The (noncommutative) structure of spacetime

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August 25, 2011

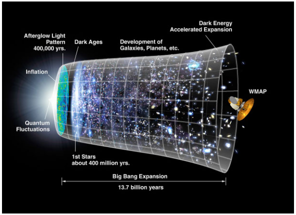
IMAPP

Radboud University Nijmegen

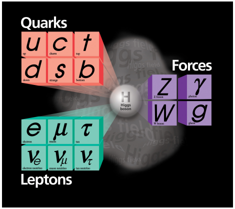
Noncommutative geometry

in physics:

Einstein's general theory of relativity

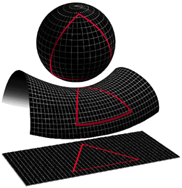


Elementary particles (Standard Model)



in mathematics

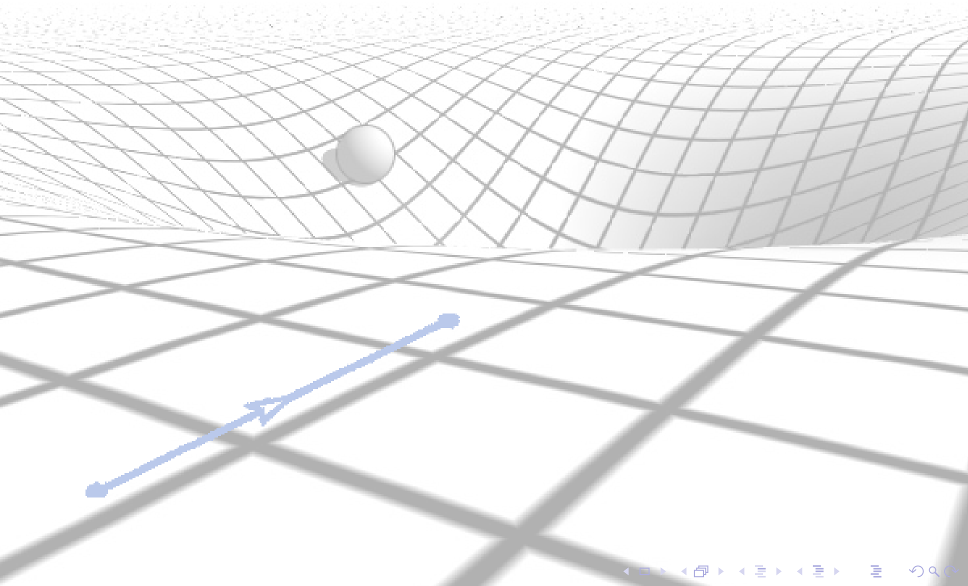
Riemannian geometry



Noncommutative geometry

$$\mathbf{x} \cdot \mathbf{y} \neq \mathbf{y} \cdot \mathbf{x}$$

A fermion in a spacetime background



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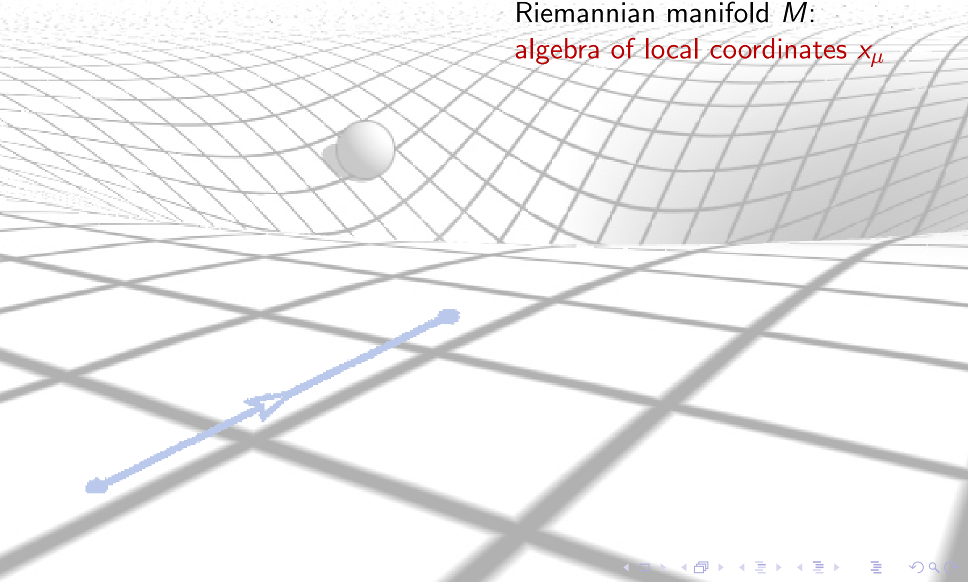

