Radio Astronomy Exercise

Ver 0.1 by Cameron Van Eck, 30 May 2013

Description of the data:

The data used in this exercise was taken by the Rothney Radio Telescope (RRT), located at the Rothney Astrophysical Observatory (RAO) in Calgary, Canada. The RRT is a 3.3m single-dish radio telescope, with a receiver system tuned to receive frequencies around the 21-cm hydrogen line at 1420.49 MHz. The RRT is aligned with the local meridian (North-South axis) and points at a fixed elevation that is chosen before observing. During an observation, the rotation of the Earth causes different parts of the sky to appear in the telescope beam over time.

The data in this exercise was taken over 48 hours starting on 31 October 2007, with the telescope fixed at an elevation of 25°. The data file RRT_data.fits contains the resulting observation. When opened in a FITS viewer, the horizontal axis is frequency (from 1419.5 MHz to 1420.7 MHz, each pixel is 1.9 kHz) and the vertical axis is time (increasing upwards, each pixel is 60 seconds). This data has been supplied courtesy of Dr. Phil Langill, director of the RAO.

Required materials:

-computer with 'kvis' FITS viewer, and access to the data.
-some note paper
-a calculator

Initial instructions:

0. Get a copy of the data
1. Open a terminal window and navigate to where the data is.
2. In the terminal, open 'kvis'.
3. In the kvis main window, select 'Files' and select the data file.
4. In the kvis browser window, make sure the data is selected, and press 'Histogram'
5. In the histogram window, press the '95%' button. This should change the color scale, making the data visible in the main window. Press 'Close' in the histogram window (NOT the Red X button, but the button labeled 'Close').
6. Take a look at the data. Scrolling in and out will let you zoom into sections of the data; left clicking on the data will re-center the main window on your mouse pointer. Left clicking and dragging will zoom in on the box you create. Become comfortable with moving around and zooming in on sections of data.

Question: What do you see in the data? What features can you identify? What do you think is causing these features?
[Answer:
Two horizontal white bands: this is emission from the sun, while it passes through the telescope beam.
Bright vertical lines: this is narrow-band RFI. The bright line at x=220 is an airport radar, the other lines are unknown.

Weak horizontal striping: this is due to gain variations in the amplifier system over time. A more sophisticated telescope would have the capability to calibrate those out.
Vertical 'S' shaped curve: This is the hydrogen line of local interstellar gas. The curve is caused by the orbital motion of the earth: when the telescope is pointed in the direction of the Earth's motion, the line is blue-shifted (higher frequency), whereas when the telescope is pointed away the line is red-shifted (lower frequency).
4 bright horizontal blobs: This is the galactic plane, which contains an abundance of hydrogen. This hydrogen has different line-of-sight velocity depending where it is in the galaxy, so the line gets smeared out in frequency. Two are observations of the inner Galaxy (the smoother pair), two are of the outer Galaxy (the more blobby pair)]

Challenge question: Twice a day the telescope beam crosses the Galactic plane, where there is a lot of hydrogen at different velocities. Due to the geometry of the telescope and the Earth's orientation with respect to the Galaxy, one of those crossings points towards the inner Galaxy and one towards the outer Galaxy.

Once the basic nature and features of the data are understood, there are many different astrophysical properties that can derived from this data. Within the time available, work on any of these that you find interesting, in any order:
1. Measure the length of the solar day versus the sidereal day. [fairly easy]
2. Measure the orbital velocity of the Earth. [easy, but difficult challenge questions]
3. Measure the velocity of hydrogen in the galaxy. [somewhat harder]

<u>Solar day versus sidereal day:</u>
Goal:
To measure the difference between a solar day and a sidereal day.

Theory:
Since the Earth is orbiting the Sun, the position of the Sun relative to the background stars moves over time. Over the course of one year, the Sun travels around a complete circle, called the ecliptic, through the sky. This means that the Sun takes a different amount of time to return to the same part of the sky compared to the stars. The period it takes the Sun to return to some reference position in the sky is called a solar day, while the period it takes the background stars to return to the reference position is a sidereal day.
Using our fixed telescope as a reference position, we can measure the time difference between two observations of the Sun as 1 solar day, and the difference between two observations of the Galactic plane as 1 sidereal day.

