Chaos

stability

Simon Portegies Zwart

Sterrewacht Leiden

AMUSE

1608 Lippershey

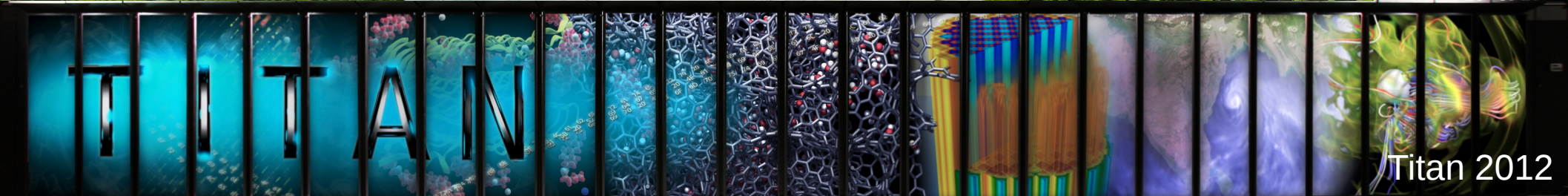
For the last 400 years
telescopes became a little larger

An ape on the shoulders of a giant,

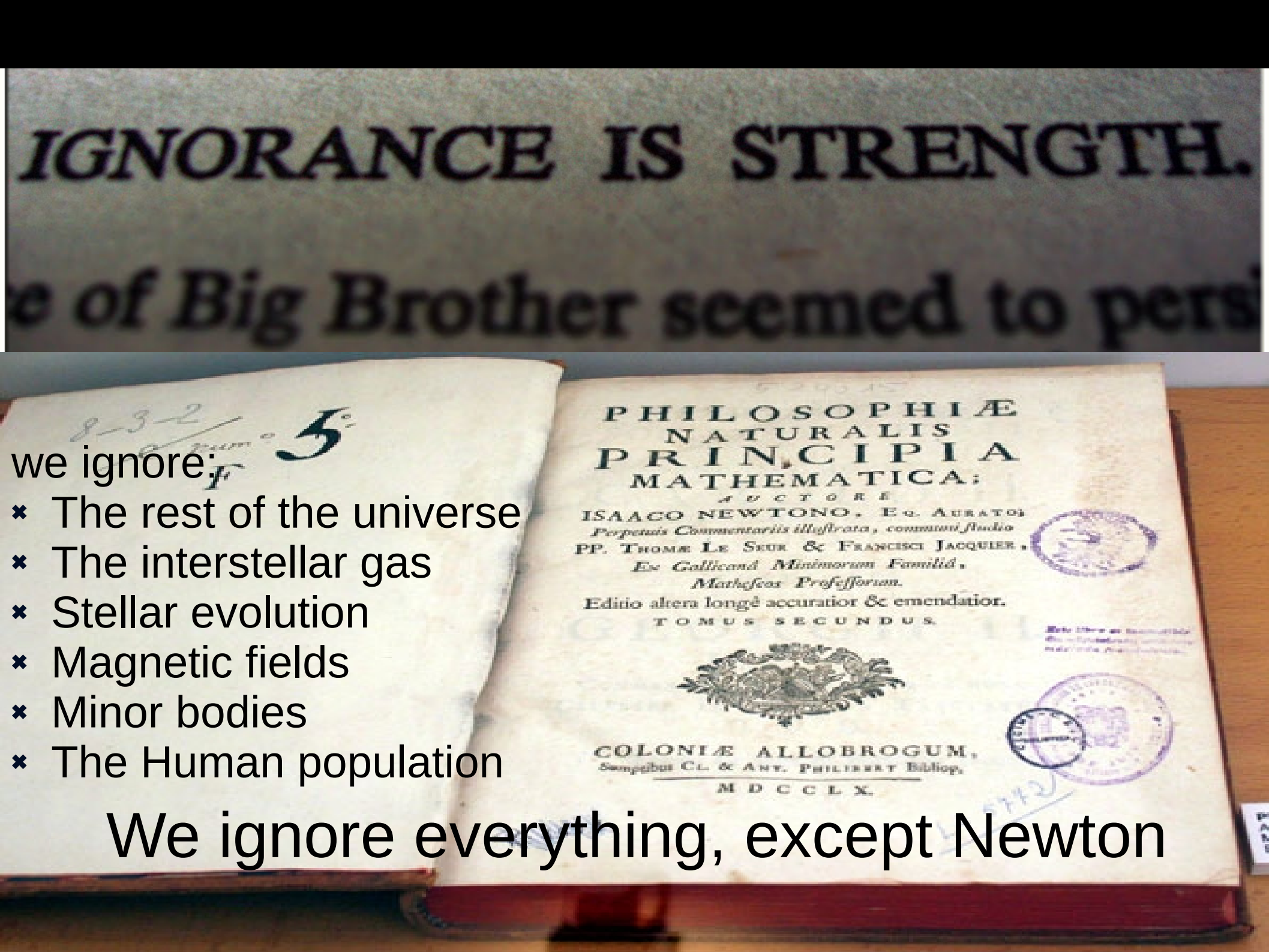
Rodin 1902

still is an ape.

Ape 33M B.C.

