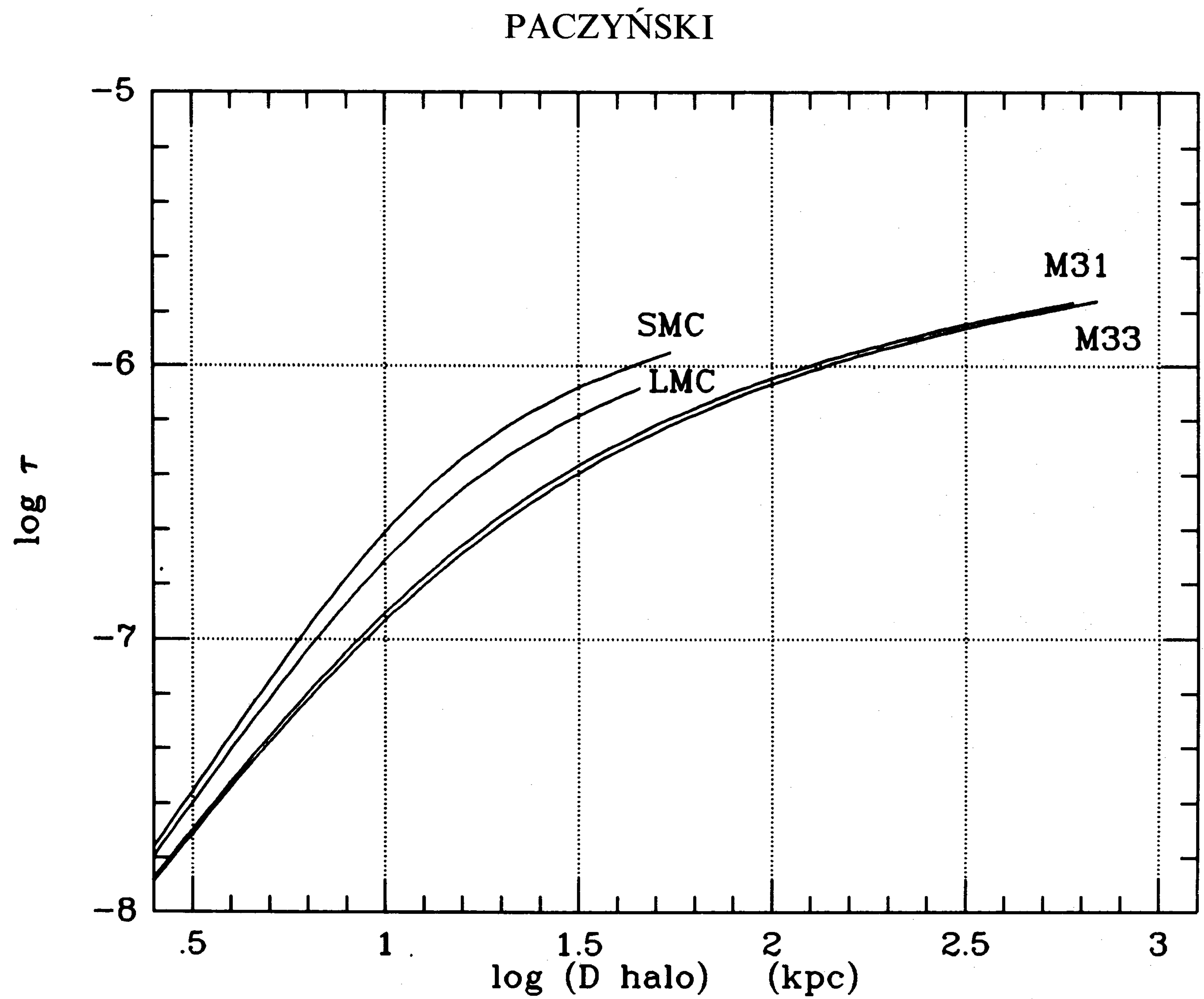
If we are at the centre of the dark halo with mass ~4×10^{11} $M_\odot$ (as suggested by the Galactic rotation curve), then

$$\Sigma_M \approx 4 \times 10^{11} M_\odot / 4\pi \approx 3 \times 10^{10} M_\odot \mathrm{sr}^{-1}$$