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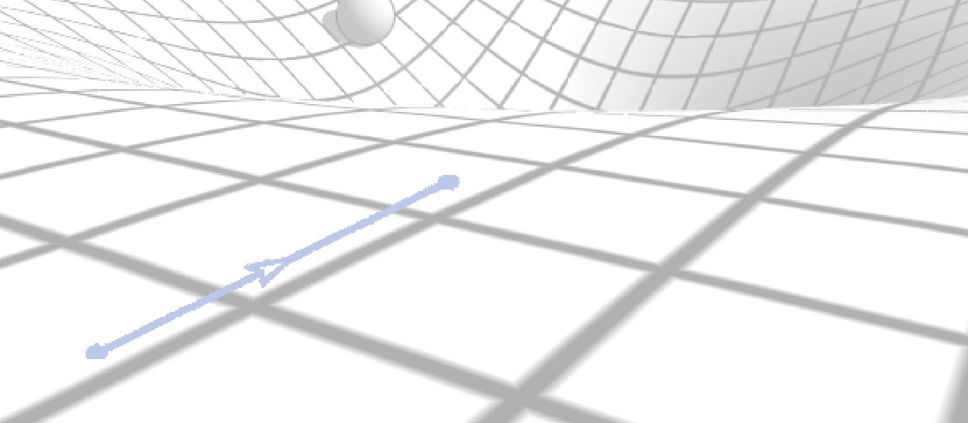

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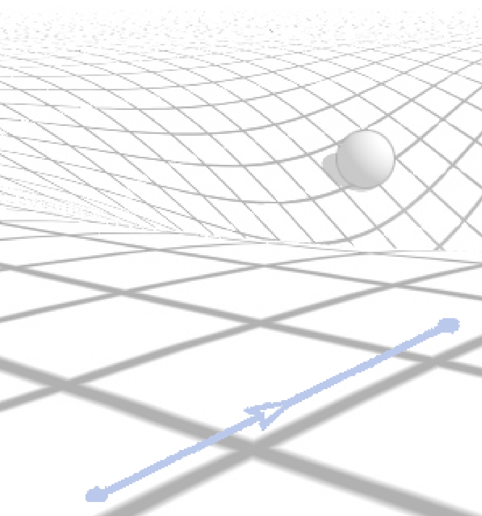

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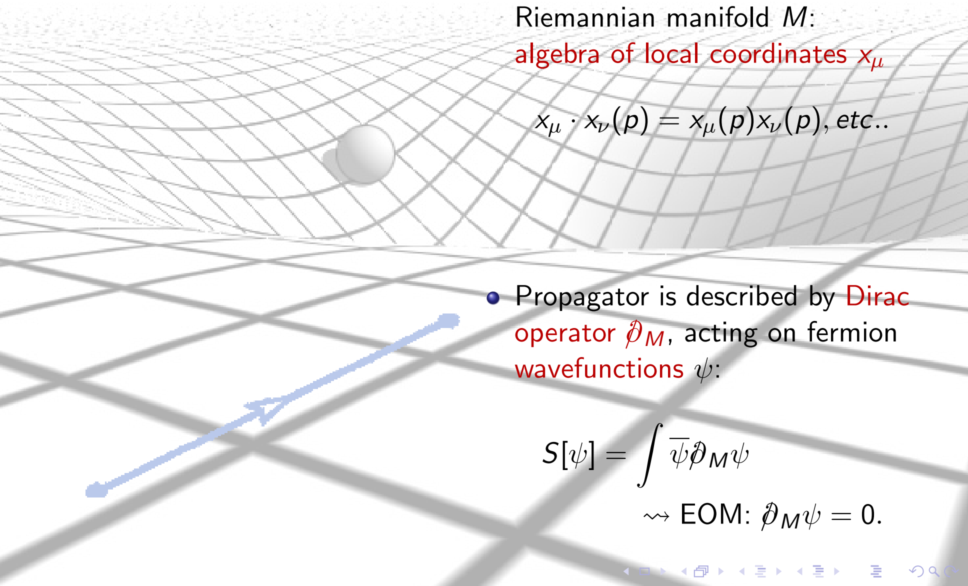

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$$S[\psi] = \int \overline{\psi} \partial\!\!\!/_M \psi$$

$$\rightsquigarrow \text{EOM: } \partial\!\!\!/_M \psi = 0.$$

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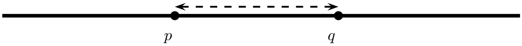