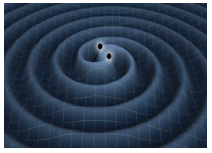


Titan 2012

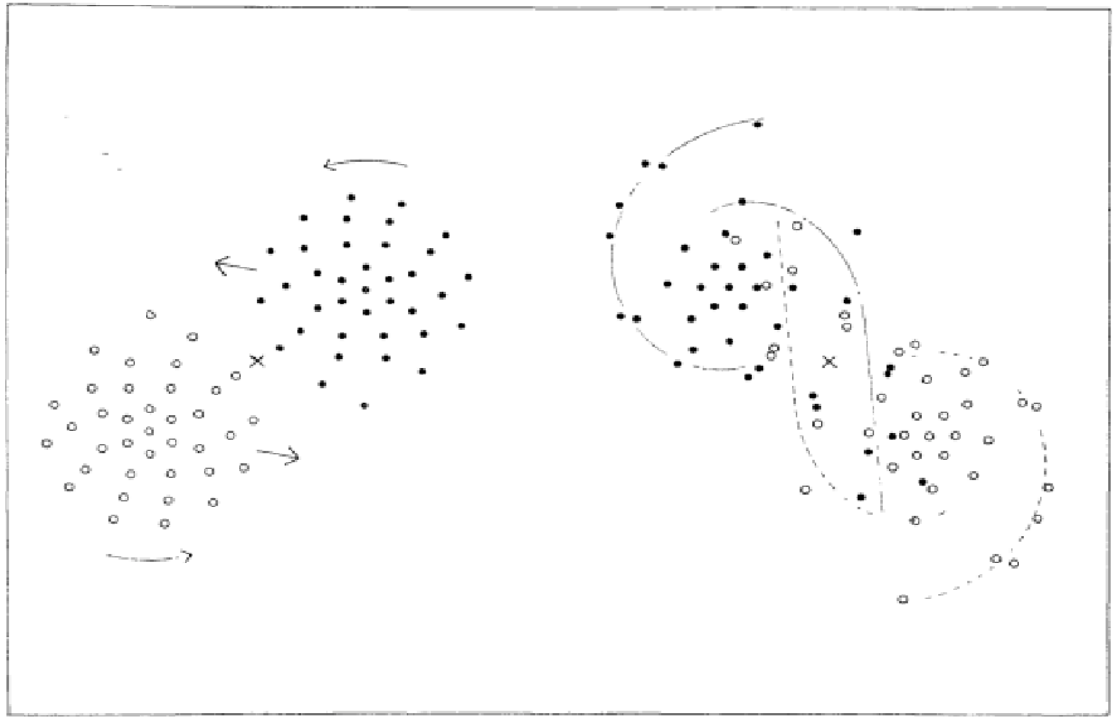

IGNORANCE IS STRENGTH.
e of Big Brother seemed to pers
we ignore;
The rest of the universe
The interstellar gas
Stellar evolution
Magnetic fields
Minor bodies
The Human population
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA:
AUCTORE
ISAACO NEWTONO, Eq. Auratô
Perpetuis Commentariis illustrata, communi studio
PP. Thomæ Le Seur & Francisci Jacquier,
Ex Gallicanâ Minimorum Familiâ,
Matheseos Professorum.
Editio altera longè accuratior & emendatior.
TOMUS SECUNDUS.
COLONIÆ ALLOBROGUM,
Sumptibus Cl. & Ant. Philibert Bibliop.
M D C C L X.
We ignore everything, except Newton

