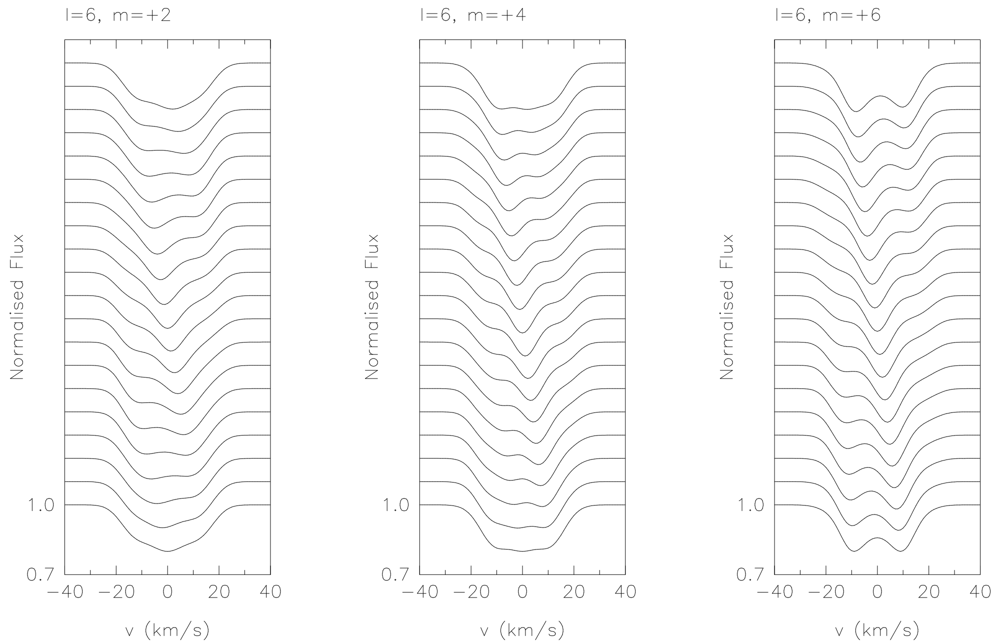


Mode Identification from Multicolour Photometry and Line-Profile Variations

1. Introduction & History

Goal of MI : essential for successful application of asteroseismology

Prior knowledge : frequency spectrum

- Up to mid-1970s: MI based on frequencies derived from photometry
- Mid-1980s: MI based on amplitude ratios from multicolour photometry
- 1971: Osaki presents a theoretical description of the calculation of LPVs :

$$\text{mode} \Rightarrow \text{velocity field} \Rightarrow \text{LPVs}$$

$$\text{LPVs} \overset{?}{\Rightarrow} \text{velocity field} \overset{?}{\Rightarrow} \text{NRP mode : identification of } (\ell, m)?$$

- Since mid-1980s: LPVs can be measured $\rightarrow$ Osaki's code can be applied

