Parameter estimation of spinning binaries using MCMC

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Introduction

Binary systems consisting of stellar-mass ($\sim 1M_\odot - 100M_\odot$) compact objects are amongst the most promising gravitational-wave sources for ground-based laser interferometers. If at least one of the binary members is a black hole, our current astrophysical understanding suggests that the black hole should be spinning at least moderately [1].

Spins strongly affect the gravitational waveforms by introducing phase and amplitude modulations, caused by the coupling of the angular momenta. For parameter estimation on an inspiral signal, it is therefore of vital importance to take into account the effects of spins.

Building on a non-spinning binary inspiral parameter-estimation code [2], we have extended this code to extract the source parameters of spinning binary inspirals. The algorithm is based on a Markov-chain Monte-Carlo (MCMC) technique [3] to compute the posterior probability density functions (posterior PDFs) of the source parameters.

The waveform

In this first stage of our study, we model the gravitational-wave signal at the restricted 1.5PN approximation [4] and the effect of spins is included in the limit of *simple precession* [5]. These simplifications allow us to calculate the waveforms analytically, thereby greatly reducing the computation time. Here we present results for a fiducial binary system of a $10\,M_\odot$ spinning black hole and a $1.4\,M_\odot$ non-spinning neutron star. The waveform is described by a twelve-dimensional parameter vector $\vec{\lambda}$, where the parameters are: chirp mass M_c, symmetric mass ratio η, spin magnitude $a_{\rm spin}$, the constant angle between spin and orbital angular momentum $\theta_{\rm SL}$, geocentric time (t_c), phase (ϕ_c) and 'precession phase' (α_c) at coalescence, distance d_L, position in the sky (R.A. and Dec.), and orientation of the total angular momentum vector (θ_{J0}, ϕ_{J0}).

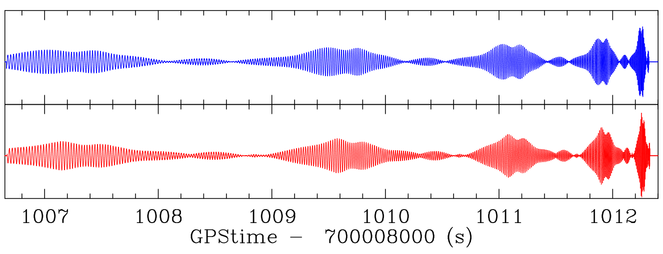

Fig. 1. A typical inspiral signal, as seen by the two 4-km LIGO interferometers Hanford (top) and Livingston (bottom).

Markov-chain Monte Carlo

The goal of this study is to evaluate the PDFs of the source parameters

$$\mathrm{PDF}(\vec{\lambda}) \propto \mathrm{prior}(\vec{\lambda}) \times \prod_{i=1}^{N} \Lambda_i(d_i|\vec{\lambda}),$$

where

$$\Lambda_i(d_i|\vec{\lambda}) \propto \exp\left(-2\int_0^\infty \frac{\left|\tilde{d}_i(f) - \tilde{m}_i(\vec{\lambda}, f)\right|^2}{S_{n,i}(f)}\, df\right)$$

is the likelihood for detector i and N is the number of detectors in the network. In the above expression, $\tilde{m}_i(\vec{\lambda}, f)$ and $\tilde{d}_i(f)$ are the waveform and data, respectively. $S_{n,i}(f)$ is the noise spectral density, f the frequency. Using an MCMC technique we produce the marginalised posterior PDFs.

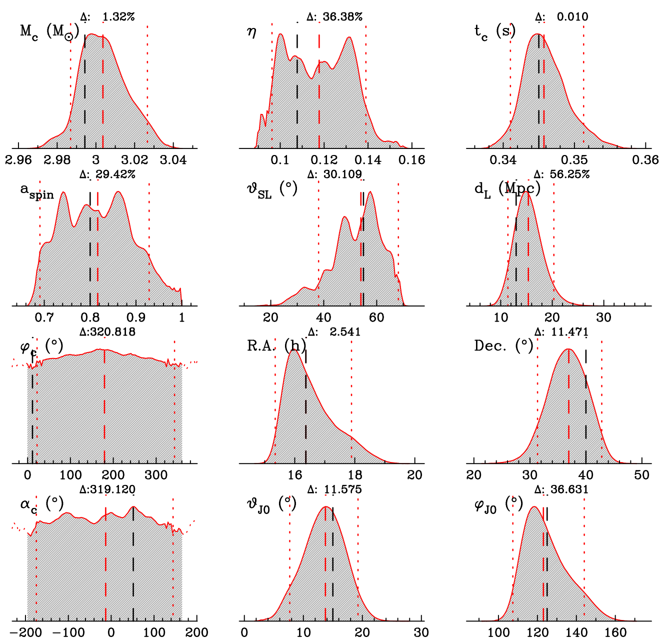

Fig. 2. Posterior PDFs for the twelve source parameters, in observations carried out with the LIGO Hanford 4-km interferometer. Ten parallel chains of length $\sim 3\times10^6$ were computed and the first 5×10^5 members of each chain were discarded as *burn-in*. At the moment, we start the chains at the true parameter values. The black dashed lines identify the true parameter values, the red dashed lines show the median of each PDF and the red dotted lines show the smallest 90% probability interval. The quantity Δ at the top of each panel provides the width of this range, either in % or in the units of the plot.