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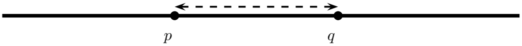
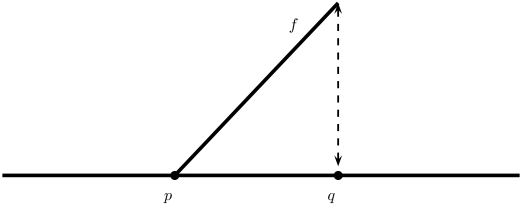


- Another way of writing the distance on a line is by maximizing over functions of slope ≤ 1:

$$d(p,q) = \sup\{|f(p) - f(q)| : |df/dx| \leq 1\}$$



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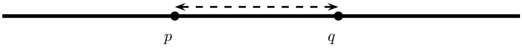

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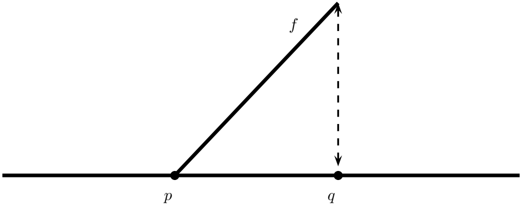


- This generalizes to any spacetime manifold, using the Dirac operator $\not{\partial}_M$ and its spectrum of eigenvalues to make sense of 'functions of slope ≤ 1'.

Intermezzo: History of the meter

Meter defined in 1791 as 10^{-7} times one quarter of the meridian of the Earth.

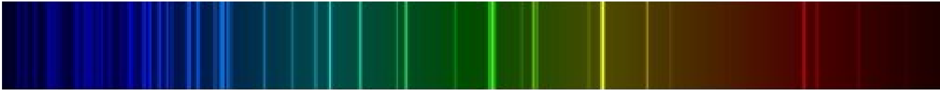
Expedition in 1792: measuring the arc of the meridian between Barcelona-Duinkerken, at the beginning of the French revolution... [a]

[a] Adler (2002)

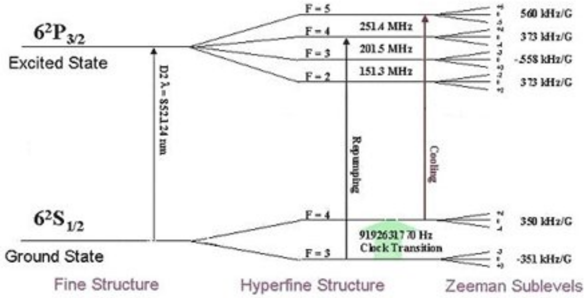
Meter made concrete by platinum bar “mètre-étalon”, saved (from 1889) in Pavillon de Breteuil near Paris:

Practical objections mètre-étalon (natural variations):

- 1960: meter defined as a multiple of a transition wavelength in Krypton 86Kr:



- 1967: **second** = 9192631770 periods of a transition radiation between two hyperfine levels in Caesium-133.



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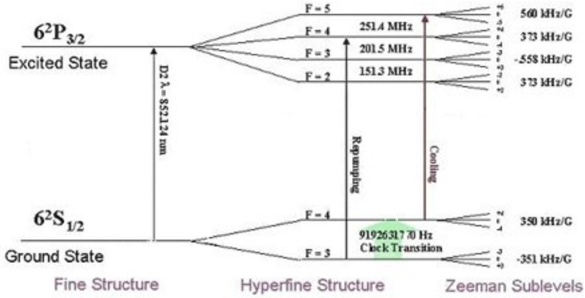
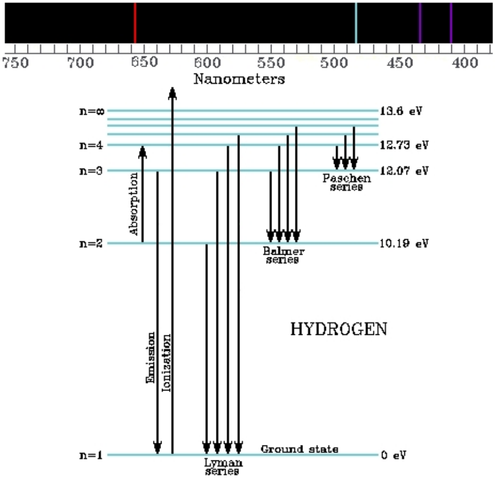


- 1983: Definition of the **meter** as the distance that light travels in 1/299792458 second...

So, measuring distances by looking at spectra



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- Spectral definition of Riemannian distance generalizes to such noncommutative spacetimes.