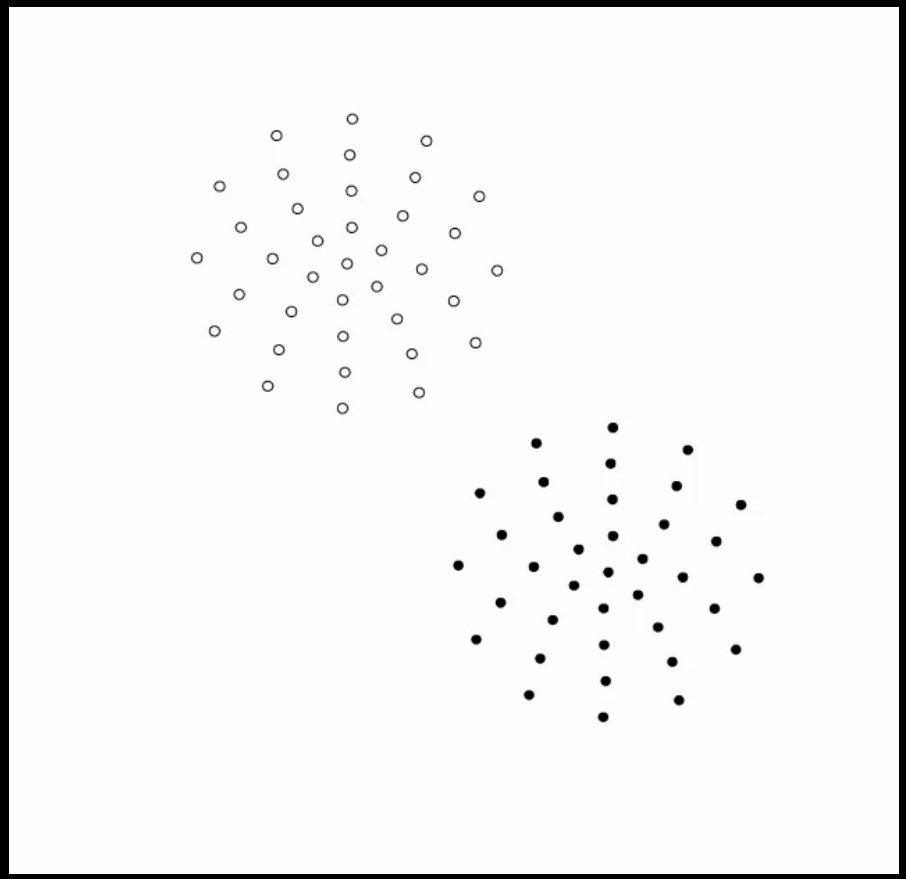
Gravity's complexities

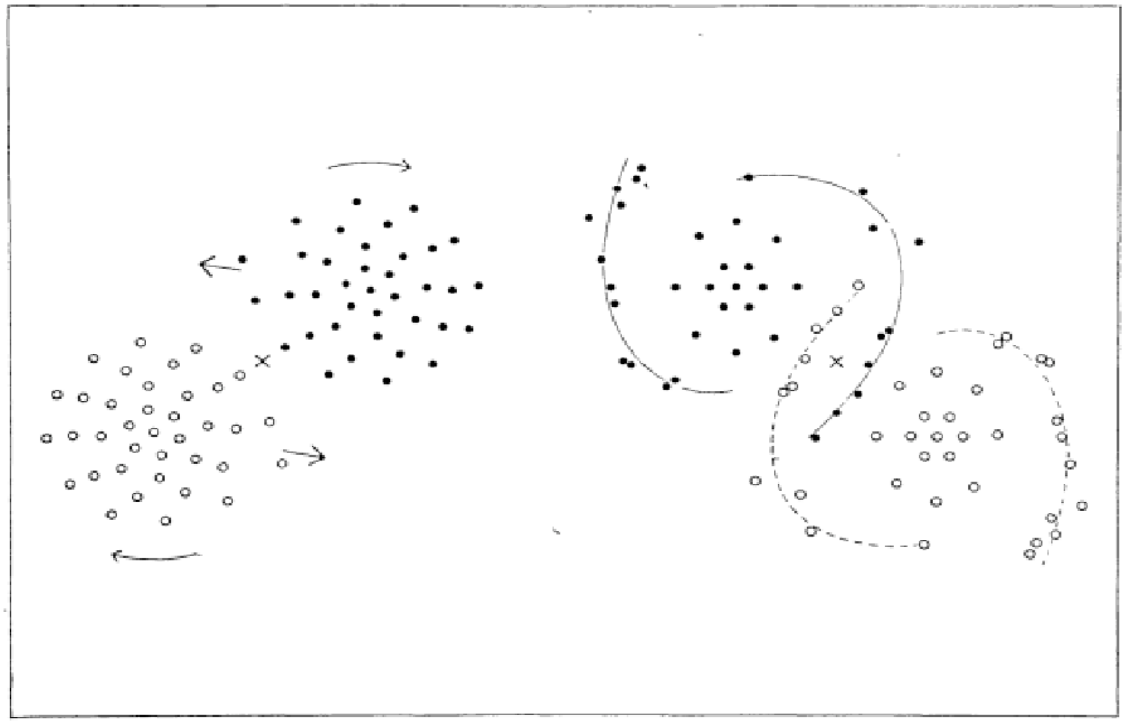
- Gravity has a negative heat capacity.
- The force calculation is an N*N operation.
- Gravity is global aware: There is no shielding in gravity, such as in molecular dynamics.
- At small distances the force grows limitless.

 $E=3.6\ 10^{56}$ erg/s ~ 1million supernovae

- The equations of motion are intrinsically chaotic (but we do not know why).







Erik Holmberg
1908-2000

Pollock1954

"*Errors in calculations of n-body systems grow exponentially … and may therefore invalidate the results …*" (Miller 1964)