Results

We are carrying out a thorough exploration of the parameter space for spinning binary systems. Here we present the results for a fiducial source characterised by $a_{\rm spin} = 0.8$, $\theta_{\rm SL} = 55^\circ$ and $d_L = 13\,\mathrm{Mpc}$ (the SNR is ~ 20 at each detector). We show the marginalised PDFs of selected parameters, for observations carried out with the LIGO instrument at Hanford only (Fig. 2) and with both LIGO 4-km interferometers at Hanford and Livingston (Fig. 3).

The most striking result is that due to the additional modulations induced by the spins, even with one detector one can fully resolve the physical *and* geometrical parameters of the binary, including the source location in the sky, although the uncertainties are fairly large. As expected, the accuracy of the measurements increases substantially when instruments are added to the network. As an example, we show in Fig. 4 the error box in the sky.

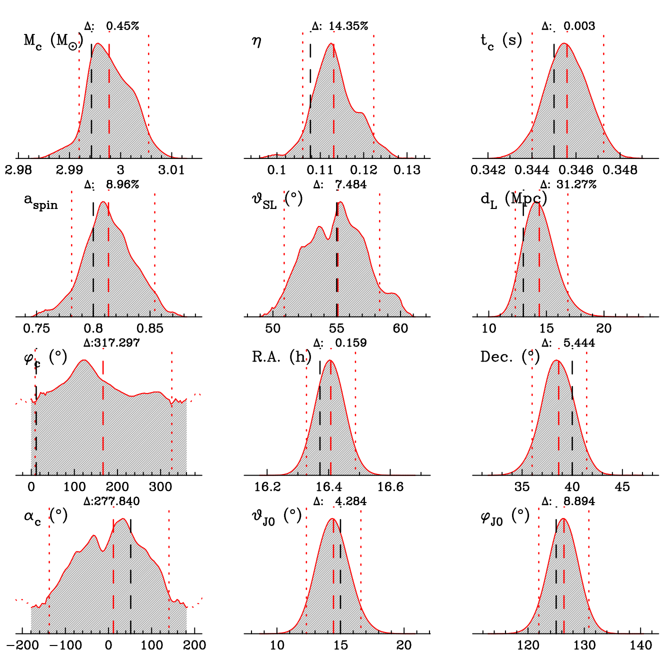

Fig. 3. Posterior PDFs for the same signal as in Fig. 2., but now observed with *both* LIGO Hanford and Livingston 4-km interferometers. The lines and numbers have the same meaning as in Fig. 2. Notice the difference in scale (and hence the width of the 90%-probability range) between the two figures.

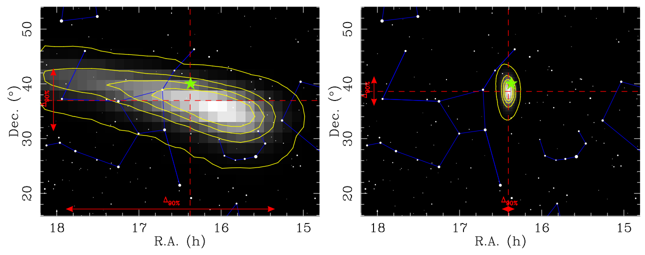

Fig. 4. Two-dimensional PDFs of the source sky coordinates (right ascension and declination) plotted over a sky map, for the case of one detector (left) and two detectors (right). The two panels have the same scale. The true position of the source is indicated by the green star. The red dashed lines show the medians of the distributions, the red arrows show the 90%-probability ranges. Lighter grey scales indicate larger probability, the yellow contours follow constant probability. Besides increased accuracy, the right-hand side shows a much weaker correlation between the two coordinates (9% vs. 67%).

Correlations

Observations with multiple detectors help in reducing the correlations amongst signal parameters: however, some of them are intrinsic to the problem at hand. Typical examples, shown in Figure 5 are those between the mass parameters and the spin parameters. Such correlations affect the efficiency with which the chains are able to explore the posterior PDF if they are not taken into account in the proposal distributions used in the MCMC algorithm.

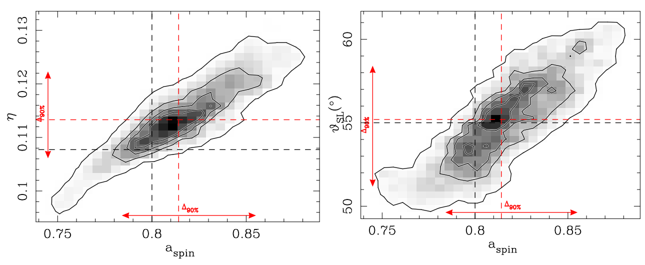

Fig. 5. Two-dimensional PDFs of η vs. $a_{\rm spin}$ (left) and $\theta_{\rm SL}$ vs. $a_{\rm spin}$ (right), for an MCMC run using two detectors. The red dashed lines and arrows have the same meaning as in Fig. 4, the black dashed lines indicate the true parameter values.

Conclusions and future work

- The binary geometrical parameters are resolvable even in observations with one detector (though the uncertainties are large).
- The quality of astronomy improves (as expected) with the number of detectors, and we are already exploring the effect of including VIRGO into the network along with LIGO.
- We are modifying the proposal distributions used in the MCMC to take into account correlations between parameters and make the code substantially more efficient.
- We will carry out a systematic investigation of the parameter space, starting our chains from offset values.
- We plan to include more realistic waveforms and test the code on injections into real interferometer data.

References

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