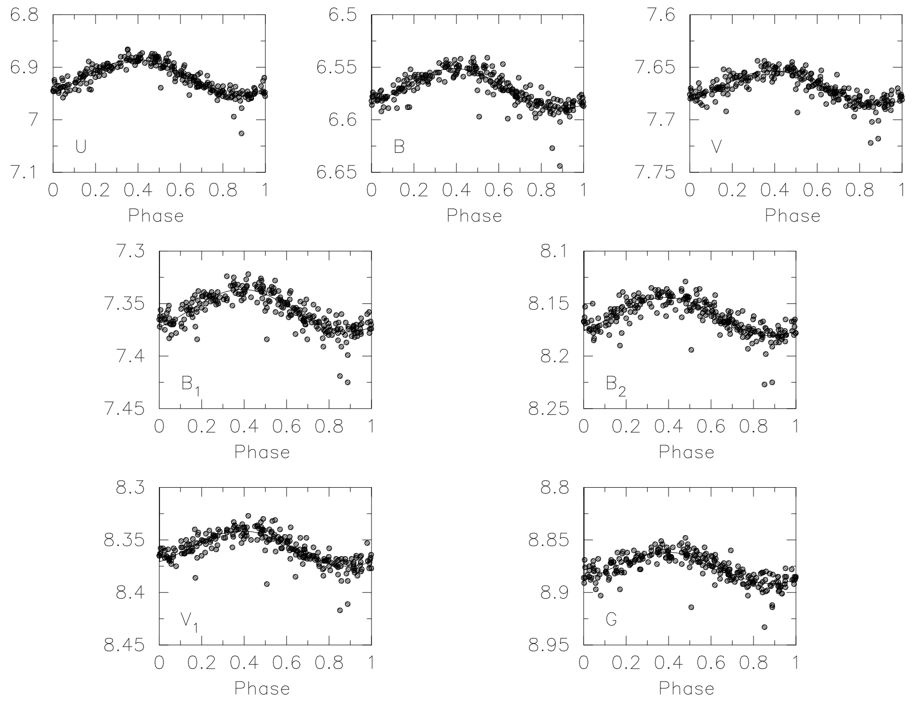
$$\Downarrow$$

MI based on line-profile fitting

- End-1980s: new objective methods to perform MI are developed

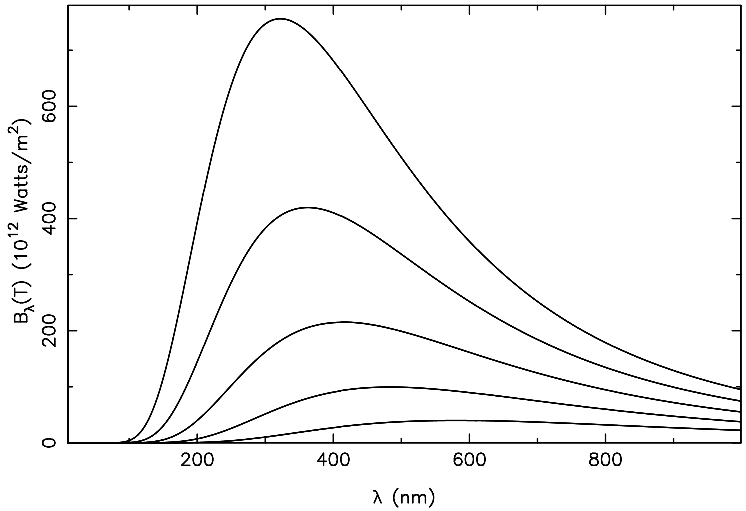
2. Identification from Multicolour Photometry

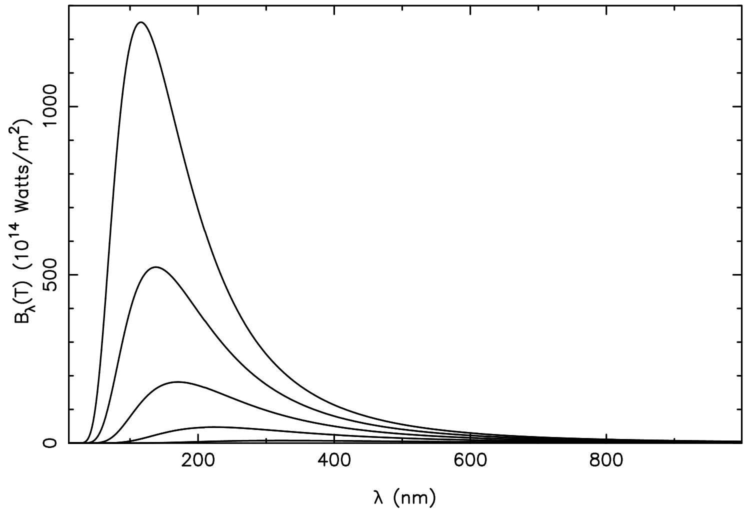
Phase diagram of the Geneva data for the seven filters of the star HD 71913 for the frequency 4.8596 c/d: amplitude is $\neq$ for $\neq \lambda$



	U	B	V	B_1	B_2	V_1	G
λ_0 (Å)	3464	4227	5488	4015	4476	5395	5807
μ (Å)	159	282	296	188	163	202	200

Variation as function of wavelength:





Due to shape of Black Body Radiation:
pulsation amplitude will always be larger in blue than in red

Determine the monochromatic amount of energy radiated by the star as measured by a distant observer: $E(\lambda,t) = A(\lambda)\,\exp(-i\omega t)$,
where the equilibrium value is defined as

$$E(\lambda) = \frac{R^2}{2\pi d^2}\int_0^1\int_0^{2\pi} F_\lambda^+ h_\lambda(\mu')\mu' \mathrm{d}\mu' \mathrm{d}\phi'$$

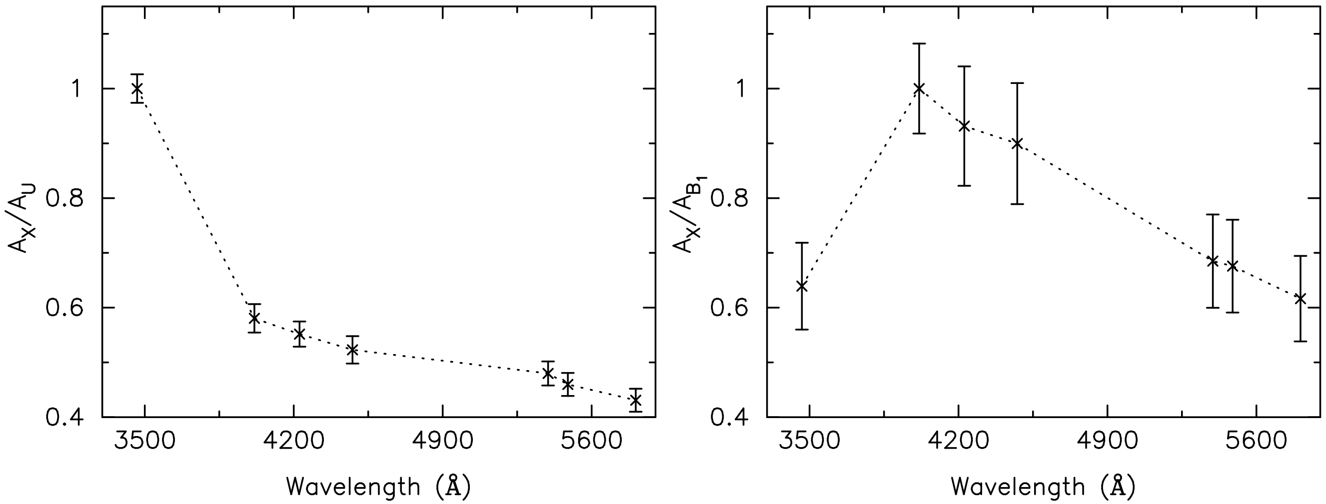
Linear approximation for the perturbed quantities:

$$F^{+}_{\lambda,0} + \delta F^{+}_{\lambda}(\theta,\phi,t) = F^{+}_{\lambda}\left[T_{\mathrm{eff},0} + \delta T_{\mathrm{eff}}(\theta,\phi,t), g_0 + \delta g_{\mathrm{e}}(\theta,\phi,t)\right]$$

In the linear approximation, we have:

$$\begin{aligned} \frac{\delta F^{+}_{\lambda}}{F^{+}_{\lambda,0}} &= \left(\frac{\partial \ln F^{+}_{\lambda}}{\partial \ln T_{\mathrm{eff}}}\right)\frac{\delta T_{\mathrm{eff}}}{T_{\mathrm{eff},0}} + \left(\frac{\partial \ln F^{+}_{\lambda}}{\partial \ln g_{\mathrm{e}}}\right)\frac{\delta g_{\mathrm{e}}}{g_0} \qquad (1) \\ &\equiv \alpha_{T,\lambda}\frac{\delta T_{\mathrm{eff}}}{T_{\mathrm{eff},0}} + \alpha_{g,\lambda}\frac{\delta g_{\mathrm{e}}}{g_0}. \end{aligned}$$

Examples of observed amplitude ratios:



Amplitude ratios depend on the kind of mode, but more importantly also on the effective temperature of the star (strong flux dependence!)

Final result of long theoretical computation:

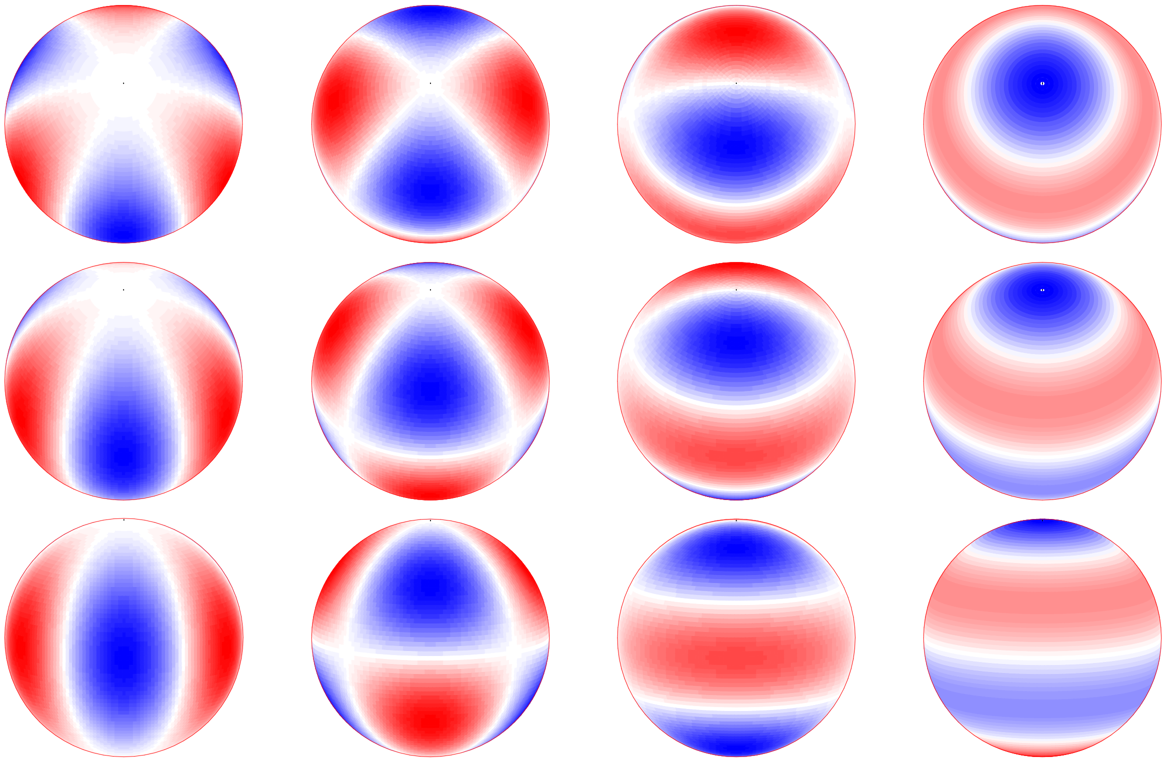
$$\begin{aligned} \delta m_\lambda &= -\frac{2.5}{\ln 10}\sqrt{4\pi}\,\frac{\xi_r(R)}{R}\,P_l^m(\cos i)b_{l,\lambda}\left[-(l-1)(l-2)\cos(\omega t)\right. \\ &+ \left. f_T\cos(\psi_T+\omega t)(\alpha_{T,\lambda}+\beta_{T,\lambda}) - f_g\cos(\omega t)(\alpha_{g,\lambda}+\beta_{g,\lambda})\right], \end{aligned} \tag{2}$$