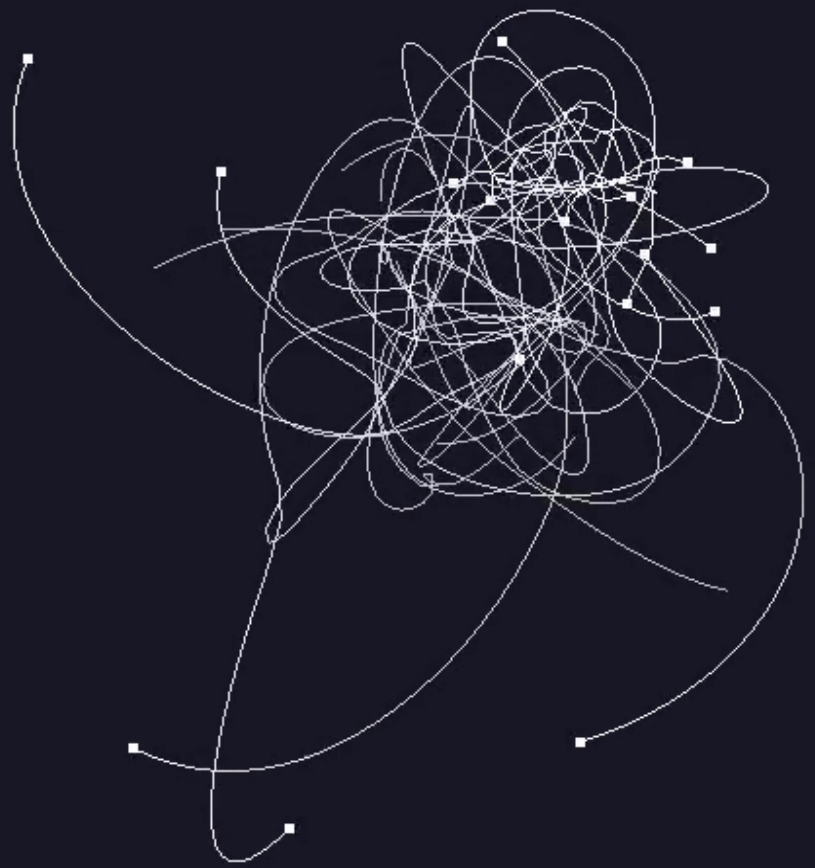
Assahoryu vs Baruto

BRUTUS

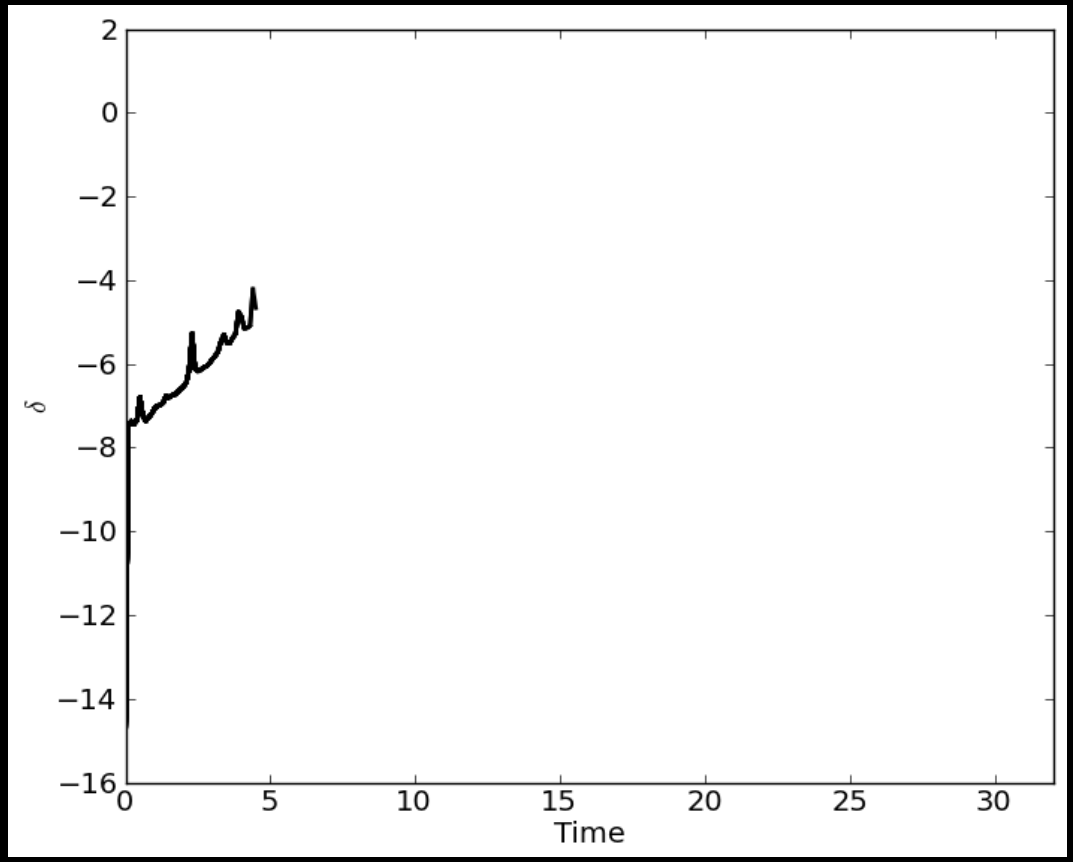
a brute force arbitrary-precision N-body code

- Two ingredients:
  - Gragg-Bulirsch-Stoer method
    - Modified midpoint method
    - Richardson extrapolation
    - Tolerance parameter
  - Arbitrary-Precision arithmetic
    - Number of significant digits