Instructions:

1. Pick a reference point on one of the observations of the Sun. The easiest choice of reference is either the top or bottom of the bright band.
2. Measure the y coordinate of the corresponding reference point on the other observation of the Sun.
3. Calculate the difference between the two pixel values.
4. Repeat the measurement for one matched set of Galactic plane crossings (a matched set is a pair of crossings that look the same, since there are two types).
5.(optional) To improve the accuracy of your measurement, repeat this process several times with different reference points.

Question: If each pixel is exactly 1 minute, what is the length of a Solar day? What is the length of the sidereal day?

[Answer: When I tried this, I got within 1 pixel of 24 hours for the Solar day. My (single) estimate for the sidereal day was 23 hours 53 minutes (actual: 23 hours 56 minutes), but this is highly subject to which reference point was used.]

Challenge Questions: Why is there a difference between the Solar and sidereal day? Can you calculate how long the sidereal day should be? Hint: A Solar day is defined as exactly 24 hours and the Earth orbits the Sun in the same direction as it rotates.

[Answer: The rotation of the Earth around the Sun means that the Sun's position in the sky changes with respect to the background stars. See http://en.wikipedia.org/wiki/Sidereal_day for a good diagram.

The simplest way to derive the sidereal day: in one year, there are 365 Solar days (neglecting leap years), but the Earth makes one additional rotation relative to the stars. 365/366 * 24 hours = 23 hours 56 minutes.]

Measure the orbital velocity of the Earth

Goal: To estimate the orbital speed of the Earth from the Doppler shift of local hydrogen.

Theory: A Doppler shift is produced whenever the receiver of a signal is moving relative to the signal source, so a velocity measurement from an observed Doppler shift includes both the motion of the source and the receiver. This means that a Doppler measurement of an astrophysical signal can be affected by the rotation of the Earth, the orbit of the Earth about the Sun, and the orbit of the Sun about the Galaxy.
The 'S' shaped curve visible in the data is the 21cm hydrogen line, being Doppler shifted by the orbital motion of the Earth. By measuring the change in the measured frequency, it is possible to calculate the velocity of the Earth.

Necessary equations:
Doppler shift:
f= observed frequency
f_0 = emitted frequency

$$f = \left(1 + \frac{\Delta v}{c}\right) f_0$$

Δv = velocity
c = speed of light (300 000 km/s)
Instructions:
1. Locate the position on the 'S' curve where the frequency is highest (furthest to the right). Record the frequency at this position (this is 'f' in the Doppler equation).
2. Using the Doppler shift formula above, calculate the velocity between the telescope and the hydrogen. Use f_0 = 1420.406 MHz for the emitted frequency of the hydrogen emission.
3. (optional) To improve the accuracy of your measurement, repeat with the other maximum in the 'S' curve.

Question: Assuming that the Earth is in a circular orbit of 149 million km, and takes 365 days to make one orbit, what is the Earth's orbital velocity? How does this compare to the measured Doppler velocity?

[Answer:
observed maximum frequency = 1420.49 MHz (x-coord ~ 507)
Measured Doppler speed: 18 km/s
Orbital speed: 29.7 km/s
The observed Doppler speed is much lower than the orbital speed. This is due to the fact that the telescope is not perfectly aligned with the orbital velocity vector. Also, I'm not sure I entirely trust the frequency calibration of this telescope.]

Challenge question: Using the information that the telescope is located at 50° N latitude, and was pointing south with an elevation of 25° above the horizon, can you estimate the angle between the telescope and the orbital velocity vector (at the time of your Doppler measurement)? Include the 23.5° axial tilt of the Earth (assume the tilt is in the direction of the orbital velocity at the time of the observation). What would the expected Doppler velocity be for a measurement at this angle?

[Answer: If the telescope is at 50°N, the south horizon is at declination -40° (50°-90°). If the telescope is at an elevation of 25°, this is declination -15°. The axial tilt moves the telescope down 23.5 degrees, for a total angle of 38.5° between the telescope and the orbital velocity vector. The expected velocity is then 30 km/s * cos(38.5°) = 23.5 km/s, which should be closer to the measurement.]