with

$$b_{l,\lambda} = \int_0^1 \mu'\, h_\lambda(\mu')\, P_l \mathrm{d}\mu',\ \beta_{T,\lambda} = \frac{\partial \ln b_{l,\lambda}}{\partial \ln T_{\mathrm{eff}}},\ \beta_{g,\lambda} = \frac{\partial \ln b_{l,\lambda}}{\partial \ln g}$$

See Cugier & Daszyńska (2001), Dupret (2002), Townsend (2002), Dupret et al. (2003), Daszyńska-Daszkiewicz et al. (2003), and Randall et al. (2005).

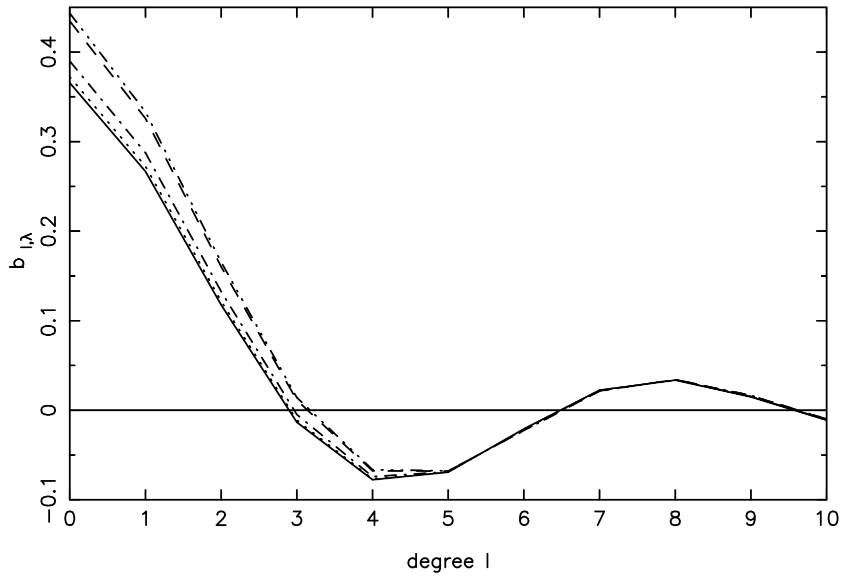
Geometrical cancellation due to $P_l^m(\cos i)$:



Inclination Angles of Complete Cancellation:

(l,m)	IACC						IALC
$(1,0)$						90°	0°
$(2,0)$			54.7°				0°
$(3,0)$		39.2°				90°	0°
$(4,0)$		30.6°			70.1°		0°
$(5,0)$	25.0°		57.4°			90°	0°
$(1,1)$	0°						90°
$(2,2)$	0°						90°
$(3,3)$	0°						90°
$(4,4)$	0°						90°
$(5,5)$	0°						90°
$(2,1)$	0°					90°	45.0°
$(3,1)$	0°			63.4°			31.1°
$(3,2)$	0°					90°	54.7°
$(4,1)$	0°	49.1°				90°	23.9°
$(4,2)$	0°			67.8°			40.9°
$(4,3)$	0°					90°	60.0°
$(5,1)$	0°	40.1°			73.4°		19.4°
$(5,2)$	0°		54.7°			90°	32.9°
$(5,3)$	0°				70.5°		46.9°
$(5,4)$	0°					90°	63.4°

Partial cancellation due to $b_{l,\lambda}$:



$b_{l,\lambda}$ for different l. Lower 3 curves: $T_{\rm eff} = 6000$ K, $\log g = 4.0$ at U (full), B (dotted) and V (dashed-dot); 2 upper curves: $T_{\rm eff} = 25000$ K, $\log g = 4.0$ at U and B (indistinguishable, dashed) and V (dashed-dot-dot-dot).

Observations: magnitudes for particular filters j with transmission curves $w_j(\lambda)$ and a wavelength range from $\lambda_{j,\mathrm{blue}}$ to $\lambda_{j,\mathrm{red}}$:

$$\delta m_j = \frac{\int_{\lambda_{j,\mathrm{blue}}}^{\lambda_{j,\mathrm{red}}} \delta m_\lambda \, w_j(\lambda) \, \mathrm{d}\lambda}{\int_{\lambda_{j,\mathrm{blue}}}^{\lambda_{j,\mathrm{red}}} w_j(\lambda) \mathrm{d}\lambda}$$

for comparisons with observations.