Example: electroweak theory

Algebra describing F is $\mathbb{C} \oplus \mathbb{H}$:

- A complex number z
- A quaternion $q = q_0 + iq_k\sigma^k$; in terms of Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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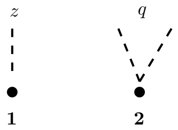

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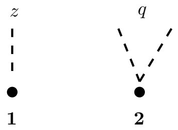

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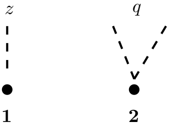

Matrix $\not{\partial}_F^+ = \left(\begin{smallmatrix} \varphi_1 & \varphi_2 \\ -\bar{\varphi}_2 & \bar{\varphi}_1 \end{smallmatrix}\right)$.
Distance $d(\mathbf{1}, \mathbf{2}) =$ inverse of largest eigenvalue of $\not{\partial}_F$

But how to get physics from this?

Given such a space $M \times F$, with Dirac operators $\not{\partial}_M$ and $\not{\partial}_F$, a Lagrangian is given by an extremely simple formula[1,2]

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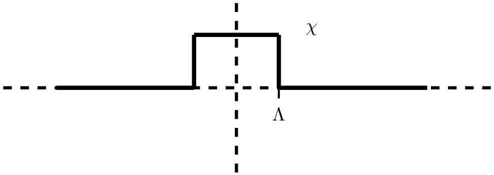
for some cut-off function χ, say, of the form



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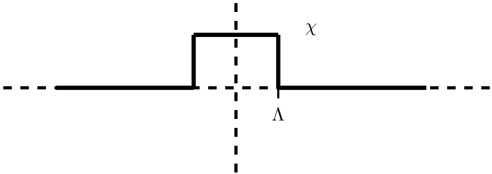

The function χ gives rise to the **coupling constants** for the physical theory described by the Lagrangian.

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$$+ \frac{\chi_0}{320\pi^2} \int_M \sqrt{g}\, C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} dx + \mathcal{O}(\Lambda^{-1})$$

which is (in red) the **Einstein-Hilbert action** of general relativity, with EOM:

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For this, we identify $\frac{\Lambda^2 \chi_2}{24\pi^2} = \frac{1}{16\pi G}$ (thus, cutoff $\Lambda \sim$ Planck energy).

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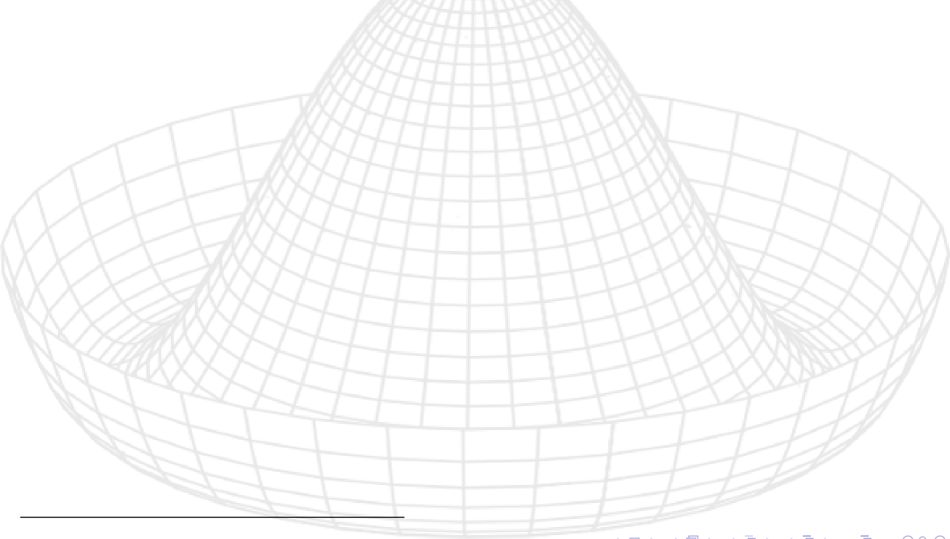
- Slow-roll inflation[3][4]

[3] De Simone-Hertzberg-Wilczek hep-ph/0812.4946

[4] Marcolli-Pierpaoli. arXiv:0903.3683

Noncommutative geometry of the Standard Model

Let the noncommutative space F be described by the noncommutative algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$



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- The three coupling constants are all expressed in terms of χ_0, implying the relation:

$$g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

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$$\lambda \sim \frac{4}{3} g_3^2$$