For deflectors at D_L=10 kpc, sources at D_S=50 kpc (LMC)

$$\tau_L \approx 1.5 \times 10^{-6}$$

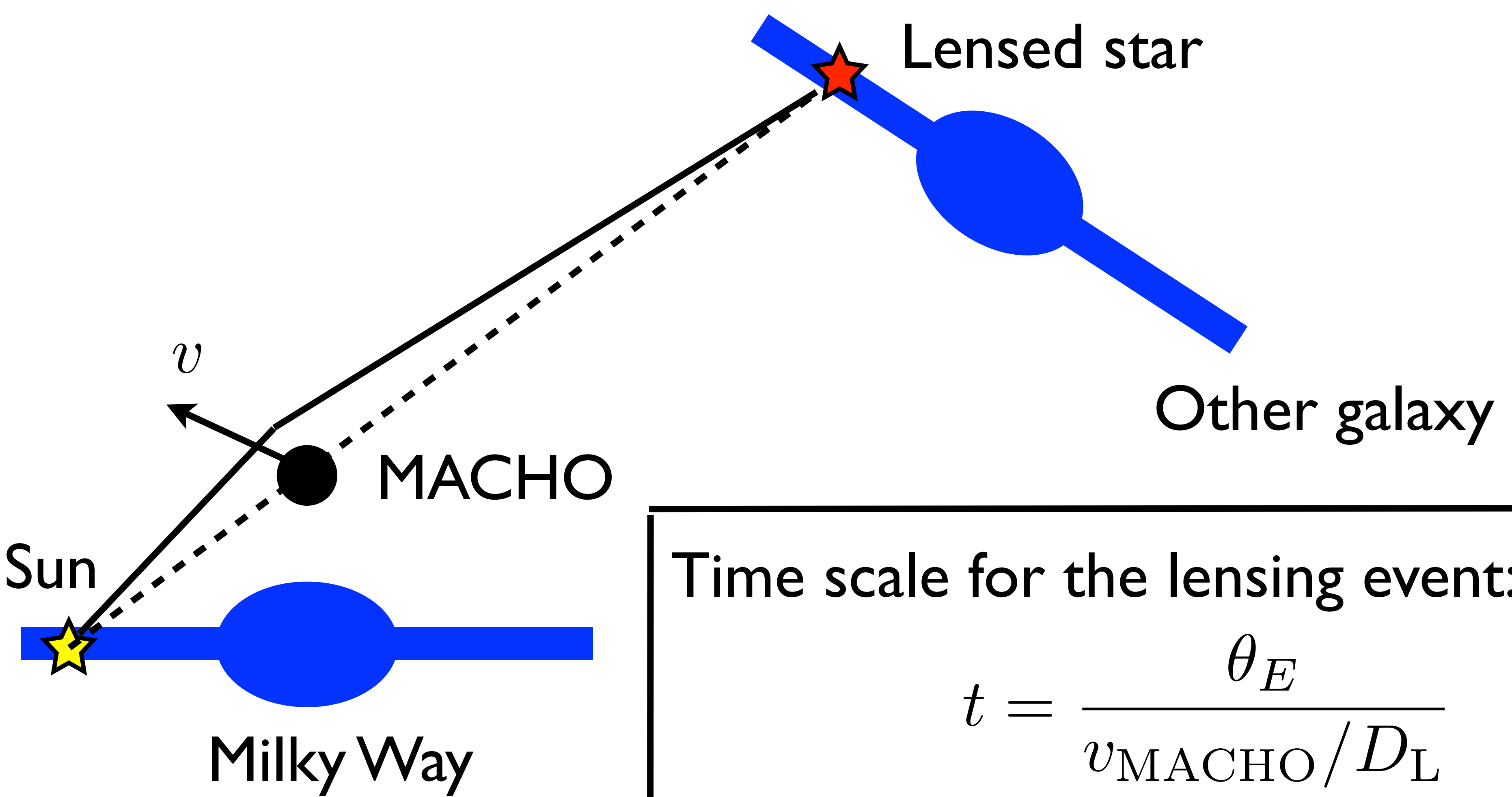
Optical depths for more realistic halo model