Tjarda Boekholt 2015

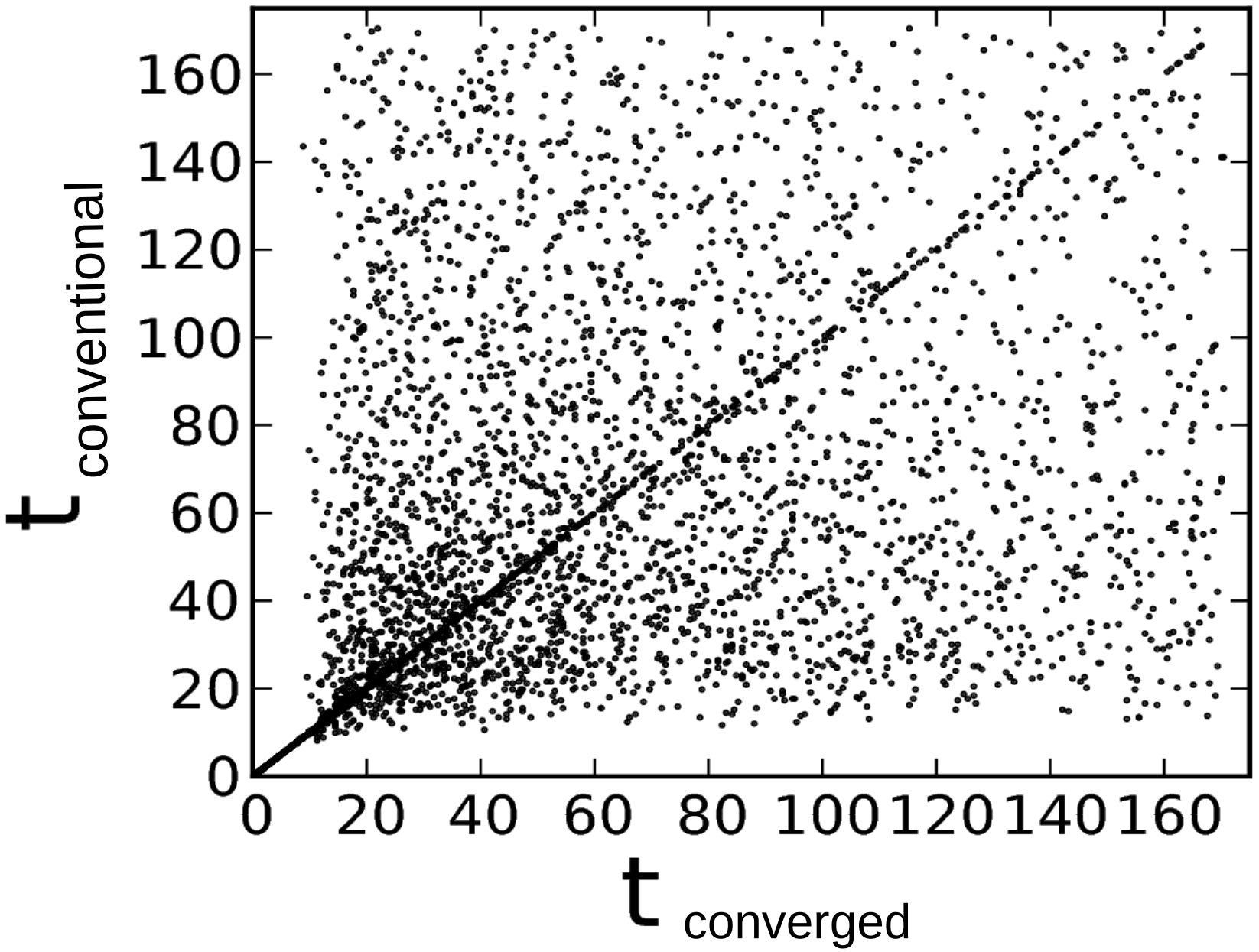


Exponential divergence

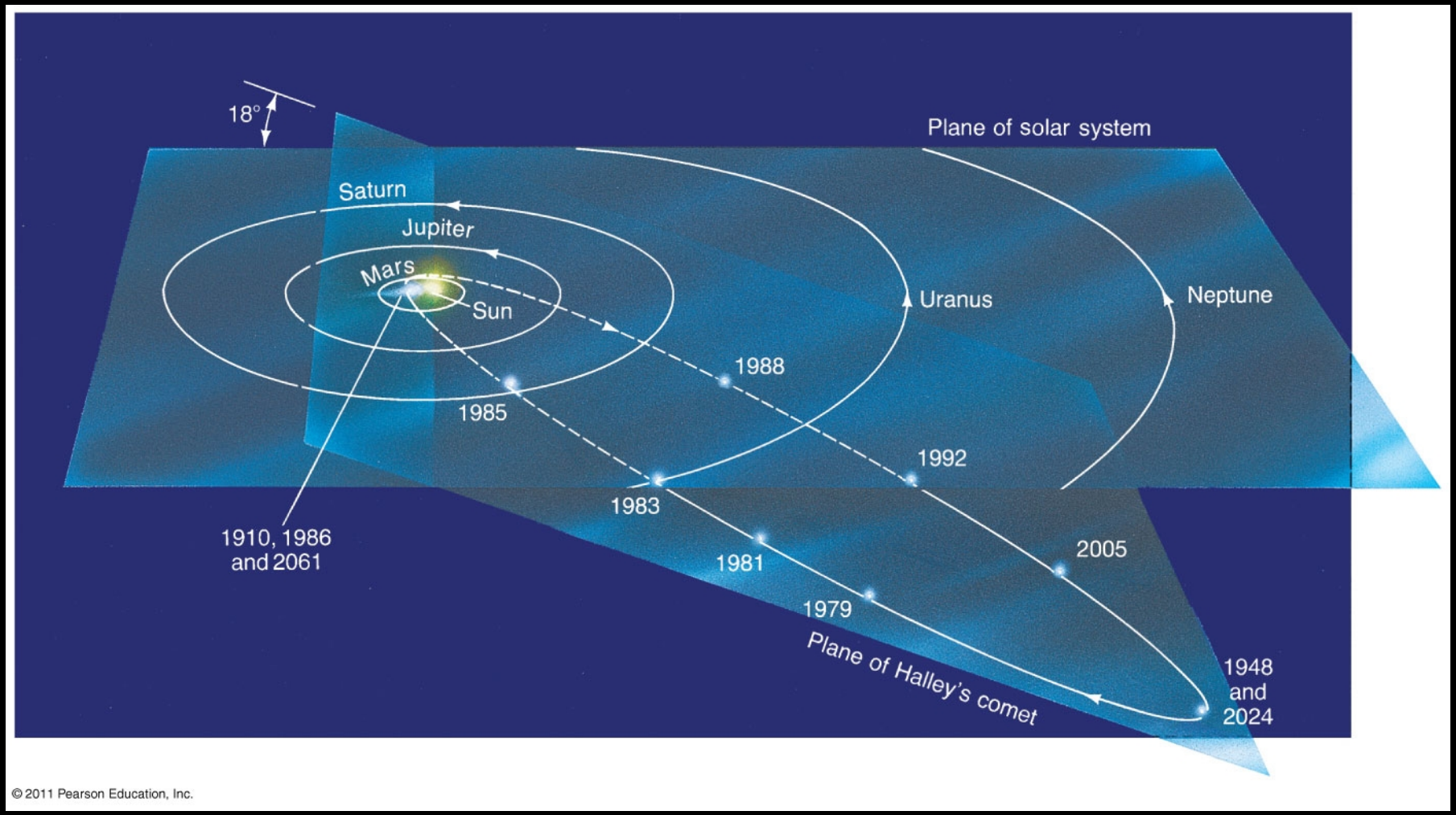