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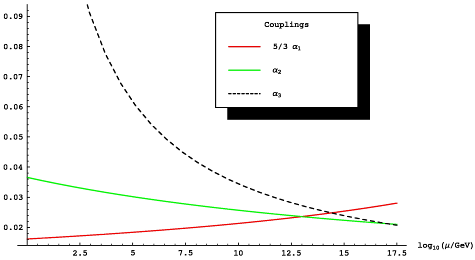
Relation $g_3^2 = g_2^2 = \frac{5}{3} g_1^2$ almost holds at GUT-scale:



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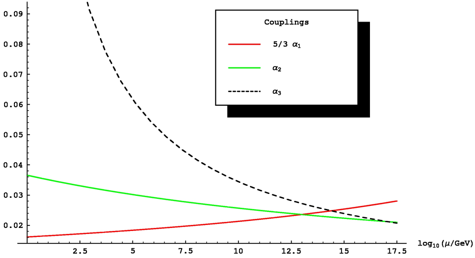

We interpret the NC model as the SM at GUT-scale. Then, the relation $\lambda \sim 4g_3^2/3$ can be RG-run down to give a prediction of the mass of the Higgs:

$$m_H^2 = 8\lambda \frac{M_W^2}{g_2}$$

which gives $167 \text{ GeV} \lesssim m_H \lesssim 176 \text{ GeV}$.[6]

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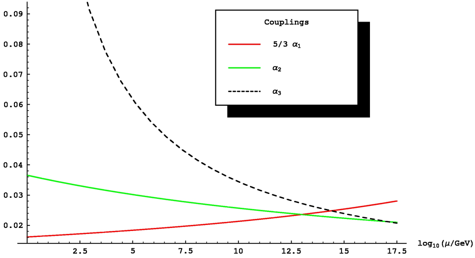

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- Similarly, obtain postdiction: $m_t < 180 \ \mathrm{GeV}$.

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Gravitational waves

As in GR, the noncommutative model describes gravitational waves:

$$-3\left(\frac{\dot{a}}{a}\right)^2 + 2\left(\frac{\dot{a}}{a}\right)\dot{h} + \ddot{h} - \frac{1}{2}\nabla^2 h - \frac{\alpha\kappa}{6a^2}\nabla^2(\partial_t^2 - \nabla^2)h = \kappa^2 T_{00}.$$

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- $a \sim 1$: deviation from rate of energy loss in (circular) binary pulsars.[7]

[7] Nelson-Ochoa-Sakellariadou. arXiv:1005.4276

Gravitational waves

As in GR, the noncommutative model describes gravitational waves:

$$-3\left(\frac{\dot{a}}{a}\right)^2 + 2\left(\frac{\dot{a}}{a}\right)\dot{h} + \ddot{h} - \frac{1}{2}\nabla^2 h - \frac{\alpha\kappa}{6a^2}\nabla^2(\partial_t^2 - \nabla^2)h = \kappa^2 T_{00}.$$

with $g_{\mu\nu} = a(t)^2 \mathrm{diag}\{-1, \delta_{ij} + h_{ij}\}$, $h = h^i_i$. The constants α and κ are the coefficients of the conformal and Einstein–Hilbert term, respectively. Two relevant examples:

- $a \sim 1$: deviation from rate of energy loss in (circular) binary pulsars.[7]
- Dominant κ (varying with Λ) and $a \sim \Lambda^{-1} = t^{-1/2}$:

$$h(t) = \frac{4\pi^2 T_{00}}{288\chi_2} t^3 + B + A\log t + \frac{3}{8}(\log t)^2$$

as opposed to conventional $h(t) = 2\pi G T_{00} t^2 + \cdots$.[8]

[7] Nelson-Ochoa-Sakellariadou. arXiv:1005.4276

[8] Marcolli-Pierpaoli. arXiv:0903.3683

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Suggesting for the following improvements:

1. Introduce a variant of the NC SM model (eg. SUSY)
2. Describe the quantized Standard Model in terms of NCG.
   $\rightsquigarrow$ quantization of (nc) gravity

NCG and SUSY

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- Towards MSSM: Gauge symmetry requires F of SM unchanged, sfermions appear in $\not{\partial}_F$.
- Potential reduction of free parameters in MSSM, as before in SM.
- SUSY appears to be automatically broken.

Outlook

- NCG of the MSSM.
- F has 'spin dimension' 6: relation to Calabi–Yau compactifications?
- Quantization: since NCG is based in mathematics, hard problem (quantization of even Yang–Mills theory not well-defined).
- Interesting applications to cosmology to further explore: Higgs coupling to scalar curvature, higher-order and conformal gravity.