D = maximum extent of halo

Paczynski (1986)

Lensing by MACHOs in the Galactic halo



Time scale for the lensing event:

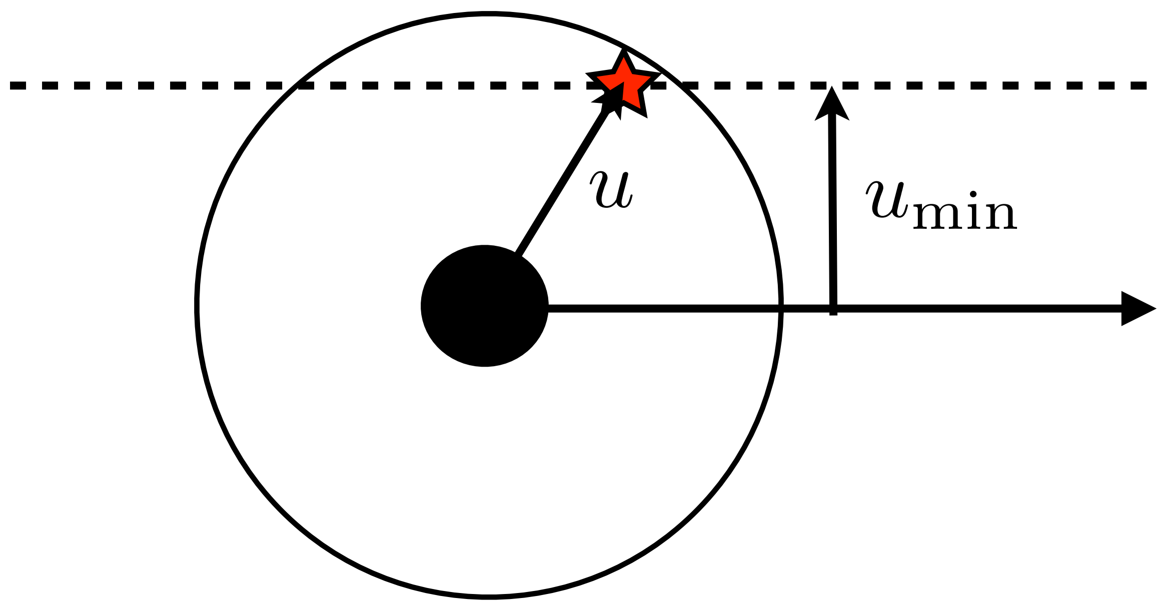
$$t = \frac{\theta_E}{v_{\rm MACHO}/D_{\rm L}}$$

$$= \frac{D_{\rm L}}{v_{\rm MACHO}} \sqrt{\frac{4GM}{c^2} \left(\frac{D_{\rm LS}}{D_{\rm S} D_{\rm L}} \right)}$$

For D_L~10 kpc, v_{MACHO}~200 km/s, D_S~50 kpc (LMC):

$$t \approx 70 \left(\frac{M}{M_\odot} \right)^{1/2} \text{days}$$

Microlensing light curves



$$\mu_{1+2} = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$$u(t) = \sqrt{\left(\frac{tv}{\theta_E D_L}\right)^2 + u_{\min}^2}$$