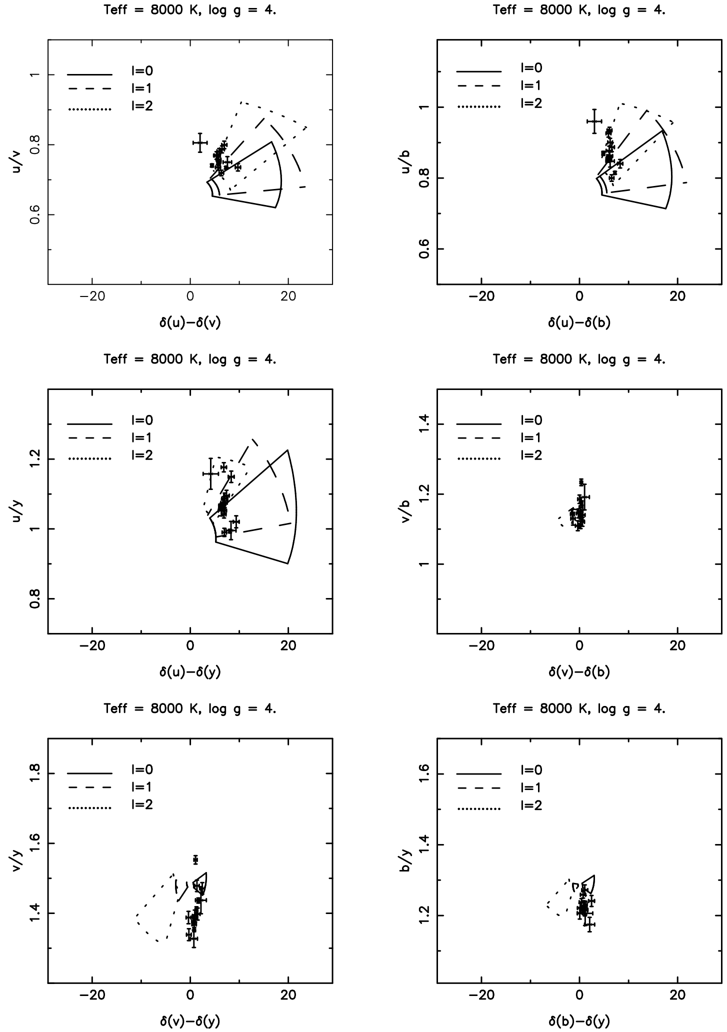
Eliminate the common factor

$$-(2.5/\ln 10)\sqrt{4\pi}(\xi_r(R)/R)P_l^m(\cos i)$$

by considering amplitude ratios for different photometric bands. We lose information on i and m ...

For highest S/N, take ampl. ratios w.r.t. filter with lowest relative error.

Early applications for δ Sct stars:



Mode identification schemes using only amplitudes

1. Compute stellar models that cover observational error box in $(T_{\text{eff}}, \log g)$.

2. Perform non-adiabatic computations to derive f_T, ψ_T, f_g for modes with frequencies close to the observed ones, for different degree l, for all the models that pass through the observational error box. ($l = 0, \ldots, 4$)

3. For each filter j and for each degree l, compute the theoretical amplitude while omitting common factor:

$$A_{j,\mathrm{th}} = \frac{\int_{\lambda_{\mathrm{blue}}}^{\lambda_{\mathrm{red}}} |b_{l,\lambda}| \, |T_1 + T_2 + T_3| \, w_j(\lambda) \mathrm{d}\lambda}{\int_{\lambda_{\mathrm{blue}}}^{\lambda_{\mathrm{red}}} w_j(\lambda) \mathrm{d}\lambda},$$

with

$$\begin{aligned} T_1 &\equiv (1-l)(l+2). \\ T_2 &\equiv f_T \; \exp(-\mathrm{i}\psi_T) \; (\alpha_{T,\lambda} + \beta_{T,\lambda}), \\ T_3 &\equiv -f_g \; (\alpha_{g,\lambda} + \beta_{g,\lambda}). \end{aligned}$$