$$\delta = 0.5 \log_{10} 1/(6N) \sum (x_2-x_1)^2 + (v_2-v_1)^2$$

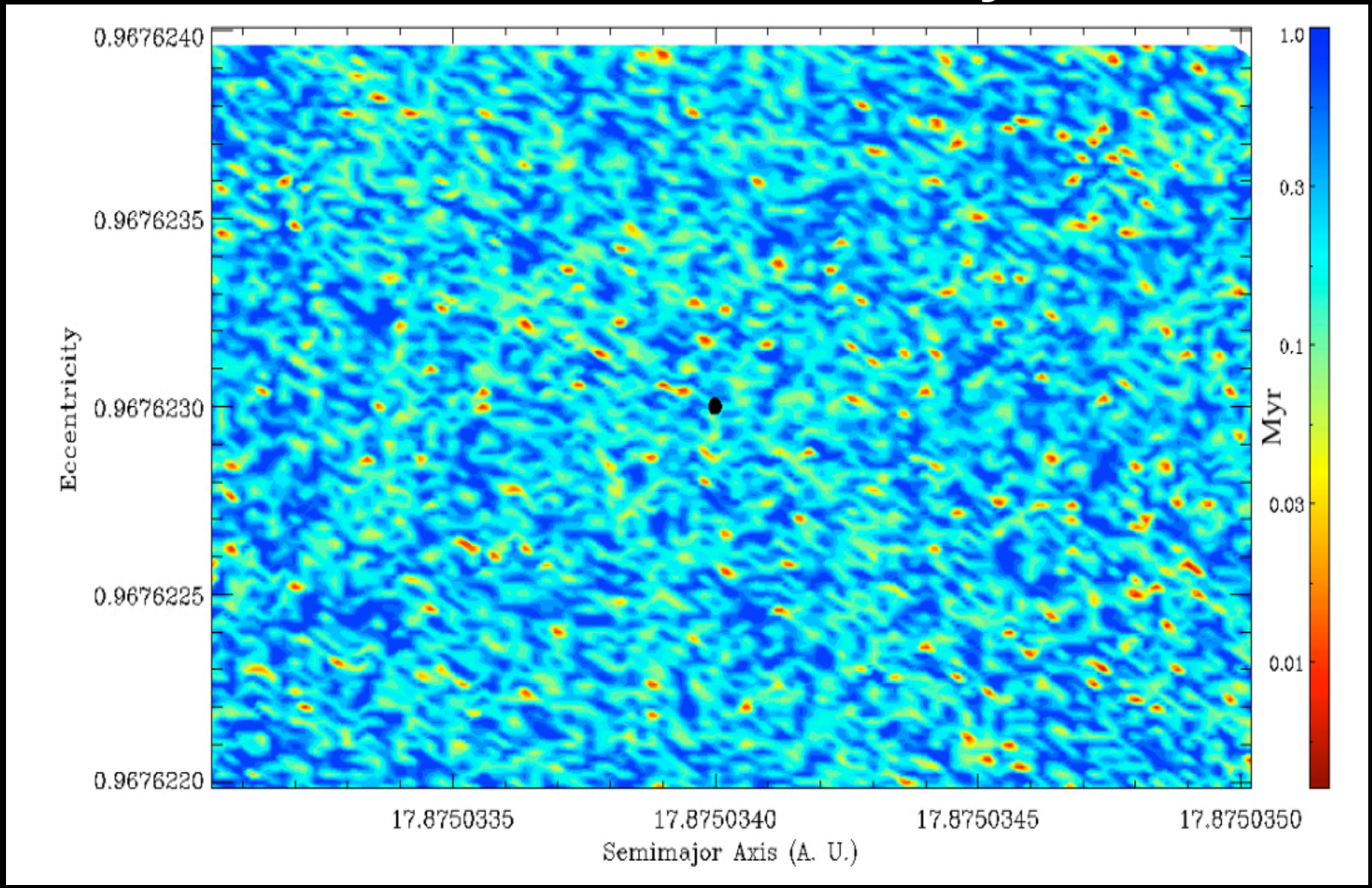
10,000 realizations of N=3 give no systematic bias