Solution degenerate in v, D_L and $\theta_E(M)$

Characteristic *achromatic* light curve

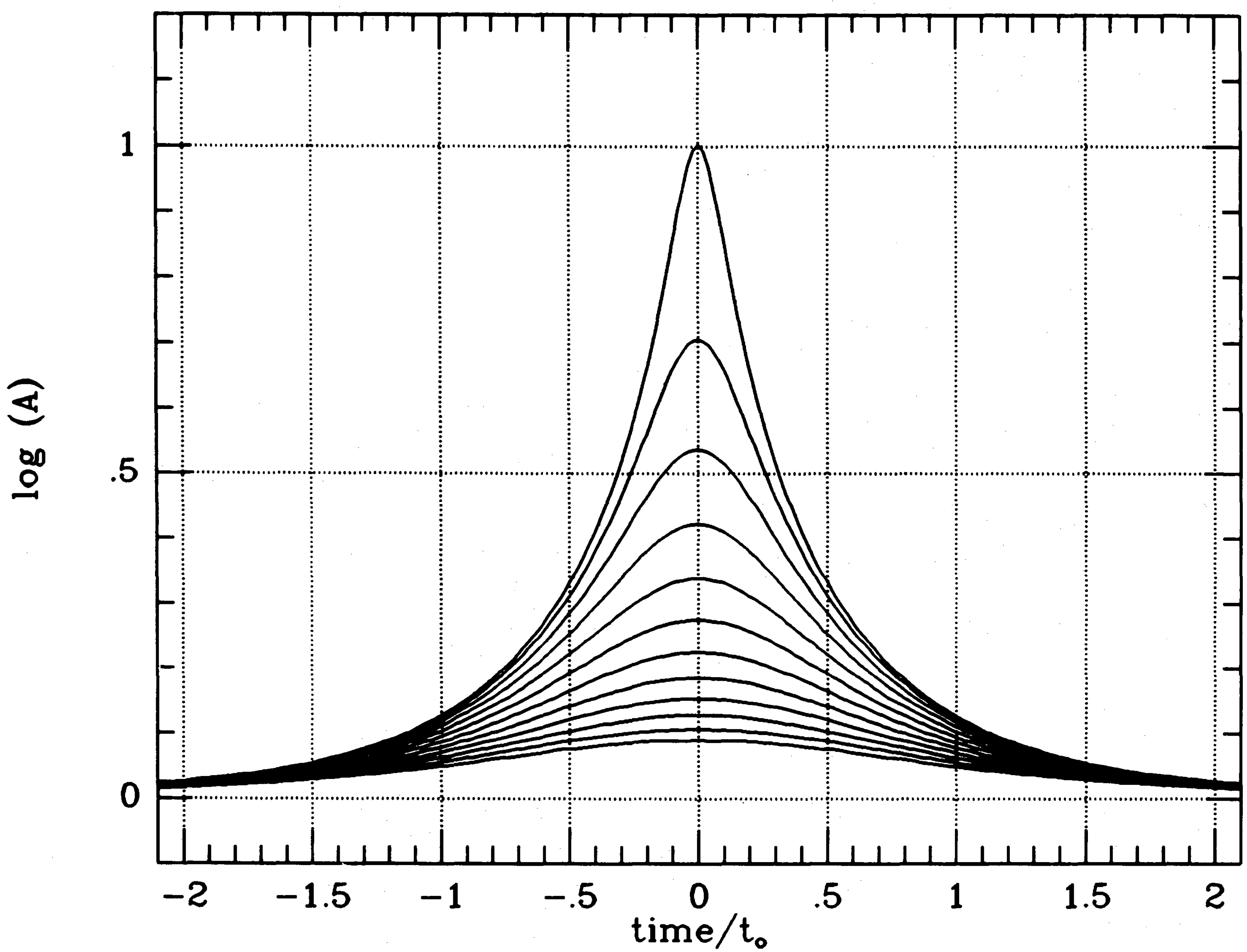


FIG. 2.—Time variation of the amplification due to gravitational microlensing for events with the impact parameter d/R_0 equal 0.1, 0.2, ..., 1.1, 1.2. The largest amplitude corresponds to the smallest impact parameter. The unit of time is given as $t_0 \equiv R_0/v$, where R_0 is the radius of ringlike image formed when the source, the lensing mass, and the observer are perfectly aligned (see eq. [2] and [16]) and v is the relative tangential velocity of the lensing object.

Paczynski (1986)