4. Choose reference filter $A_{\text{ref,th}}$ to compute the amplitude ratios.

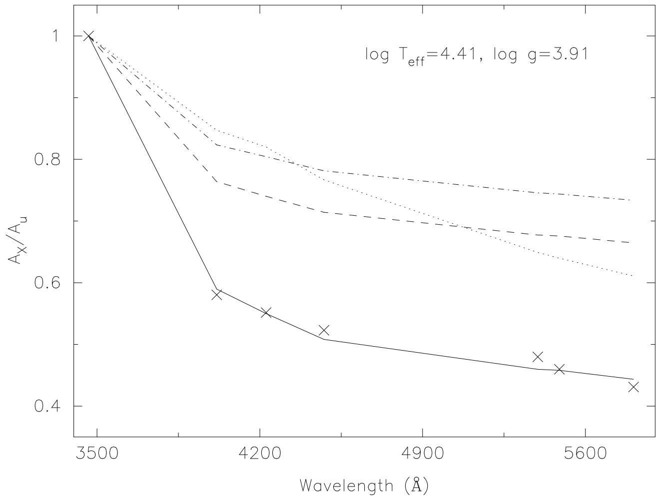
5. Compare theoretical $A_{j,\text{th}}/A_{\text{ref,th}}$ with observed $A_{j,\text{obs}}/A_{\text{ref,obs}}$, *for all stellar models through error box* in $(T_{\text{eff}}, \log g)$ by visual inspection or

$$\chi^2(l) = \sum_{j=1}^{\#\text{filters}} \left(\frac{A_{j,\text{th}}/A_{\text{ref,th}} - A_{j,\text{obs}}/A_{\text{ref,obs}}}{\sigma_{j,\text{obs}}} \right)^2,$$

where $\sigma_{j,\text{obs}}$ =s.e. of observed amplitude ratio for filter j and the reference filter.

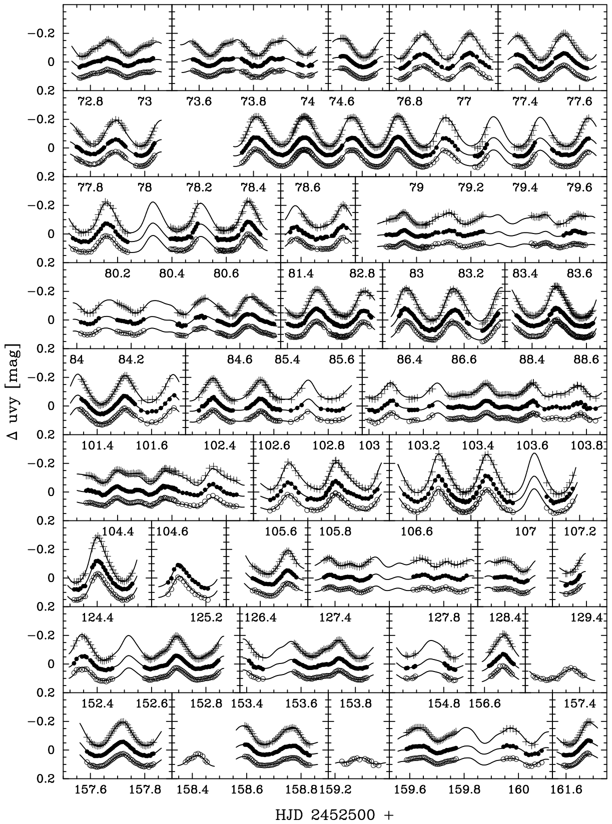
Example of HD 71913 = star with SpT B

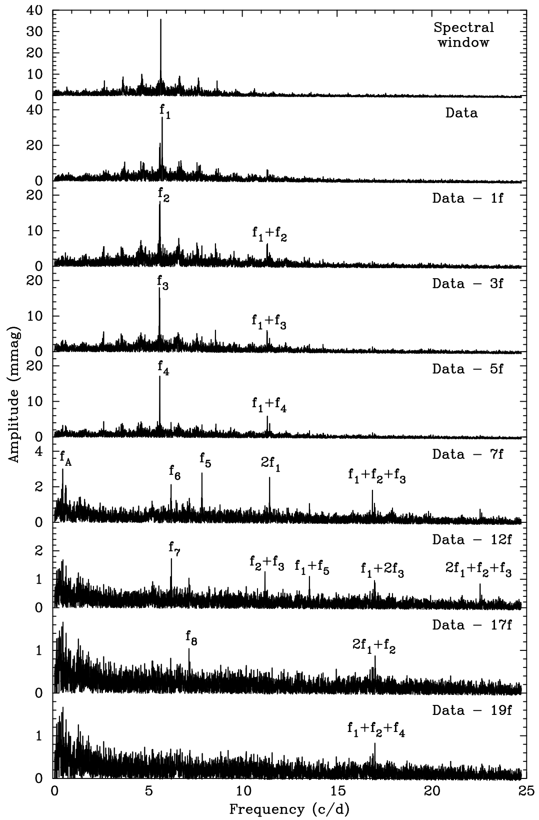
filter	amplitude	ratio	v.r.
U	0.035	1.00	82%
B_1	0.020	0.57	70%
B	0.019	0.54	72%
B_2	0.018	0.51	66%
V_1	0.017	0.49	67%
V	0.016	0.46	70%
G	0.015	0.43	63%



Amplitude ratios as a function of λ for the 4 best solutions of HD 71913. Full line: $\ell = 0$, dashed: $\ell = 1$, dotted: $\ell = 3$, dot-dashed: $\ell = 2$.