<u>Measure the velocity of Galactic hydrogen</u>
Goal: To measure the Doppler shift of interstellar hydrogen and estimate its distance from Earth.

Theory: While most of the gas in the Galaxy has roughly the same orbital speed, gas at different locations in the Galaxy is moving in different directions, and thus has velocity relative to the Sun. By measuring the Doppler shift of a gas cloud in space, and knowing what direction in the Galaxy the telescope is pointing during the measurement, we can estimate the distance to the cloud, and thus determine where in the Galaxy the gas cloud is.
However, care must be taken when measuring the Doppler shift, since the Earth's orbital velocity is also contributing to the Doppler shift. The easiest way to remove this effect is to measure the Doppler shift relative to the local hydrogen (the 'S' curve in the data), since this local hydrogen should be moving in the same Galactic orbit as the Sun.

Instructions:
1. In the main kvis window, press the 'View' button. This will open the View Control window. Within the View Control window, go to the 'Profile Axis:' button and select 'X'. Then, do to the 'Profile Mode:' button and select 'Line'. After the Profile window opens, press 'Close' in the View Control window.
2. In the Profile window, select the 'Auto V Zoom' checkbox. Move your cursor around the image in the main window and observe how the Profile window changes. The Profile window is showing a horizontal slice through one line of pixels in the image. This lets you see the radio spectrum in a graph form, for a single time. Within the Profile window, you can zoom in to different parts of the profile using the same commands as in the main window (scroll wheel, left click, etc.). You can also use the spacebar key in the main window to freeze the Profile window on the current time (press spacebar again to unfreeze). Become comfortable using the Profile window to look at the spectrum at different times.
3. Select one of the observations of the Galactic plane to study. Look at the 'S' curve before and after the Galactic plane observation, and identify the corresponding peak in the Profile window.
4. Move the middle of the Galactic plane observation and freeze the Profile window at that time. Record the x-coordinate of the peak of the 'S' curve line (the local gas).
5. Measure the x-coordinates of all other peaks present in the spectrum. These are the distant gas clouds. (Except for the RFI spike at x=220, ignore that.)
6. Calculate the different in x-coordinate between the gas clouds and the local gas. Using the information that the local gas signal is emitted at a frequency of 1.420406 GHz, and the width of one pixel is $1.9\text{x}10^{-3}$ MHz, calculate the observed frequency of the distant gas.
7. Using the Doppler shift equation above, calculate the velocity of the gas clouds.
8. Using the equations below, calculate the distance to the gas clouds in km/s.

Equations:

Orbital radius:

R= distance from Galactic center to gas cloud

R_0= distance from Sun to Galactic center (8.5 kpc)

$V_{Doppler}$=measured Doppler velocity

$V_{rotation}$=orbital velocity of gas (220 km/s)

l=Galactic longitude of cloud (17° for inner Galaxy, 228° for outer Galaxy

$$R = \frac{R_0}{\frac{V_{\rm Doppler}}{V_{\rm rotation}\sin(l)} + 1}$$

Outer galaxy distance formula:

d=distance from Earth

$$d = R_0 \cos(l) + \sqrt{R^2 - R_0^2 \sin^2(l)}$$

Inner galaxy distance formula:

$$d = R_0 \cos(l) \pm \sqrt{R^2 - R_0^2 \sin^2(l)}$$

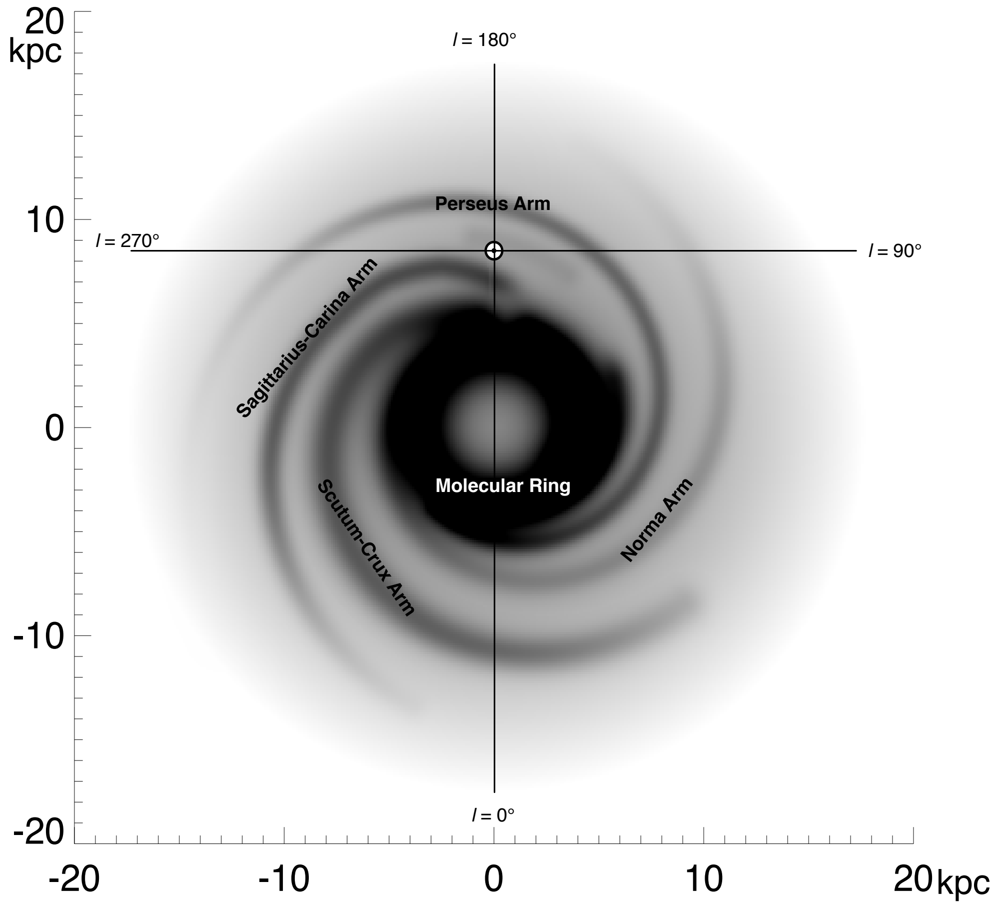
This formula gives two answers, depending on the choice of plus or minus sign. The geometry of the observation means that one of the answers is correct, but there is not enough information to determine which.

Question: Using the image below, where in the Galaxy do you think your gas clouds are?

[Answer: For the inner galaxy, I found the local emission at x=420, and peaks at x=380 and x=480. This corresponds to Doppler velocities of 16 km/s and -24 km/s respectively, which corresponds to Galactic radii of 6.8 and 13.6 kpc. For the first, this gives a distance of either 1.8 or 14.5 kpc. For the second, either -5.2 or 21.4 kpc; the negative is obviously wrong (no distance ambiguity for negative Doppler shifts in inner galaxy). Many students may forget the negative sign on the second, giving a smaller distance.

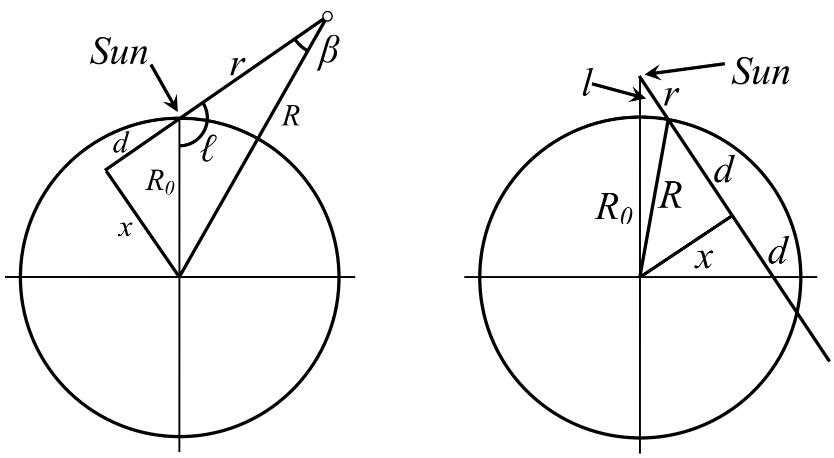
For locations: if the first is at 1.8 kpc, that would put it around the Sagittarius spiral arm, 14.5 would put it between the Perseus and Norma arms. The 21.4 kpc cloud is somewhere beyond the Scutum-Crux arm, more or less.