Orbit of comet Halley

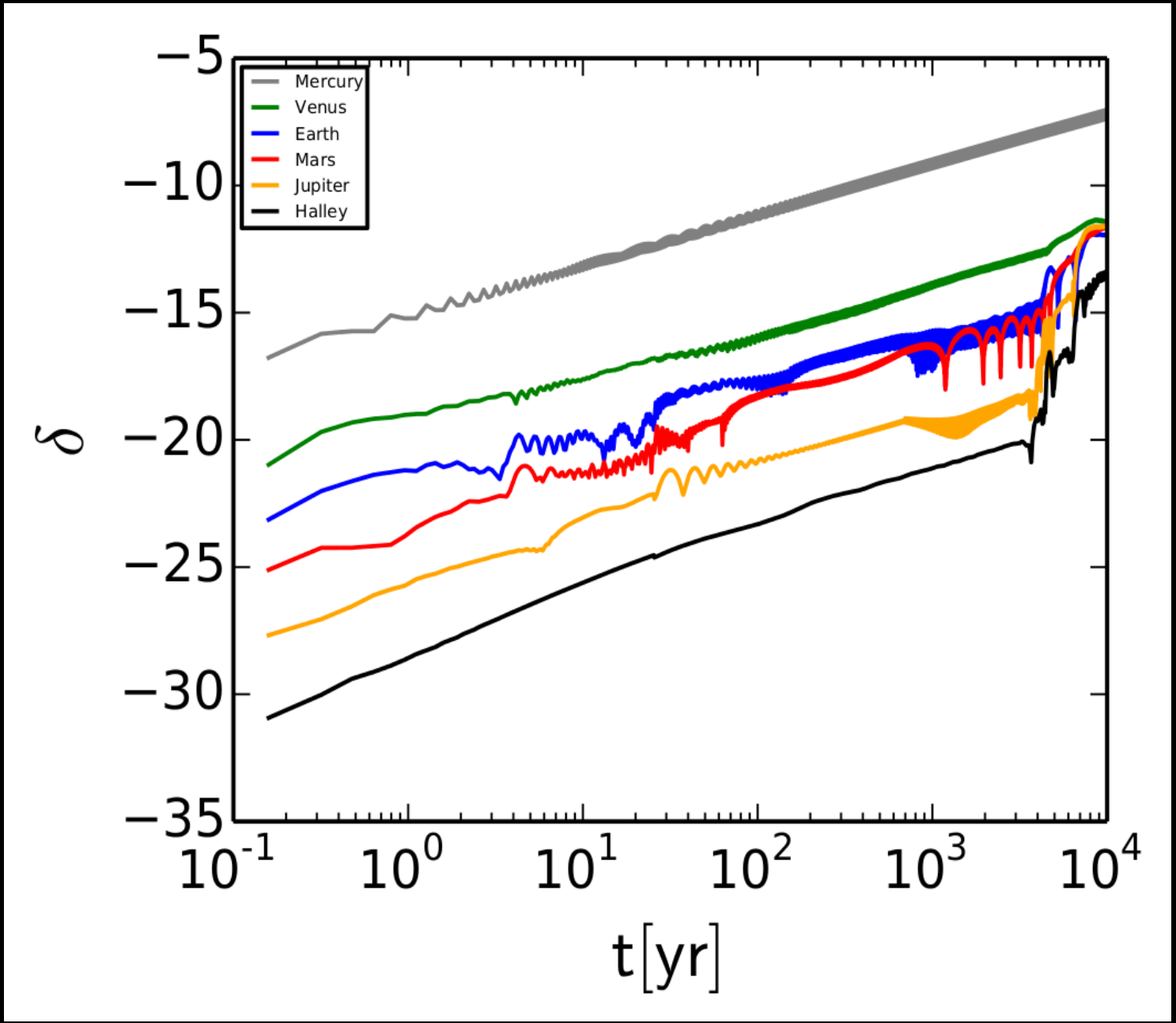


Chaos in Halley

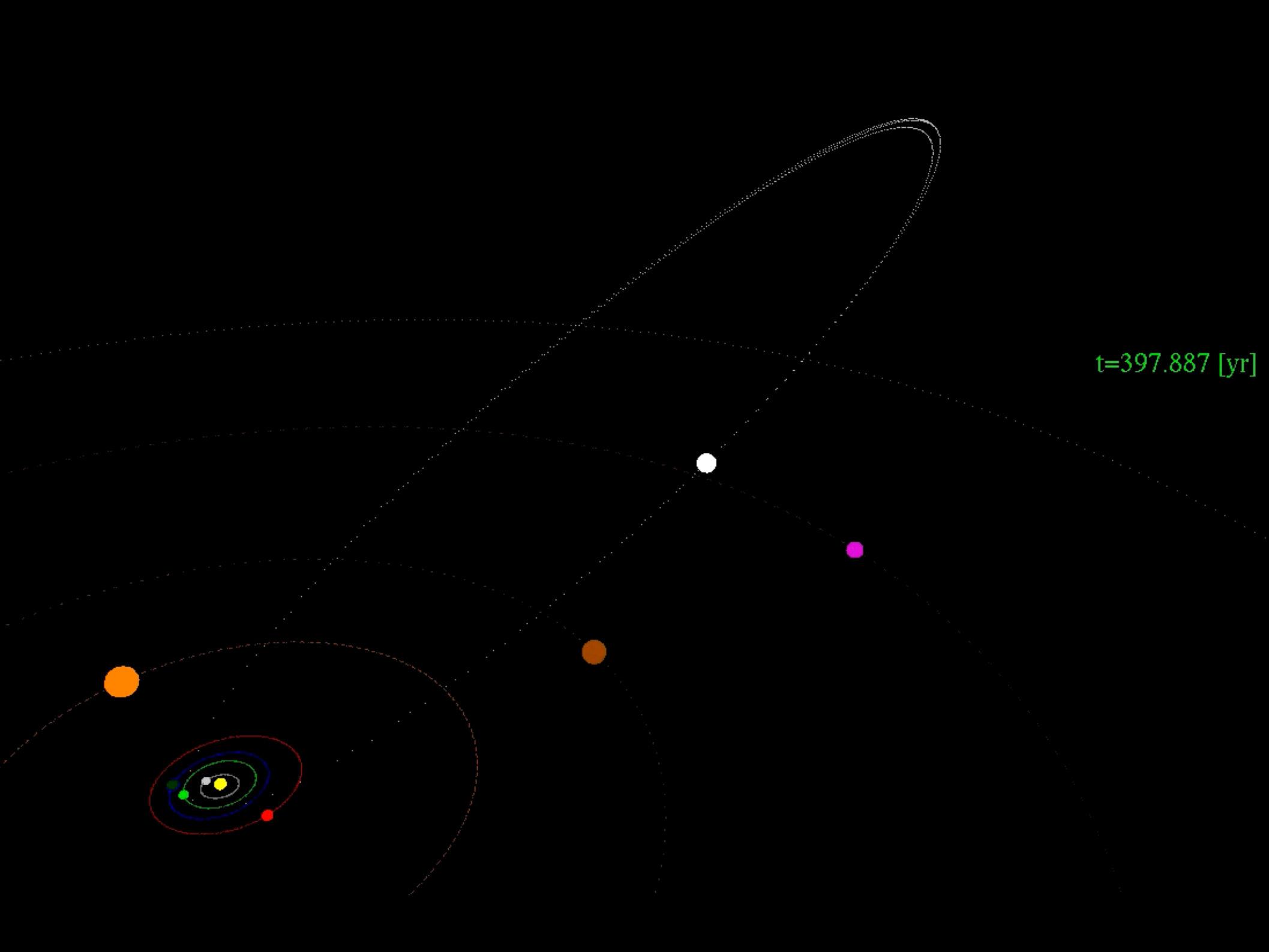


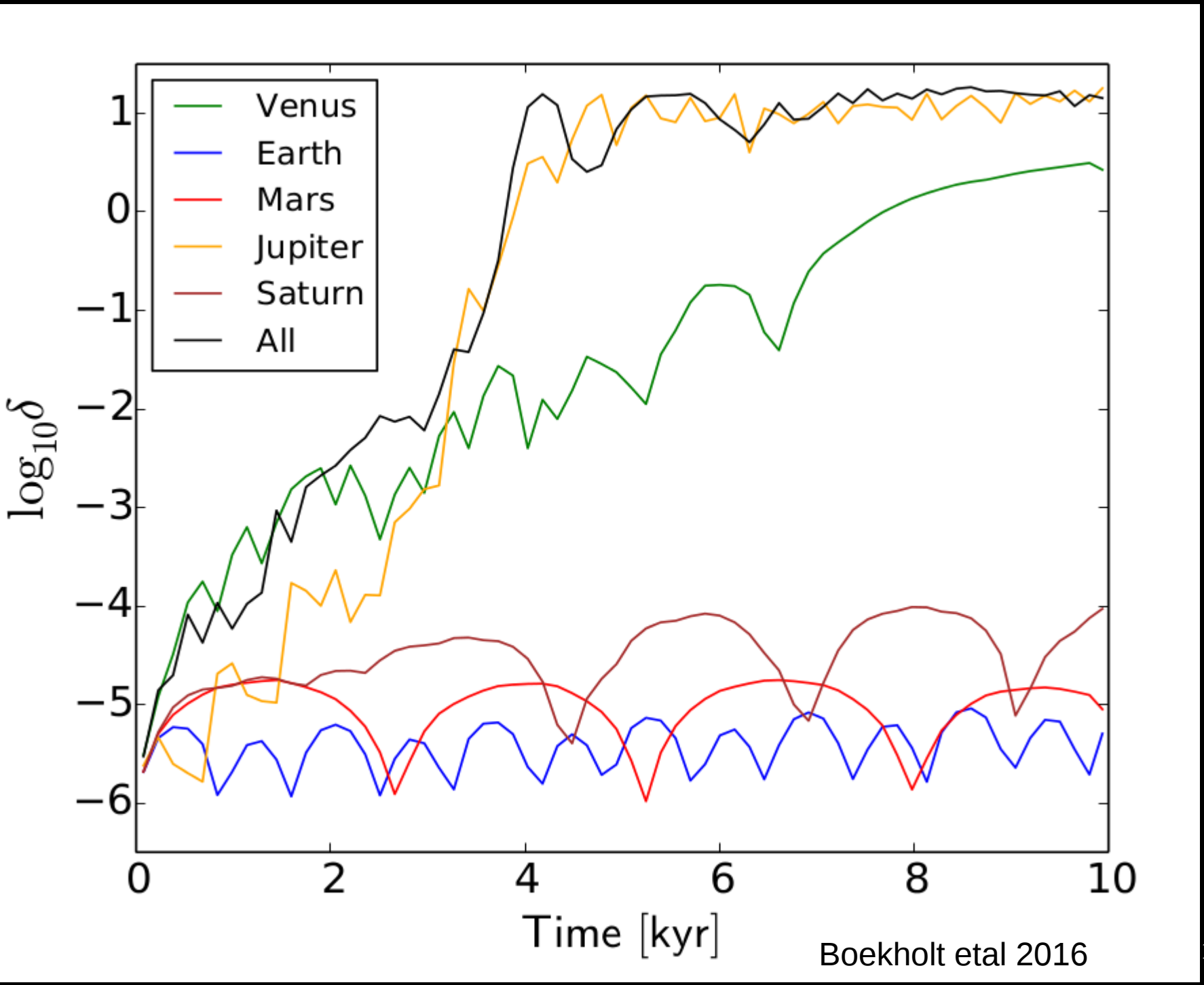
Munoz-Gutierrez etal 2014

Growth of the error



$$\delta = 0.5 \log_{10} 1/(6N) \sum (x_2 - x_1)^2 + (v_2 - v_1)^2$$


t=397.887 [yr]



Boekholt etal 2016

Messages

• Chaos prevents accurate calculations, but a statistical ensamble of simulations still gives the correct phase-space characteristics of the physical system.

• The comet Halley's orbital chaos is currently driven by Jupiter, but about 3000 years from now Venus will become the dominant perturbing body.

Solar System chaos

- The inner Solar System is surely chaotic, although it is bounded chaos and the major four planets are probably not affected.

- It is not known if the outer Solar System is chaotic.