Application to β Cep star ν Eri (Handler et al. 2004):

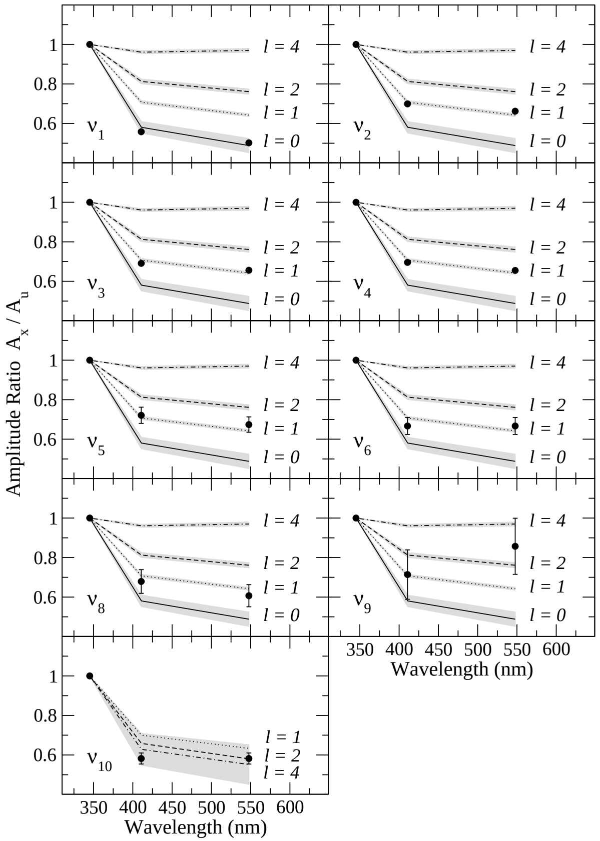




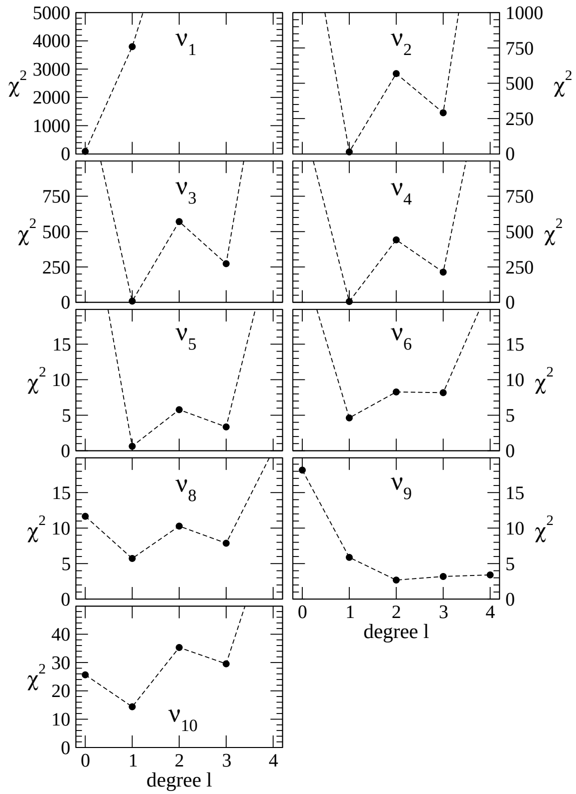
Application to β Cep star ν Eri (De Ridder et al. 2004):

ID	Frequency (d^{-1})	Amplitude ($km\,s^{-1}$)	Amplitude (mmag)	Degree l
ν_1	5.7633	22.4	73.5	0
ν_2	5.6539	8.9	37.9	1
ν_3	5.6201	8.1	34.6	1
ν_4	5.6372	7.9	32.2	1
ν_5	7.898	1.0	4.3	1
ν_6	6.244	1.0	3.9	1
ν_7	6.223	0.3	–	–
ν_8	6.262	0.8	2.8	1
ν_9	7.200	–	1.4	–
ν_{10}	0.432	–	5.5	–