Outer galaxy: Exactly answers will depend on which time-slice is used. I get local emission at x=455, and peaks at x=290, 365, and 420. This gives velocities of 66, 36, and 14 km/s. This gives Galactic radii of 12.4, 10.2, and 9.1 kpc, and distances of 5.0 kpc, 2.3 kpc, and 0.9 kpc. This puts the first cloud beyond the Perseus arm, the second within the Perseus arm, and the third closer to the local arm (which isn't labeled in the diagram).]



Challenge Question 1: Using trigonometry, can you derive the equations for d in the inner and outer Galaxy? Can you explain why there are two solutions for the inner Galaxy?

[Answer: Not enough time to do a full write up. Use this figure for reference (not that it uses 'r' where I use 'd'), and apply law of cosines:



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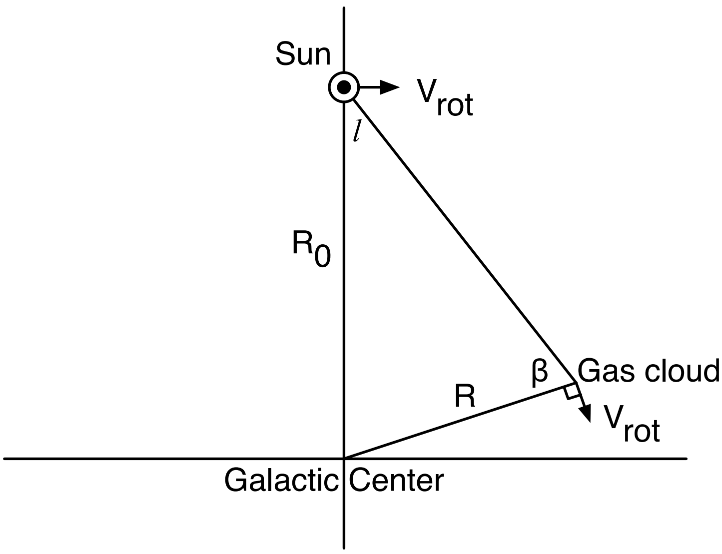
Challenge Question 2: Can you derive the relationship between Doppler velocity and distance from the Galactic center?
Hint: The measured Doppler velocity is the difference between the line-of-sight component of the gas cloud orbital velocity and the line-of-sight component of the Sun's orbital velocity:
$V_{\text{Dop}} = V_{\text{rot}} \cos(90° - \beta) - V_{\text{rot}} \cos(90° - l)$



[Answer:
(Proof the angle between the gas cloud's velocity and line-of-sight is 90°-β is left as an exercise to the reader.)
Trigonometric identity: cos(90°-x)=sin(x)

$$V_{\text{Dop}} = V_{\text{rot}}(\sin(\beta) - \sin(l))$$

Law of sines: $$\frac{\sin(\beta)}{R_0} = \frac{\sin(l)}{R}$$

$$\begin{aligned}
V_{\text{Dop}} &= V_{\text{rot}}\left(R_0\frac{\sin(l)}{R} - \sin(l)\right) \\
&= V_{\text{rot}}\sin(l)\left(\frac{R_0}{R} - 1\right) \\
\frac{V_{\text{Dop}}}{V_{\text{rot}}\sin(l)} &= \frac{R_0}{R} - 1 \\
\frac{V_{\text{Dop}}}{V_{\text{rot}}\sin(l)} + 1 &= \frac{R_0}{R} \\
R &= R_0\left(\frac{V_{\text{Dop}}}{V_{\text{rot}}\sin(l)} + 1\right)^{-1}
\end{aligned}$$

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[Concepts that it would be nice to discuss with the students before the exercise:
-21 cm line of hydrogen
-Earth's location in the Galaxy + definition of inner vs outer Galaxy
-rotation curve of Galaxy
-Doppler shift (with mention of line-of-sight velocity vs. true velocity)
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