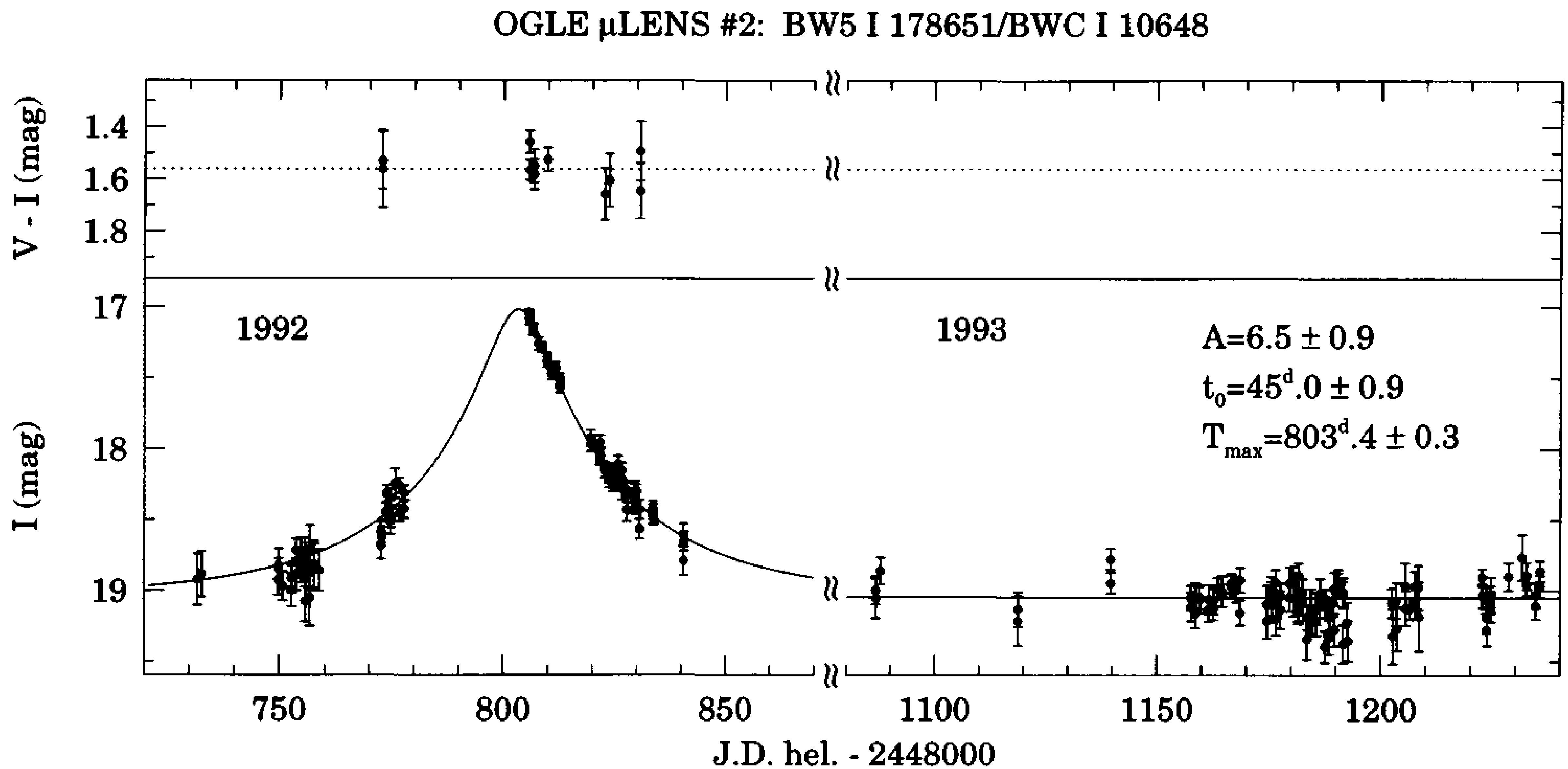
Finding MACHOs

- Now we know what to look for:
- Monitor at least a few million stars
- Look for achromatic, symmetric light curves

Searching for MACHOs

- MACHO collaboration:
  - Monitoring of LMC and Galactic bulge, using 50 inch telescope at Mt Stromlo, Australia
  - 8 million stars in LMC, 10 million in Bulge
- OGLE collaboration:
  - Using 1 m telescope at Las Campanas, also LMC + bulge
- EROS collaboration:
  - La Silla (bulge, LMC, SMC)

Results



Paczynski (1996)

Results

- Many events have been found towards bulge (>100), relatively few towards LMC/SMC (~15)
- Most likely masses 0.15 - 0.9 $M_\odot$
- Not enough MACHOs to account for mass of dark matter halo, at most ~20%
- The search for DM goes on..