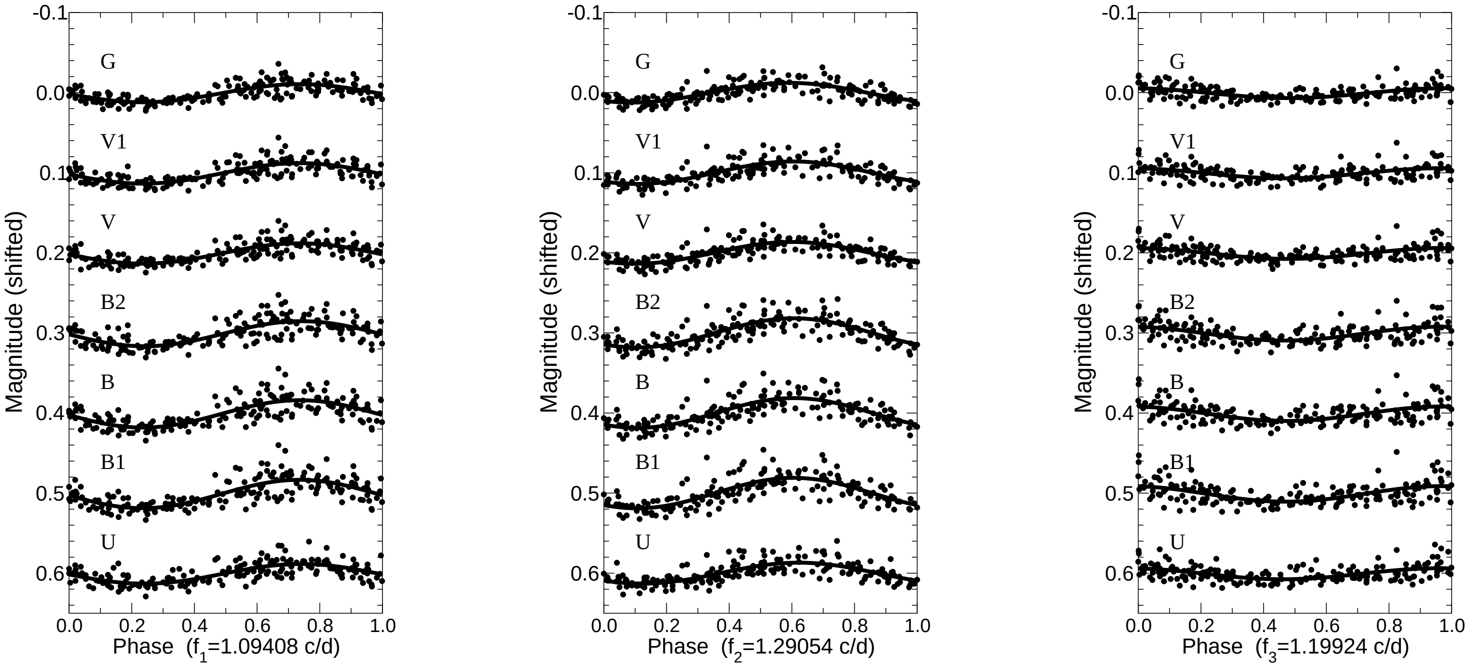
Application to β Cep star ν Eri (De Ridder et al. 2004):



Application to β Cep star ν Eri (De Ridder et al. 2004):

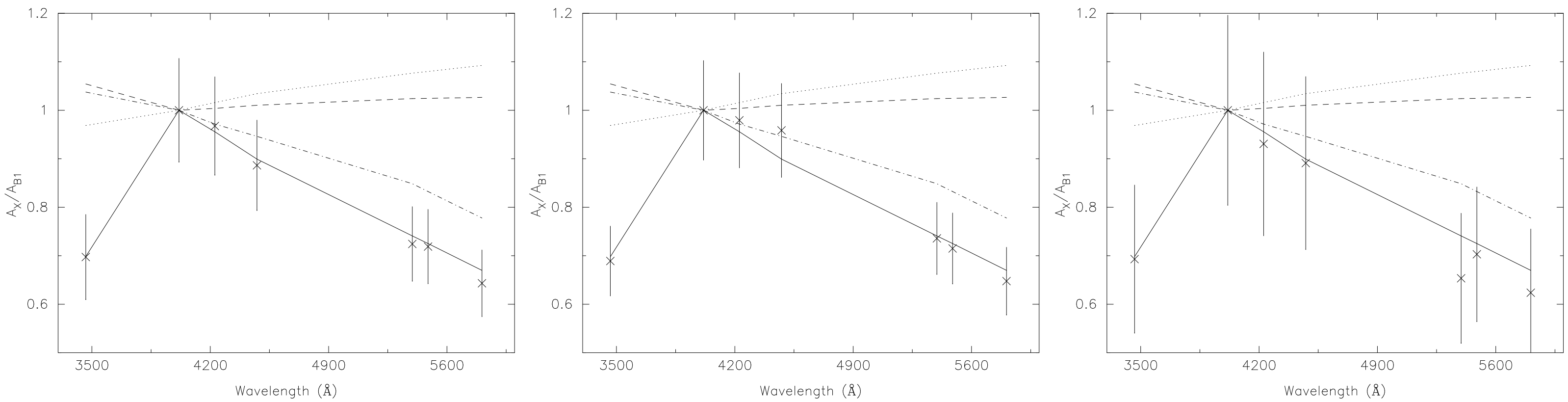


Example of HD 48501 = star with SpT F (Aerts et al. 2004)



Phase diagrams of the seven-colour photometry of the γ Doradus star HD 48501 for the indicated frequencies.

Example of HD 48501 = star with SpT F (Aerts et al. 2004)



Theoretically derived amplitude ratios versus observed ones with respect to the B_1 filter for $\ell = 1$ (full line), 2 (dashed), 3 (dotted) and 4 (dot-dashed). The left, middle and right panels are for respectively f_1, f_2, f_3.

Use the theoretical amplitude ratios in these slides to interprete your data !